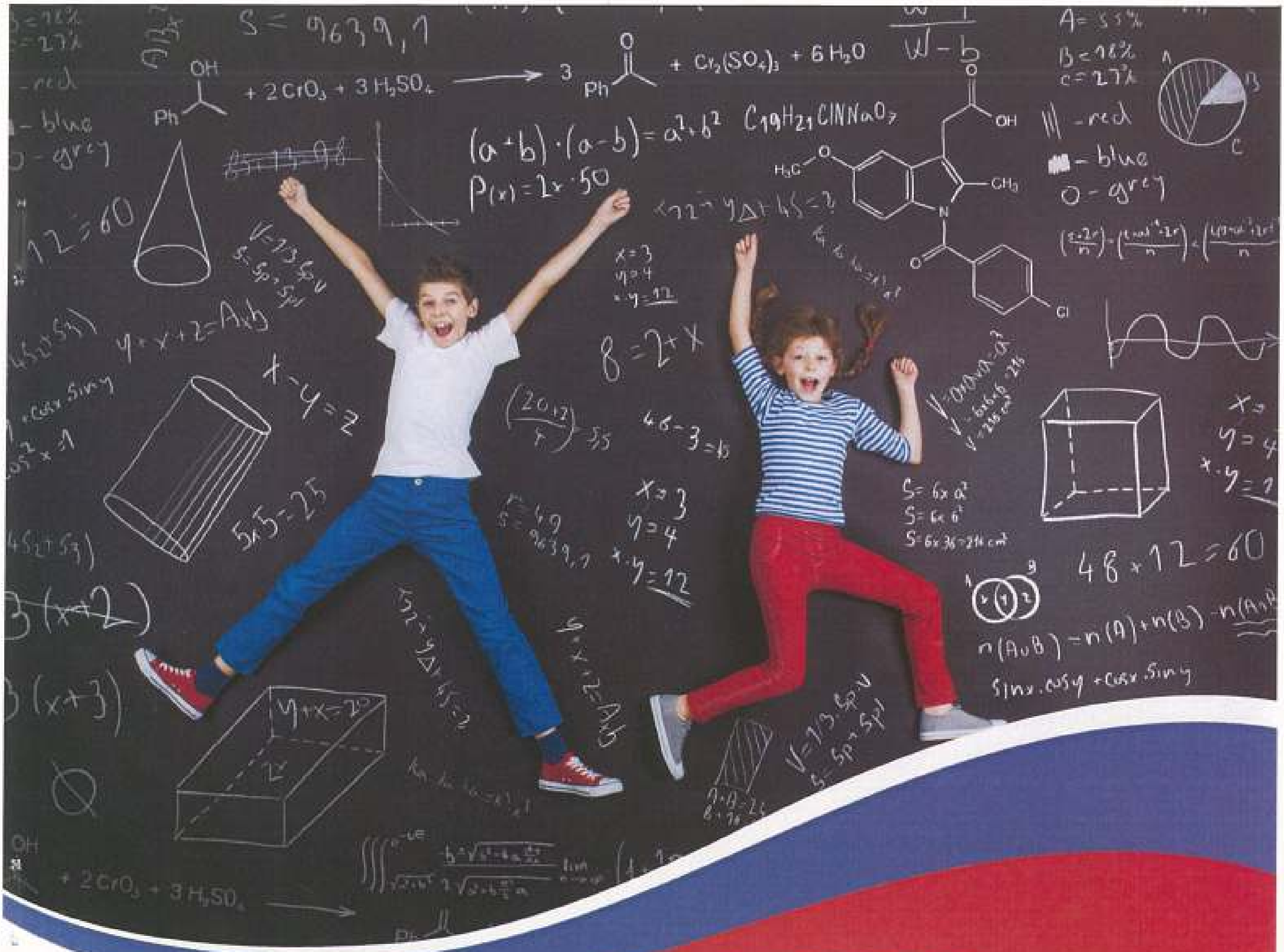

GRADE 9

MATHEMATICS OLYMPIAD

Official Guide

International Olympiad Foundation

International Olympiad Foundation

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1. Number System

Learning Objective :

In this chapter we shall learn about :

*Natural numbers

*Whole numbers

*Rational numbers

*Irrational numbers and real numbers

Natural Numbers

Counting numbers 1, 2, 3, ... are known as natural numbers.

Thus 1, 2, 3, 4, 5, 6, 7, ... are natural numbers. It is denoted by N. Hence, N = {1, 2, 3, 4, ...}

Whole Numbers

All natural numbers along with zero are called whole numbers. It is denoted by W.

Hence W = {0, 1, 2, 3, 4, ...}

Integers

All natural numbers, zero and negatives of natural numbers form the set integers.

Example: 0, 1, –1, 2, –2, 3, –3, ... etc., are integers

$\therefore$ Natural numbers $\in$ Whole numbers $\in$ Integers

Rational Numbers

The numbers of the form $\frac{p}{q}$, where p, q are integers and $q \neq 0$ are known as rational numbers.

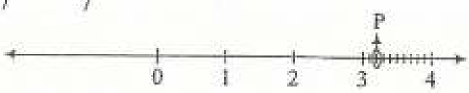
Example: $\frac{-1}{2}, \frac{3}{2}, \frac{7}{9}, \frac{-7}{6}, \frac{8}{9}$ .... etc., are rational numbers.

Example 1: Write 3 rational numbers equivalent to $\frac{6}{5}$.

Solution: We have $\frac{6}{5} = \frac{6\times2}{5\times2} = \frac{6\times5}{5\times5} = \frac{6\times3}{5\times3} = \frac{12}{10} = \frac{30}{25} = \frac{18}{15}$

Example 2: Represent $3\frac{2}{7}$ on real line.

Solution: We have $3\frac{2}{7} = 3 + \frac{2}{7}$



Divide the portion between 3 and 4 to 7 equal parts and mark the second spot, i.e., P.

P will represent $3\frac{2}{7}$ on real line.

Example 3: Insert five rational numbers between 6 and 8.

Solution: $d = \frac{y-x}{n+1} = \frac{8-6}{5+1} = \frac{2}{6} = \frac{1}{3}$

$\therefore$ Five rational numbers between 6 and 8 are $\left(6+\frac{1}{3}\right), \left(6+\frac{2}{3}\right), \left(6+\frac{3}{3}\right), \left(6+\frac{4}{3}\right), \left(6+\frac{5}{3}\right)$

$= \left(\frac{19}{3}\right), \left(\frac{20}{3}\right), \left(\frac{21}{3}\right), \left(\frac{22}{3}\right), \left(\frac{23}{3}\right)$

Example 4: Find four rational numbers between $\frac{1}{2}$ and 1.

Solution: We have $d = \frac{1-\frac{1}{2}}{4+1} = \frac{1}{10}$

$\therefore$ Four rational numbers between $\frac{1}{2}$ and 1 are $\left(\frac{1}{2}+\frac{1}{10}\right), \left(\frac{1}{2}+\frac{2}{10}\right), \left(\frac{1}{2}+\frac{3}{10}\right), \left(\frac{1}{2}+\frac{4}{10}\right)$.

$= \left(\frac{6}{10}\right), \left(\frac{7}{10}\right), \left(\frac{8}{10}\right), \left(\frac{9}{10}\right)$

Example 5: Write nine rational numbers between 0 and 3.

Solution: Here $d = \frac{3-0}{9+1} = \frac{3}{10}$

$\therefore$ Nine rational numbers between 0 and 3 are

$\left(0+\frac{3}{10}\right), \left(0+\frac{6}{10}\right), \left(0+\frac{9}{10}\right), \left(0+\frac{12}{10}\right)$ ...., $\left(0+\frac{27}{10}\right)$

Required rational numbers are $\frac{3}{10}, \frac{6}{10}, \frac{9}{10}, \frac{12}{10}, \frac{15}{10}, \frac{18}{10}, \frac{21}{10}, \frac{24}{10}$ and $\frac{27}{10}$.

Germinating Decimal

Every fraction $\frac{p}{q}$ can be expressed as a decimal if the decimal terminates, i,e, comes to an end then the decimal is said to be terminating.

Example: $\frac{1}{8} = 0.125$, $\frac{1}{4} = 0.25$, $\frac{1}{2} = 0.5$, etc

Repeating (Recurring Decimals)

A decimal in which a digit or a set of digits repeats periodically, is called a repeating or a recurring decimal.

Example: (i) $\frac{1}{3} = 0.3333$ ------ $= 0.\overline{3}$ (ii) $\frac{15}{7} = 2.\overline{142857}$ (iii) $\frac{2}{3} = 0.6666$ ------ $= 0.\overline{6}$

Terminating decimals have their denominators of the form $2^m \times 5^n$, where, m and n are natural numbers or even m, n is (are) zero.

Example 6: Find which of the following rational numbers are terminating decimals, without actual division,

(a) $\frac{5}{30}$ (b) $\frac{12}{125}$ (c) $\frac{11}{500}$

Solution: (a) Given denominator $= 30 = 2 \times 5 \times 3$

$\because$ denominator has an extra term than 2 and 5. Therefore, decimal is non-terminating.

(b) $125 = 5 \times 5 \times 5 = 2^0 \times 5^3$

$\therefore$ Decimal is terminating.

(c) $500 = 2 \times 5 \times 5 \times 2 \times 5 = 2^2 \times 5^3$

$\because$ Denominator has 2 and 5 as its factors.

$\therefore$ Decimal is terminating.

Example 7: Express each of the following decimals as a fraction in the simplest form :

(a) $0.\overline{36}$ (b) $0.\overline{54}$ (c) $0.\overline{324}$ (d) $0.\overline{123}$

Solution: (a) Let $x = 0.\overline{36} = 0.363636\ldots$ ...(i)

$100\,x = 36.3636$ ...(ii)

Using eq. (i), and eq. (ii)

$$99x = 36$$

$$\Rightarrow \quad x = \frac{36}{99} = \frac{4}{11}$$

(b) Let $x = 0.54444$ ...(i)

$10x = 5.4444$ ...(ii)

$100x = 54.4444$ ...(iii)

Using eq .(iii) and eq. (ii)

$$90x = 49$$

$$\Rightarrow \quad x = \frac{49}{90}$$

(c) $x = 0.324324324$ ...(i)

$1000\,x = 324.324324$ ...(ii)

Using eq. (i) and eq. (ii)

$$999\,x = 324$$

$$x = \frac{324}{999} = \frac{36}{111} = \frac{12}{37}$$

(d) $x = 0.1232323$ ...(i)

$10x = 1.232323$ ...(ii)

$100x = 123.232323$ ...(iii)

$1000x = 123.232323$ ...(iv)

Using eq. (iv) and eq. (ii)

$$9990x = 122$$

$$\Rightarrow \qquad x = \frac{122}{9990} = \frac{61}{4995}$$

Irrational Numbers

A number which can neither be expressed as a terminating decimal nor as a repeating decimal , is called an irrational number.

Example: $\sqrt{2}, \sqrt{3}, \sqrt{7}, \sqrt{5}$ etc.

Properties of Irrational Numbers

(a) Irrational numbers satisfy the commutative, associative and distributive laws for addition and multiplication.

(b) Sum of two irrationals can be or cannot be irrational.

Example: $\sqrt{3}+\sqrt{2}$ will be irrational, but

$\left(2-\sqrt{2}\right)+\left(2+\sqrt{3}\right)=4$, which is rational.

(c) Multiplication of two irrationals need not be irrational. The division of two irrationals also behaves same.

Example:

$\sqrt{2}\times\sqrt{3}=\sqrt{6} \rightarrow$ Irrational

$\sqrt{3}\times\sqrt{3}=3 \rightarrow$ Rational

$\frac{\sqrt{3}}{\sqrt{2}}=\sqrt{\frac{3}{2}} \rightarrow$ Irrational

$\frac{2\sqrt{3}}{\sqrt{3}}=2 \rightarrow$ Rational

(d) Any operation between a rational and an irrational number will always result in irrational number.

(e) The square root of all positive numbers is not always irrational, same is for the cube root of positive and negative numbers.

Example:

$\sqrt{3}=1.732$ .... irrational

$\sqrt{2}=1.414$ .... irrational

$\sqrt{4}=2$ .... rational

$\sqrt[3]{8}=2$ .... rational

Real Numbers

A number whose square is non-negative zero or positive is called real number.

Or

The set of rational and irrational numbers together is called real numbers.

Completeness Property

On number line, each point corresponds to an unique real number.

Density Property

Between any two real numbers, there exist infinitely many real numbers.

Properties of Real Numbers

(i) **Closure property of addition and multiplication:** The sum or the product of two real numbers will result in a real number.

(ii) **Associative law:** $a + (b + c) = (a + b) + c$, and $a(bc) = (ab)c$, where a, b and c are real numbers.

(iii) **Commutative law:** $a + b = b + a$ and $ab = ba$, where a, b are any real numbers.

(iv) **Existence of additive and multiplicative identities:**

Additive Identity $\Rightarrow$ $a + 0 = 0 + a = a$

Here 0 is additive identity.

Multiplicative Identity $\Rightarrow$ $a.1 = 1.\ a = a$ for every real number 'a'

Here 1 is multiplicative identity.

(v) **Existence of additive and multiplicative inverse:**

$(-a)$ is additive inverse of 'a' and $\frac{1}{a}$ is multiplicative inverse of a.

(vi) **Distributive laws of multiplication over addition:**

$$(a + b)\,c = ac + bc, \text{ and, } a(b + c) = ab + ac$$

where, a, b and c are real numbers.

Example 1: Add $\left(2\sqrt{3}+\sqrt{2}\right)$ and $\left(7\sqrt{2}-\sqrt{3}\right)$.

Solution: We have $\left(2\sqrt{3}+\sqrt{2}\right) + \left(7\sqrt{2}-\sqrt{3}\right) = 8\sqrt{2}+\sqrt{3}$

Example 2: Multiply $\left(5+\sqrt{6}\right)$ and $\left(5-\sqrt{6}\right)$.

Solution: $\left(5+\sqrt{6}\right)\left(5-\sqrt{6}\right) = (5)^2 - \left(\sqrt{6}\right)^2 = 25 - 6 = 19$

Example 3: Simplify $\left(\sqrt{3}+\sqrt{5}\right)^2$.

Solution: $\left(\sqrt{3}+\sqrt{5}\right)\left(\sqrt{3}+\sqrt{5}\right)$

$= \sqrt{3}\ \left(\sqrt{3}+\sqrt{5}\right)+\sqrt{5}\left(\sqrt{3}+\sqrt{5}\right)$

$= 3 + \sqrt{15}+\sqrt{15}+5 = 8 + 2\sqrt{15}$

Rationalisation

The process of correcting an irrational denomination to a rational number by multiplying its numerator and denominator by a suitable number is called rationalisation and the number used is called rationalising factor.

To rationalise the denomination of $\frac{1}{\sqrt{x}+y}$, we multiply it by $\frac{\sqrt{x}-y}{\sqrt{x}-y}$, where x, y are integers.

Example 4: Simplify $\frac{2}{\sqrt{3}}$ by rationalising the denominator.

Solution: $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Law of Radicals

If m and n are rational numbers and a is a positive real number then

(i) $a^m . a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$

(iv) $a^p \times b^p = (ab)^p$

Example 5: Simplify $\frac{1}{2+\sqrt{3}}$.

Solution: We have $\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{(2)^2 - \left(\sqrt{3}\right)^2} = 2 - \sqrt{3}$

Example 6: Solve $\frac{1}{4-\sqrt{15}}$.

Solution: We have $\frac{1}{4-\sqrt{15}} = \frac{4+\sqrt{15}}{(4)^2 - (\sqrt{15})^2} = 4+\sqrt{15}$

Example 7: If $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$, then find the value of 'a' and 'b'.

Solution: We have $\frac{3+\sqrt{2}}{3-\sqrt{2}} = \frac{3+\sqrt{2}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$

$$= \frac{\left(3+\sqrt{2}\right)^2}{(3)^2 - \left(\sqrt{2}\right)^2} = \frac{9+2+6\sqrt{2}}{7} = \frac{11}{7} + \frac{6\sqrt{2}}{7} = a + b\sqrt{2}$$

$$\therefore \quad a = \frac{11}{7}, b = \frac{6}{7}$$

Example 8: If $x = 2 + \sqrt{3}$, then find the value of $x^2 + \frac{1}{x^2}$.

Solution: Given $x = 2 + \sqrt{3}$

$$x^2 = \left(2+\sqrt{3}\right)^2 = 4 + \left(\sqrt{3}\right)^2 + 4\sqrt{3} = 7 + 4\sqrt{3}$$

$$\therefore \quad \frac{1}{x^2} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{7-4\sqrt{3}}{(7)^2 - \left(4\sqrt{3}\right)^2} = \frac{7-4\sqrt{3}}{49-48} = 7 - 4\sqrt{3}$$

$$\therefore \quad x^2 + \frac{1}{x^2} = 7+4\sqrt{3} + 7 - 4\sqrt{3} = 14$$

Example 9: $\frac{1}{3-\sqrt{8}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{8}-\sqrt{7}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}=x$ then $x=?$

Solution: $\frac{1}{3-\sqrt{8}}=\frac{3+\sqrt{8}}{(3)^2-(\sqrt{8})^2}=3+\sqrt{8}$

$\frac{1}{\sqrt{7}-\sqrt{6}}=\sqrt{7}+\sqrt{6},\ \frac{1}{\sqrt{8}-\sqrt{7}}=\sqrt{8}+\sqrt{7}$

$\frac{1}{\sqrt{6}-\sqrt{5}}=\sqrt{6}+\sqrt{5}$, $\frac{1}{\sqrt{5}-2}=\sqrt{5}+2$

Using all these and putting it in expression, we have

$=3+\sqrt{8}+\sqrt{7}+\sqrt{6}-(\sqrt{8}+\sqrt{7})-(\sqrt{6}+\sqrt{5})+(\sqrt{5}+2)$

$=3+\sqrt{8}+\sqrt{7}+\sqrt{6}-\sqrt{8}-\sqrt{7}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$

$=(3+2)=5$

Example 10: If $(16)^{\frac{3}{2}}=x$ then what is the value of 'x'?

Solution: Here $x=(16)^{\frac{3}{2}}=\left[(4)^2\right]^{\frac{3}{2}}=(4)^{2\times\frac{3}{2}}=(4)^3=64$

Example 11: Simplify $(125)^{\frac{-1}{3}}$.

Solution: We have $(125)^{\frac{-1}{3}}=\left(\frac{1}{125}\right)^{\frac{1}{3}}=\left[\left(\frac{1}{5}\right)^3\right]^{\frac{1}{3}}=\left(\frac{1}{5}\right)^{3\times\frac{1}{3}}=\frac{1}{5}$

Example 12: Simplify $(81)^{\frac{-1}{4}}$.

Solution: $(81)^{\frac{-1}{4}}=\left(\frac{1}{81}\right)^{\frac{1}{4}}=\left[\left(\frac{1}{3}\right)^4\right]^{\frac{1}{4}}=\left(\frac{1}{3}\right)^{4\times\frac{1}{4}}=\frac{1}{3}$

Example 13: Simplify $(625)^{0.16}\times(625)^{0.09}$.

Solution: $(625)^{0.16+0.09}=(625)^{0.25}=\left[(5)^4\right]^{0.25}=(5)^{4\times0.25}=(5)^1=5$

Example 14: If $x=7+4\sqrt{3}$, then $x+\frac{1}{x}=?$

Solution: Given $x=7+4\sqrt{3}$, $\frac{1}{x}=\frac{1}{7+4\sqrt{3}}\times\frac{7-4\sqrt{3}}{7-4\sqrt{3}}=\frac{7-4\sqrt{3}}{(7)^2-(4\sqrt{3})^2}=7-4\sqrt{3}$

$\therefore\quad x+\frac{1}{x}=(7+4\sqrt{3})+(7-4\sqrt{3})=14$

Example 15: Evaluate $\left[(64)^{-2}\right]^{\frac{1}{4}}$.

Solution: $\left((64)^{-2}\right)^{\frac{1}{4}} = (64)^{-2\times\frac{1}{4}} = (64)^{-\frac{1}{2}} = \frac{1}{8}$

Multiple Choice Questions

1. Choose the correct statement :
   (a) Every whole number is a natural number.
   (b) Every integer is a rational number.
   (c) Every integer is a whole number.
   (d) Every rational number is an integer
2. Which of the following number is irrational?
   (a) $\frac{7}{8}$ (b) $\sqrt{\frac{9}{125}}$ (c) $\frac{93}{300}$ (d) $\frac{190}{30}$
3. Which of the following decimal is terminating?
   (a) $\frac{3}{11}$ (b) $\frac{11}{6}$ (c) $\frac{11}{16}$ (d) $\frac{15}{7}$
4. $x = 0.\overline{57}$ Express 'x' in fractional form the requires fraction will be
   (a) $\frac{26}{44}$ (b) $\frac{27}{45}$ (c) $\frac{26}{45}$ (d) $\frac{57}{100}$
5. $0.2\overline{45}$ in the simplest form will be equal to :
   (a) $\frac{49}{20}$ (b) $\frac{27}{110}$ (c) $\frac{22}{10}$ (d) $\frac{243}{9900}$
6. Which of the following number is rational?
   (a) π (b) $\frac{22}{7}$
   (c) $\sqrt{7}+2$ (d) 0.141141114...
7. If $\frac{\sqrt{3}+1}{2-\sqrt{3}} = x + y\sqrt{3}$, then x, y have values equal to
   (a) 3,5 (b) 5,3 (c) 3,4 (d) 3,6
8. $\left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right) \times 50$ equals
   (a) 1000 (b) 200
   (c) 500 (d) 1500
9. If $x = 2+\sqrt{3}$, then $x + \frac{1}{x}$ is equal to :
   (a) $2\sqrt{3}$ (b) 4
   (c) 14 (d) 7
10. The value of the expression $\frac{16\times 2^{n+1} - 4\times 2^{n}}{16\times 2^{n+2} - 2\times 2^{n+2}}$ is
    (a) 2 (b) 2^n (c) $\frac{1}{2}$ (d) 4
11. Find the value of $x^3 - 2x^2 - 7x + 5$, if $x = \frac{1}{2-\sqrt{3}}$.
    (a) 1 (b) 0 (c) 2 (d) 3
12. If $\frac{\left(x^{a+b}\right)^2\left(x^{b+c}\right)^2\left(x^{c+a}\right)^2}{\left(x^a x^b x^c\right)^4} = y$ then y is equal to
    (a) x^{a+b+c} (b) 1 (c) x^{c+a} (d) 2
13. If $5^{x-3} \cdot 3^{2x-8} = 225$, $x = ?$
    (a) 4 (b) 3 (c) 5 (d) 6
14. $\sqrt{13-m\sqrt{10}} = \sqrt{8}+\sqrt{5}$, then $m =$
    (a) –2 (b) –5 (c) –6 (d) –4
15. $\left[2-3(2-3)^3\right]^3 = x$ then the value of $x = ?$
    (a) 125 (b) –125 (c) 25 (d) 625
16. $10^x = 64$, then the value of $10^{\frac{x}{2}+1}$ is
    (a) 8 (b) 6.4 (c) 640 (d) 80
17. $4^x - 4^{x-1} = 24$, then $(2x)^x$ is equal to
    (a) $\sqrt{5}$ (b) $125\sqrt{5}$ (c) $25\sqrt{5}$ (d) $5\sqrt{5}$
18. If $x^2 + \frac{1}{x^2} = 83$, then $x^3 + \frac{1}{x^3} =$
    (a) 756 (b)256 (c)729 (d) 702

19. If $x^2+\frac{1}{x^2}=98$, then $x+\frac{1}{x}=?$

(a) 10 (b) 12 (c) $7\sqrt{2}$ (d) 11

20. If $\frac{x}{y}+\frac{y}{x}=-1$, then $x^3-y^3=$

(a) -1 (b) $\frac{1}{2}$ (c) 1 (d) 0

21. If $x=7+4\sqrt{3}$ and $xy=1$ then $\frac{1}{x^2}+\frac{1}{y^2}=?$

(a) 64 (b) 194 (c) $\frac{1}{49}$ (d) 134

22. If $x^{-2}=64$, then $x^0+x^{\frac{1}{3}}$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 2 (d) 3

23. $\left\{(23+2^2)^{\frac{2}{3}}+(140-19)^{\frac{1}{2}}\right\}^2$, is

(a) 324 (b) 400 (c) 196 (d) 289

24. The positive square root of $7+4\sqrt{3}$ is

(a) $7+\sqrt{3}$ (b) $7+2\sqrt{3}$
(c) $3+\sqrt{2}$ (d) $2+\sqrt{3}$

25. If $\sqrt{2}=1.4142$, then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to

(a) 2.4142 (b) 0.4142
(c) 5.8282 (d) 0.1718

26. If $\frac{3^{5x}}{3^{2x}}\times 81^2\times 6561=3^7$ then x

(a) 3 (b) $\frac{1}{3}$ (c) -3 (d) $-\frac{1}{3}$

27. $\frac{5^{n+2}-6\times 5^{n+1}}{13\times 5^n-2\times 5^{n+1}}$ is equal to

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $-\frac{3}{5}$ (d) $-\frac{5}{3}$

28. If $x=1-\sqrt{2}$, then the value of $\left(x-\frac{1}{x}\right)^3$ is

(a) 4 (b) 27 (c) 8 (d) -8

29. The value of $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{7}-\sqrt{6}}+\frac{1}{\sqrt{5}-2}$ is

(a) 5 (b) -5 (c) 4 (d) -4

30. If $x=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ then $x+y+xy=$

(a) 5 (b) 7
(c) 9 (d) 17

31. The square root of $5+2\sqrt{6}$ is

(a) $\sqrt{3},\sqrt{2}$ (b) $\sqrt{3},\sqrt{2}$
(c) $\sqrt{5},\sqrt{6}$ (d) $\sqrt{5},\sqrt{6}$

32. The value of 'm' for which $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{\frac{-1}{3}}\right]^{\frac{1}{4}}=7^m$ is

(a) -3 (b) 2 (c) $\frac{-1}{3}$ (d) $\frac{1}{4}$

33. If $2^{-m}\times\frac{1}{2^m}=\frac{1}{4}$ then $\frac{1}{14}\left\{(4^m)^{\frac{1}{2}}+\left(\frac{1}{5^m}\right)^{-1}\right\}$ is equal to

(a) 2 (b) $\frac{1}{2}$ (c) 4 (d) $-\frac{1}{4}$

34. If $x=\sqrt{6}+\sqrt{5}$ then $x^2+\frac{1}{x^2}$

(a) $2(\sqrt{6}+1)$ (b) $2\sqrt{5}+2$
(c) 20 (d) 22

35. $\frac{5-\sqrt{3}}{2+\sqrt{3}}=a+b\sqrt{3}$ then the respective values of a and b are

(a) 13, −7 (b) 13, 7
(c) −13, 7 (d) −13, −7

36. If $m - n = 1$ then $\frac{9^n \times 9 \times \left(3^{\frac{-n}{2}}\right)^{-2} - (27)^n}{3^{3m} \times 2^3} = \left(\frac{1}{3}\right)^x$ then $x =$

(a) 2 (b) −2 (c) 3 (d) −3

37. If $t = 8^2$ then $K = t^{\frac{2}{3}} + 4t^{\frac{-1}{2}}$ then $K =$

(a) $\frac{33}{2}$ (b) 1 (c) $\frac{257}{16}$ (d) $\frac{31}{2}$

38. $\left(\frac{243}{32}\right)^{-0.8} = t$, then the value of '$t$' will be

(a) $\frac{4}{9}$ (b) $\frac{2}{3}$ (c) $\frac{8}{27}$ (d) $\frac{16}{81}$

39. If $\sqrt{5} = k$, then $\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$ has value equal to

(a) $k(\sqrt{2}+1)$ (b) $k(\sqrt{2}-1)$
(c) $k(\sqrt{2}+3)$ (d) $k(2+\sqrt{2})$

40. If $x = \frac{\sqrt{3}+1}{2}$, then the value of $4x^3 + 2x^2 - 8x + 7$ is

(a) 0 (b) 10 (c) 5 (d) 15

41. If $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$ then the value of $\frac{6}{\sqrt{5}-\sqrt{3}}$ is

(a) 11.904 (b) 10.904
(c) 3.968 (d) 8.968

Answer Key

1. (b)	2. (b)	3. (c)	4. (c)	5. (b)	6. (b)	7. (b)	8. (c)	9. (b)	10 (c)
11. (d)	12. (b)	13. (c)	14. (d)	15. (a)	16. (d)	17. (c)	18. (a)	19. (a)	20. (d)
21. (b)	22.(b)	23. (b)	24. (d)	25. (b)	26. (c)	27. (d)	28. (c)	29. (a)	30. (c)
31. (b)	32. (c)	33. (b)	34. (d)	35. (a)	36. (c)	37. (a)	38. (d)	39. (a)	40. (b)
41. (a)									

Hints and Solutions

1. **(b)** Zero is a whole number which is not a natural number. Every integer is a rational number. Every whole number is a integer but converse is false.

2. **(b)** Since,

$$\frac{7}{8} = 0.875 \text{ (Terminating decimal)}$$

$$\sqrt{\frac{9}{125}} = \frac{3}{5\sqrt{5}} \text{ (Irrational)}$$

$$\frac{93}{300} = \frac{31}{100} = 0.31 \text{ (Terminating decimal)}$$

$$\frac{190}{30} = 6.\bar{3} \text{ (Repeating decimal)}$$

∵ Repeating and terminating decimals are rational numbers.

3. **(c)** ∵ All the fractions are in their simplest form.

∴ The fraction having the denominator in the form $2^m \times 5^n$ will be terminating.

$\therefore$ Just analysing the denominators, we have 11,6 and 7 cannot be expressed in $2^m \times 5^n$ form , but $16 = 2^4 \times 5^0$.

$\therefore \frac{11}{16}$ will be a terminating decimal

4. **(c)** Given $x = 0.\overline{57} = 0.5777$ ...(i)

then $10x = 5.777$ ...(ii)

and $100x = 57.777$ ...(iii)

Subtracting equation (ii) from equation (iii), we have

$$90x = 52$$

$$\Rightarrow \quad x = \frac{26}{45}$$

5. **(b)** Given $n = 0.2\overline{45}$ then $x = 0.24545$

$10x = 2.4545$ ...(i)

and $100x = 24.54545$

and $1000x = 245.454545$ ...(iii)

Subtracting eq (i) and eq (iii), we get

$$990x = 243$$

$$\Rightarrow \quad x = \frac{243}{990} = \frac{27}{110}$$

6. **(b)** $\pi = 3.14157......$

(Non-repeating non-terminating decimal)

$$\frac{22}{7} = 3.142871$$

$\therefore \frac{22}{7}$ is a rational number.

7. **(b)** Rationalising the denominator we have

$$\frac{\sqrt{3}+1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{(\sqrt{3}+1)(\sqrt{3}+2)}{(2)^2-(\sqrt{3})^2}$$

Let $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$= 3 + 2 + 3\sqrt{3} = 5 + 3\sqrt{3} = x + y\sqrt{3}$$

$\therefore x = 5, y = 3$

8. **(c)** Here

$$x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{3+2+2\sqrt{6}}{(\sqrt{3})^2-(\sqrt{2})^2} + \frac{3+2-2\sqrt{6}}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$= 5 + 2\sqrt{6} + 5 - 2\sqrt{6} = 10$$

$\therefore$ Required value $= 10 \times 50 = 500$

9. **(b)** Given $x = 2+\sqrt{3}$ then

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$$\therefore x + \frac{1}{x} = (2+\sqrt{3}) + (2-\sqrt{3}) = 4$$

10. **(c)** We have $\frac{16 \times 2^{n+1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{n+2}}$

$$= \frac{(2)^4 \times 2^{n+1} - (2)^2 \times 2^n}{(2)^4 \times (2)^{n+2} - 2 \times 2^{n+2}}$$

$$= \frac{2^{n+2}\{(2)^3-1\}}{2^{n+3}\{(2)^3-1\}} = \frac{1}{2}$$

11. We have $x = \frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2+\sqrt{3}$

$$\Rightarrow \quad x - 2 = \sqrt{3}$$

Squaring both sides

$$(x-2)^2 = 3$$

$$\Rightarrow \quad x^2 + 4 - 4x = 3$$

$$\Rightarrow \quad x^2 - 4x + 1 = 0 \quad ...(i)$$

$$x^3 - 2x^2 - 7x + 5$$

$$= x(x^2 - 4x + 1) + 2(x^2 - 4x + 1) + 3$$

$$= x \times 0 + 2 \times 0 + 3 = 0 + 3 = 3 \text{ (using eq (i))}$$

12. **(b)** Here $y = \frac{(x^{a+b})^2 (x^{b+c})^2 (x^{c+a})^2}{(x^a x^b x^c)^4}$

$$= \frac{(x^{a+b+b+c+c+a})^2}{(x^{a+b+c})^4}$$

$$= \frac{x^{4(a+b+c)}}{x^{4(a+b+c)}} = 1$$

13. **(c)** Given $5^{x-3} . 3^{2x-8} = 225 = 5^2 . 3^2$

$\therefore \quad x-3 = 2x-8 = 2$

$\Rightarrow \quad x = 5$

14. **(d)** Here $\sqrt{13-m\sqrt{10}} = \sqrt{8}+\sqrt{5}$

Squaring both sides, we have

$13 - m\sqrt{10} = \left(\sqrt{8} \quad \sqrt{5}\right)$

$\Rightarrow \quad 13 - m\sqrt{10} = 8+5+2\sqrt{40}$

$\Rightarrow \quad -m\sqrt{10} = 2 \times \sqrt{4\times 10}$

$\Rightarrow \quad -m\sqrt{10} = 2\times 2\sqrt{10}$

$\Rightarrow \quad m = -4$

15. **(a)** Here $\left[2-3(2-3)^3\right]^3 = \left[2-3(-1)^3\right]^3$

$= [2+3]^3 = (5)^3 = 125 = x$

16. **(d)** $\because\ 10^x = 64 \Rightarrow (10^x)^{\frac{1}{2}} = (64)^{\frac{1}{2}} = 8$

$\therefore\ 10^{\frac{x}{2}} = 8 \Rightarrow 10^{\frac{x}{2}+1} = 10^{\frac{x}{2}}.10 = 8.10 = 80$

17. **(c)** We have $4^x - 4^{x-1} = 24$

$\Rightarrow \quad 4^{x-1}(4-1) = 24$

$\Rightarrow \quad 4^{x-1} = 8$

$\Rightarrow \quad \frac{4^x}{4} = 8$

$\Rightarrow \quad 4^x = 32$

$\Rightarrow \quad (2)^{2x} = (2)^5$

$\Rightarrow \quad x = \frac{5}{2}$

$\therefore \quad (2x)^x = \left(2\times\frac{5}{2}\right)^{\frac{5}{2}} = (5)^{\frac{5}{2}}$

$= (5)^{\frac{4}{2}} \cdot (5)^{\frac{1}{2}} = 25\sqrt{5}$

18. **(a)** $x^2 + \frac{1}{x^2} = 83$

$\Rightarrow \left(x-\frac{1}{x}\right)^2 = x^2 + \frac{1}{x} - 2 = (83-2) = 81$

$\Rightarrow \quad \left(x-\frac{1}{x}\right) = 9$

$\therefore \quad \left(x-\frac{1}{x}\right)^3 = (9)^3$

$\Rightarrow \quad x^3 - \frac{1}{x^3} - 3x.\frac{1}{x}\left(x-\frac{1}{x}\right) = 729$

$\Rightarrow \quad x^3 - \frac{1}{x^3} - 3(9) = 729$

$\Rightarrow \quad x^3 - \frac{1}{x^3} = 729 + 27 = 756$

19. **(a)** Here $\left(x+\frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$= (98+2) = 100$

$\Rightarrow \quad x + \frac{1}{x} = \sqrt{100} = 10$

20. **(d)** We have $\frac{x}{y} + \frac{y}{x} = -1$

$\Rightarrow \quad x^2 + y^2 = -xy$

$\Rightarrow \quad x^2 + y^2 + xy = 0 \qquad \text{...(i)}$

We know that,

$x^3 - y^3 = (x-y)(x^2+y^2+xy) = (x-y)(0)$ [Using (i)]

$= 0$

21. **(b)** Here $xy = 1$

$$y = \frac{1}{x} = \frac{1}{7+4\sqrt{3}} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$= \frac{7-4\sqrt{3}}{(7)^2 - (4\sqrt{3})^2} = \frac{7-4\sqrt{3}}{1} = 7-4\sqrt{3}$$

$\therefore \quad \frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{x^2} + x^2$

$$= \left(x+\frac{1}{x}\right)^2 - 2 = (7+4\sqrt{3}+7-4\sqrt{3})^2 - 2$$

$$= (14)^2 - 2 = 196 - 2 = 194$$

22. **(b)** $\because \quad x^{-2} = 64$

$\Rightarrow \quad x^{-1} = 8$

$\Rightarrow \quad x = \frac{1}{8}$

$\Rightarrow \quad x^{\frac{1}{3}} = \left(\frac{1}{8}\right)^{\frac{1}{3}} = \frac{1}{2}$

$x^0 + x^{\frac{1}{3}} = 1 + \frac{1}{2} = \frac{3}{2}$

23. **(b)** The given equation can be written as

$$\left\{(23+4)^{\frac{2}{3}} + (121)^{\frac{1}{2}}\right\}^2$$

$$= \left\{(27)^{\frac{2}{3}} + (121)^{\frac{1}{2}}\right\}^2$$

$$= \left\{(3)^2 + 11\right\}^2 = \{9+11\}^2$$

$$= [20]^2 = 400$$

24. **(d)** Let $7+4\sqrt{3} = \left(a+b\sqrt{3}\right)^2$

$\Rightarrow \quad 7+4\sqrt{3} = a^2 + 3b^2 + 2ab\left(\sqrt{3}\right)$

$\Rightarrow \quad \left(a^2+3b^2\right) = 7,\ ab = 2$

$\therefore a = 2, b = 1.$

$\Rightarrow \quad \sqrt{7+4\sqrt{3}} = 2+\sqrt{3}$

25. **(b)** Here $\frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$$= \frac{\left(\sqrt{2}-1\right)^2}{\left(\sqrt{2}\right)^2 - (1)^2}$$

$$= \frac{2+1-2\sqrt{2}}{2-1} = 3 - 2\sqrt{2}$$

Let, $\sqrt{3-2\sqrt{2}} = a + b\sqrt{2}$

$\Rightarrow 3-2\sqrt{2} = a^2 + b^2 \cdot 2 + 2ab\sqrt{2}$

$\Rightarrow a^2 + 2b^2 = 3,\ ab = -1$

Solving these two equations, we have

$a = -1, b = +1$

$\therefore$ The required value $= \sqrt{2} - 1 = 1.4142 - 1$
$= 0.4142$

26. **(c)** $(3)^{5x-2x} \times (81)^2 \times 6561 = 3^7$

$\Rightarrow \quad (3)^{3x} \times (3)^8 \times 81 \times 81 = 3^7$

$\Rightarrow \quad (3)^{3x} \times (3)^8 \times (3)^8 = 3^7$

$\Rightarrow \quad (3)^{3x+8+8} = 3^7$

$\Rightarrow \quad 3x + 16 = 7$

$\Rightarrow x = \frac{7-16}{3} \quad \frac{-9}{3} = -3$

27. **(d)** Here $\frac{5^{n+1}(5-6)}{5^n(13-10)} = \frac{5^{n+1}(-1)}{5^n(3)} = \frac{-5}{3}$

28. **(c)** $\frac{1}{x} = \frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$

$$= \frac{1+\sqrt{2}}{(1)^2 - \left(\sqrt{2}\right)^2} = \frac{1+\sqrt{2}}{1-2} = -\left(1+\sqrt{2}\right)$$

$\Rightarrow \quad x - \frac{1}{x} = (1-\sqrt{2}) + 1 + \sqrt{2} = 2$

$\Rightarrow \quad \left(x - \frac{1}{x}\right)^3 = (2)^3 = 8$

29. **(a)** Here $\frac{1}{3-\sqrt{8}} = \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}}$

$$= \frac{3+\sqrt{8}}{(3)^2 - (\sqrt{8})^2} = 3 + \sqrt{8}$$

$\frac{1}{\sqrt{8}-\sqrt{7}} = \frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}$

$= \sqrt{8} + \sqrt{7}$

Similarly

$\frac{1}{\sqrt{6}-\sqrt{5}} = \sqrt{6}+\sqrt{5},\ \frac{1}{\sqrt{7}-\sqrt{6}} = \sqrt{7}+\sqrt{6}$

$$\frac{1}{\sqrt{5}-2}=\sqrt{5}+2$$

Rearranging all the terms in the required pattern, we have

$$\left(3+\sqrt{8}\right)-\left(\sqrt{8}+\sqrt{7}\right)-\left(\sqrt{6}+\sqrt{5}\right)+\left(\sqrt{7}+\sqrt{6}\right)+\left(\sqrt{5}+2\right)$$

$$=3+\left(\sqrt{8}-\sqrt{8}\right)+\left(-\sqrt{7}+\sqrt{7}\right)+\left(-\sqrt{6}+\sqrt{6}\right)+\left(-\sqrt{5}+\sqrt{5}\right)+2$$

$$=3+2=5$$

30. **(c)** Given $x=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$=\frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}=\frac{8+2\sqrt{15}}{2}$$

$$y=\frac{\left(\sqrt{5}-\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}=\frac{8-2\sqrt{15}}{2}$$

Now $x+y+xy$

$$=\frac{8+2\sqrt{15}}{2}+\frac{8-2\sqrt{15}}{2}+\left(\frac{8+2\sqrt{15}}{2}\right)\left(\frac{8+2\sqrt{15}}{2}\right)$$

$$=\frac{16}{2}+\frac{64-60}{4}$$

$$=8+1=9$$

$$=8+\frac{\{5-3\}^2}{2\times2}=8+\frac{4}{4}=8+1=9$$

31. **(b)** Let $\sqrt{5+2\sqrt{6}}=\sqrt{a^2+b^2+2ab}$

$\Rightarrow \quad a^2+b^2=5, ab=\sqrt{6}$

Solving these two equations, we get

$$a=\sqrt{3}\text{ , }b=\sqrt{2}$$

32. **(c)** $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{\frac{-1}{3}}\right]^{\frac{1}{4}}=7^m$

$\Rightarrow \quad \left\{(7)^{-2}\right\}^{-2\times\frac{-1}{3}\times\frac{1}{4}}=7^m$

$\Rightarrow \quad (7)^{-2\times-2\times\frac{-1}{3}\times\frac{1}{4}}=7^{\frac{-1}{3}}=7^m$

$\Rightarrow \quad m=\frac{-1}{3}$

33. **(b)** $2^{-m}\times2^{-m}=2^{-2}$

$\Rightarrow \quad (2)^{-2m}=(2)^{-2}$

$\Rightarrow \quad m=1$

Substituting the value of m in,

$$\frac{1}{14}\left\{\left(4^m\right)^{\frac{1}{2}}+\left(\frac{1}{5^m}\right)^{-1}\right\}$$

$$=\frac{1}{14}\left\{\left(4^{\frac{1}{2}}+\left(\frac{1}{5}\right)^{-1}\right)\right\}$$

$$=\frac{1}{14}\{2+5\}=\frac{1}{2}$$

34. **(d)** Given $x=\sqrt{6}+\sqrt{5}$

$\therefore \quad \frac{1}{x}=\frac{1}{\sqrt{6}+\sqrt{5}}\times\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$

$$=\frac{\sqrt{6}-\sqrt{5}}{1}=\sqrt{6}-\sqrt{5}$$

Now $x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$

$$=\left(\sqrt{6}+\sqrt{5}+\sqrt{6}-\sqrt{5}\right)^2-2$$

$$=\left(2\sqrt{6}\right)^2-2=24-2=22$$

35 *(a)* Here $\frac{5-\sqrt{3}}{2+\sqrt{3}}=\frac{5-\sqrt{3}}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$

$$=\frac{\left(5-\sqrt{3}\right)\left(2-\sqrt{3}\right)}{(2)^2-\left(\sqrt{3}\right)^2}$$

$= 10 + 3 - 7\sqrt{3} = a + b$

$= 13 - 7\sqrt{3} = a + b\sqrt{3}$

$\Rightarrow a = 13, b = -7$

36. **(c)** $\dfrac{9^n \times 9 \times 3^n - (27)^n}{3^{3m} \times 2^3} = \left(\dfrac{1}{3}\right)^x$

$\Rightarrow \dfrac{(27)^n[9-1]}{3^{3m} \times 8} = \left(\dfrac{1}{3}\right)^x$

$\Rightarrow \dfrac{(27)^n \times 8}{(27)^m \times 8} = \left(\dfrac{1}{3}\right)^x$

$\Rightarrow (3)^{3(n-m)} = (3)^{-x}$

$\Rightarrow x = 3(m-n) = 3 \quad [m - n = 1]$

37. **(a)** Given $t = 8^2 = 64$

$\therefore K = 8^{\frac{4}{3}} + 4(64)^{\frac{-1}{2}}$

$= (2)^4 + 4 \times \dfrac{1}{8}$

$= 16 + \dfrac{1}{2} = \dfrac{33}{2}$

38. **(d)** Here $\left(\dfrac{243}{32}\right)^{-0.8} = \left(\dfrac{32}{243}\right)^{0.8} = \left(\left(\dfrac{2}{3}\right)^5\right)^{0.8}$

$= \left(\dfrac{2}{3}\right)^4 = \dfrac{16}{81}$

39. **(a)** We have $\dfrac{5 \times 3}{\sqrt{5}\left(\sqrt{2} + \sqrt{4} + \sqrt{8} - 1 - \sqrt{16}\right)}$

$= \dfrac{\sqrt{5} \times \sqrt{5} \times 3}{\sqrt{5}\left(\sqrt{2} + 2 + 2\sqrt{2} - 1 - 4\right)}$

$= \dfrac{3\sqrt{5}}{\left(3\sqrt{2} - 3\right)} = \dfrac{\sqrt{5}}{\sqrt{2} - 1} \times \dfrac{\sqrt{2} + 1}{\sqrt{2} + 1}$

$= \sqrt{5}\left(\sqrt{2} + 1\right)$

$= k\left(\sqrt{2} + 1\right)$

40. **(b)** $\because x = \dfrac{\sqrt{3} + 1}{2}$

$\therefore x^2 = \dfrac{3 + 1 + 2\sqrt{3}}{4} = \dfrac{4 + 2\sqrt{3}}{4} = \dfrac{2 + \sqrt{3}}{2}$

and $x^3 = \dfrac{\left(\sqrt{3} + 1\right)^3}{8} = \dfrac{1 + 3\sqrt{3} + 3\sqrt{3}\left(\sqrt{3} + 1\right)}{8}$

$= \dfrac{10 + 6\sqrt{3}}{8} = \dfrac{5 + 3\sqrt{3}}{4}$

$\therefore 4x^3 + 2x^2 - 8x + 7$

$= \left(5 + 3\sqrt{3}\right) + \left(2 + \sqrt{3}\right) - 8\left(\dfrac{\sqrt{3} + 1}{2}\right) + 7$

$= 14 + 4\sqrt{3} - 4\sqrt{3} - 4 = 10$

41. **(a)** We have $\dfrac{6}{\sqrt{5} - \sqrt{3}} = \dfrac{6}{\sqrt{5} - \sqrt{3}} \times \dfrac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}$

$= \dfrac{\left(\sqrt{5} + \sqrt{3}\right)6}{2}$

$= 3\left(\sqrt{5} + \sqrt{3}\right)$

$= 3(2.236 + 1.732)$

$= 3(3.968)$

$= 11.904$

2. Polynomials

Learning Objective:

In this chapter we shall learn about:

* Polynomials and their types
* Factors of polynomials

Algebraic Expression

Expression separated by + or – operation are called the terms of algebraic expression.

Example: $9x+x^2+x^3+4x^4$ is an algebraic expression and $9x$, x^2, x^3 and $4x^4$ are the terms of the algebraic expression.

Coefficients

In the polynomial $7x^3-6x^2+8x+4$, we say that coefficients of x^3, x^2 and x are 7, – 6 and 8 respectively and 4 is the constant term in it.

Polynomials

An algebraic expression in which the variables involved have only non-negative integral powers is called a polynomial.

Example: (i) $5x^3-5x^2+6x-3$ is a polynomial in one variable x.

(ii) $x^2y+y^2z+6x^3+7y^3$ is a polynomial in 3 variables, i.e, x,y and z.

Important Terms

Constants

A symbol having a fixed numerical value is called a constant.

Example: 2, 3, π, $\frac{-2}{3}$, – 6 etc. are constants.

Variables

A symbol which may be assigned different numerical values is known as a variable.

Example: In $C=2\pi r$, C and r are variables.

Degree of a polynomial in one variable or more than one variable

In case of one variable, the highest power of the variable is called the degree of the polynomial

Example: (i) $2x+5$ is a polynomial in x of degree 1.

(ii) x^2+2x+6 is a polynomial in x of degree 2

In case of more than one variable, the sum of the powers of variables is taken into account, the highest sum so obtained is treated as the degree of the polynomial.

Example: (i) $7x^3-5x^2y^2+3xy+6y+8$ is a polynomial in y and x of degree 4.

Types of Polynomial

Zero Polynomial

The constant polynomial 0 is called the zero polynomial.

Linear polynomial

A polynomial of degree one is called a linear polynomial.

Quadratic polynomial

A polynomial of degree two is called a quadratic polynomial.

Cubic polynomial

A polynomial of degree three is called a cubic polynomial.

Number of Terms in a Polynomial

(i) **Monomial:** A polynomial containing one nonzero term is called a monomial.

(ii) **Binomial:** A polynomial containing two non zero terms is called a binomial.
Example: $x-5y$, $5x^2+2zx$

(iii) **Trinomial:** A polynomial containing three non- zero terms is called a trinomial.
Example: x^3+5x^2+3y, x^2+3x+9, $xy+yz+x^2$ etc.

Constant Polynomial and Zero Polynomial

A polynomial containing one term only, i.e, constant term only is called a constant polynomial becomes equal to zero, the polynomial is said to be a zero polynomial.

Example 1: Which of the following expressions are polynomials ?

(a) x^2-5x+3 (b) $2\sqrt{x}+5$ (c) -8 (d) $3x^{\frac{2}{3}}+6$.

Solution: (a) $\because$ The expression x^2-5x+3 has all the non- negative integral powers in x.
$\therefore$ expression is a polynomial.

(b) $2\sqrt{x}+5=2x^{\frac{1}{2}}+5$

$\because$ x has a non-integral powers in n

$\therefore$ Given expression is not a polynomial.

(c) -8 is a constant term.

$\therefore$ This is a constant polynomial.

(d) $3x^{\frac{2}{3}}+6$ has non– integral powers in x

$\therefore$ This is a not a polynomial.

Example 2: x^2+5x-2 is polynomial of how many degrees and comment about number of terms in it?

Solution: x^2+5x-2 is a polynomial in x of degree 2 and it has 3 terms.
$\therefore$ This polynomial is a binomial.

Example 3: $3x^3+3x^2+8x+9$ is a polynomial in x. Classify this polynomial on the basis of degree and number of terms.

Solution: The highest power of x in the expression is 3.

$\therefore$ This polynomial $2x^3 + 3x^2 + 8x + 9$ is a cubic polynomial.

$\therefore$ Number of terms in polynomial = 4.

$\therefore$ This is conceded to be a 4 terms containing polynomial, i.e., quadronomial.

Factors of a Polynomial

Let $p(x)$ is a polynomial. If $p(a) = 0$ then 'a' is said to be a zero and $(x - a)$ is said to be a factor of polynomial $p(x)$.

Example 4: If $P(x) = x^2 + 3x + 4$, find $P(-2)$, $P(1)$.

Solution: $P(-2) = (-2)^2 + 3(x - 2) + 4 = 4 - 6 + 4 = 2$

$$P(1) = (1)^2 + 3 \times 1 + 4$$
$$= 1 + 3 + 4 = 8.$$

Example 5: Find a zero of the polynomials

(a) $2x + 9$ (b) $4x - 8$

Solution: **(a)** Let $P(x) = 2x + 9$

Now $P(x) = 0$

$$\Rightarrow 2x + 9 = 0 \quad \Rightarrow \quad x = \frac{-9}{2}$$

(b) Let $P(x) = 4x - 8$

$$\text{Now, } P(x) = 0 \quad \Rightarrow \quad 4x - 8 = 0 \quad \Rightarrow \quad x = \frac{8}{4} = 2$$

Example 6: Find the coefficient of $5x^2 + 3x + 9$ in the expression $15x^4 + 9x^3 + 27x^2$.

Solution: Coefficient of $5x^2 + 3x + 9$ in the expression $15x^4 + 9x^3 + 27x^2 = 3x^2(5x^2 + 3x + 9)$, is $3x^2$.

Factorization

Factorization is a process of representing the given polynomial as a product of its factors which are of lower degree than the given polynomial.

Example: $x^2 - 4 = (x + 2)(x + 2)$

Important Formulae

(a) $(x+y)^2 = x^2 + y^2 + 2xy$ (b) $(x-y)^2 = x^2 + y^2 - 2xy$

(c) $x^2 - y^2 = (x+y)(x-y)$ (d) $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

(e) $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ (f) $x^3 + y^3 = (x+y)(x^2 + y^2 - xy)$

(g) $x^3 - y^3 = (x-y)(x^2 + y^2 + xy)$ (h) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

(i) $(x-y+z)^2 = x^2 + y^2 + z^2 + 2(-xy - yz + zx)$

Example 7: Factorize: $x(x-y)^3+3x^2y(x-y)$.

Solution: We have $x(x-y)^3+3x^2y(x-y)$

$$= x(x-y)\{(x-y)^2+3xy\}$$
$$= x(x-y)\{x^2+y^2-2xy+3xy\}$$
$$= x[(x-y)(x^2+y^2+xy)]$$
$$= x(x^3-y^3)$$

Example 8: Factorize :

(i) $x^2+3x+3+x$ (ii) $a^2+b-ab-a$

Solution: (i) $x^2+3x+3+x$

$$= x(x+3)+1(x+3)=(x+3)(x+1)$$

(ii) $a^2+b-ab-a=a^2-ab+b-a$

$$= a(a-b)-1(a-b)$$
$$= (a-1)(a-b)$$

Example 9: Factorize:

(i) $x^2+\frac{1}{x^2}+2-2x-\frac{2}{x}$ (ii) $x^2+\frac{1}{x^2}-2-3x-\frac{3}{x}$

Solution: (i) $\therefore \left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2$

$$\therefore\ x^2+\frac{1}{x^2}+2-2x-\frac{2}{x}=\left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)$$
$$=\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}-2\right)$$

(ii) $\therefore \left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2$

$$\therefore\ x^2+\frac{1}{x^2}-2-3x+\frac{3}{x}=\left(x-\frac{1}{x}\right)^2-3\left(x-\frac{1}{x}\right)$$
$$=\left(x-\frac{1}{x}\right)\left(x-\frac{1}{x}-3\right)$$

Example 10: Factorize:

(i) $x^2-(a+b)x+ab$ (ii) $x^3-x^2+ax+x-a-1$

(iii) $(2x-3)^2-8x+12$

Solution: (i) $x^2-ax-bx+ab$

$$= x(x-a)-b(x-a)$$
$$= (x-a)(x-b)$$

(ii) $x^2(x-1)+x(a+1)-1(a+1)$

$$= x^2(x-1)+(a+1)(x-1)$$
$$= (x-1)(x^2+a+1)$$

(iii) $(2x-3)^2 - 8x + 12$

$= (2x-3)(2x-3) - 4(2x-3)$

$= (2x-3)(2x-3-4)$

$= (2x-3)(2x-7)$

Example 11: Factorize :

(i) $a^2 + 2ab + b^2 - 4c^2$ (ii) $x^2 - y^2 + 6y - 9$

(iii) $x^4 - 625$ (iv) $3x^3 - 48x$

(v) $(a+b)^3 - a - b$ (v) $(a+b)^3 - a - b$

(vi) $9 - a^2 + 2ab - b^2$

Solution: (i) $(a+b)^2 - 4c^2 = (a+b)^2 - (2c)^2$

$= (a+b+2c)(a+b-2c)$

(ii) $x^2 - (y^2 - 6y + 9) = x^2 - (y+3)^2$

$= (x-y-3)(x+y+3)$

(iii) $x^4 - 625 = (x^2) - (25)^2$

$= (x^2 - 25)(x^2 + 25)$

$= (x+5)(x-5)(x^2+25)$

(iv) $3x(x^2 - 16) = 3x(x+4)(x-4)$

(v) $(a+b)^3 - a - b = (a+b)\{(a+b)^2 - 1\}$

$= (a+b)(a+b+1)(a+b-1)$

(vi) $9 - a^2 + 2ab - b^2 = (3)^2 - (a^2 - 2ab + b^2)$

$= (3)^2 - (a-b)^2$

$= (3+a-b)(3-a+b)$

Example 12: Factorize :

(i) $2x^2 - \frac{5}{6}x + \frac{1}{12}$ (ii) $\sqrt{3}x^2 + 11x + 6\sqrt{3}$

Solution: (i) $\frac{2\times 12x^2 - 10x + 1}{12} = \frac{24x^2 - 10x + 1}{12}$

$= \frac{24x^2 - 6x - 4x + 1}{12}$

$= \frac{1}{12}(4x-1)(6x-1)$

(ii) $ax^2 + bx + c$, can be factorized as, multiply a and c, and express b as a sum of two numbers whose multiplication (product is equal to 'ac'

$\therefore\ \sqrt{3}\times 6\sqrt{3} = 18$

$11x = 9x + 2x$, also $9 \times 2 = 18$

$$\therefore\ \sqrt{3}x^2+11x+6\sqrt{3}=\sqrt{3}x^2+9x+2x+6\sqrt{3}$$
$$=\sqrt{3}x\left(x+3\sqrt{3}\right)+2\left(x+3\sqrt{3}\right)$$
$$=\left(\sqrt{3}x+2\right)\left(x+3\sqrt{3}\right)$$

Example 13: Factorize :

(i) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}y^2-8xz$ (ii) Evaluate : $(97)^2$.

Solution: (i) $\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\left(-\sqrt{2}xy+2\sqrt{2}yz-4xz\right)=\left(-\sqrt{2}x+y+2\sqrt{2}z\right)^2$

(ii) $(97)^2=(100-3)^2=(100)^2+(3)^2-2\times100\times3$
$=10000+9-600=9409$

Example 14: Expand $(3x+2)^3$, and factorize x^3+125.

Solution: $(3x+2)^3=27x^3+8+3\times3x\times2(3x+2)$
$=27x^3+8+18x\ (3x+2)=27x^3+8+54x^2+36x$

$x^3+125=(x)^3+(5)^3=(x+5)\,(x^2+25-5x)$

Example 15: Evaluate : $x^3+y^3+z^3-3xyz$

Solution: $x^3+y^3+z^3-3xyz=(x)^3+(y)^3+z^3-3xyz$

$$=\left[(x^3)+(y)^3+3xy(x+y)\right]+z\left(z^2-3xy\right)$$

Let $x+y=u$

$=[4^2-3xy4]+z\,(z^2-3xy)=4^3+z^3-3xy\,(4+z)$
$=(4+z)\,[4^2+z^2-4z-3xy]=(4+z)\,(x^2+y^2+z^2-xy-yz-zx)$
$=(x+y+z)\,(x^2+y^2+z^2-xy-yz-zx)$

Note

(i) If $x+y+z=0$, then
$x^3+y^3+z^3-3xyz=0$
$\Rightarrow\ x^3+y^3+z^3=3xyz$

(ii) $x^2+y^2+z^2-xy-yz-zx=\frac{1}{2}\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$

This can only be zero when, $x=y=z$
$\therefore$ when $x=y=z$, then also
$x^3+y^3+z^3=3xyz$

Example 16: Factorize :

(i) $(p-q)^3+(q-r)^3+(r-p)^3$

(ii) If $p+a=2$ then what is the value of $a^3+6ap+p^3$?

(iii) Find the product:

$$(3x - 5y - 4)\,(9x^2 + 25y^2 + 15xy + 12x - 20y + 16)$$

Solution: (i) $\because\ p - q + q - r + r - p = 0$

$$\therefore\ (p-q)^3 + (q-r)^3 + (r-p)^3 = 3\,(p-q)\,(q-r)\,(r-p)$$

(ii) $\because p + a = 2$

then $\qquad (p+a)^3 = (2)^3$

$\Rightarrow \qquad p^3 + a^3 + 3ap\,(a+p) = (2)^3$

$\Rightarrow \qquad p^3 + a^3 + 6ap = (2)^3 = 8 \qquad \{\because a + p = z\}$

(iii) $(3x - 5y - 4)\,(9x^2 + 25y^2 + 16 + 15xy + 12x - 20y)$

$$= (3x)^3 + (-5y)^3 + (-4)^3 - 3 \times 3x \times -5y \times (-4)$$

$$= 27x^3 - 125y^3 - 64 - 180xy$$

Remainder Theorem

If $p(x)$ is a polynomial of degree ≥ 1 and let a be one non-zero real number. When $p(x)$ is divided by $(x - a)$, then remainder is $p(a)$.

Factor Theorem

Let $p(x)$ be a polynomial of degree > 1 and let 'a' be any real number

(i) if $p(a) = 0$ then $(x - a)$ is a factor of $p(x)$

(ii) if $(x - a)$ is a factor of $p(x)$ then $p(a) = 0$

Example 17: Find the remainder when $a^2 + 2ab$ is divided by $a + 2b$.

Solution: $a^2 + 2ab = (a^2 + 2ab + b^2) - b^2 = (a+b)^2 - b^2$

Now let $(a + b) = x$

$\therefore\ p(x) = a^2 + 2ab = (a+b)^2 - b^2 = x^2 - b^2$

$a + 2b = (a + b) + b = x + b$

$\therefore \qquad p(-b) = (-b)^2 - b^2 = 0$

Example 18: Find the value of a, if $x - a$ is a factor of

$$x^3 - a^2x + x + 2.$$

Solution: If $x - a$ is a factor of $p(x)$

Then, $p(a) = 0$, i.e,

$p(x) = x^3 - a^2x + x + 2$

$\Rightarrow \qquad \pi(a) = a^3 - a^3 + a + 2 = 0$

$\Rightarrow \qquad a + 2 = 0$

$\Rightarrow \qquad a = -2$

Multiple Choice Questions

1. If $x+\frac{1}{x}=3$ then $x^4+\frac{1}{x^4}=$

(a) 79 (b) 43 (c) 47 (d) 81

2. If $x+y=12$ and $xy=35$ then $x^2+y^2=?$

(a) 74 (b) 64 (c) 84 (d) 80

3. If $a=b=c$ then $(a+b+c)^2-xa^2=0$, then $x=$

(a) 2 (b) 6 (c) 3 (d) 9

4. What will be the value of 991×1009 ?

(a) 999918 (b) 999919
(c) 999999 (d) 990019

5. $a^2+b^2+c^2-ab-bc-ca$ will have:

(a) Always negative value
(b) Always positive value
(c) Always non–negative value
(d) Insufficient data given

6. Number of terms in the expand form of $(x-y-z)^2$ will be

(a) 6 (b) 9 (c) 12 (d) 3

7. Square root of $a^2+4b^2+9c^2+6ac+4ab+12bc$ will be

(a) $a+2b+3c$ (b) $a+3b+2c$
(c) $a+2b-3c$ (d) $a-2b+3c$

8. If $a^2+b^2+c^2=16$ and $ab+bc+ca=10$, then the value of $(a+b+c)$ will be

(a) ± 7 (b) ± 8 (c) ± 4 (d) ± 6

9. The value of $9a^2+4b^2+16c^2+12ab-24ac-16bc$ for $b=1$, $c=-2$ will be

(a) 0 (b) 64 (c) 256 (d) 32

10. If $x^4+\frac{1}{x^4}=47$ then the value of $x^3+\frac{1}{x^3}=$

(a) 18 (b) 27 (c) 25 (d) 16

11. If $x+\frac{1}{x}=-3$, then the value of $x^3+\frac{1}{x^3}$ is

(a) –54 (b) –9 (c) –27 (d) –18

12. Cube root of $\frac{27}{x^3}-\frac{8}{x^6}-\frac{54}{x^4}+\frac{36}{x^5}$ will be,

(a) $\frac{3}{x}-\frac{2}{x^2}$ (b) $\frac{3}{x}-\frac{2}{x}$

(c) $\frac{3}{x}+\frac{2}{x^2}$ (d) $\frac{-3}{x}+\frac{2}{x^2}$

13. If $(x+k)^3+(x-k)^3=2x^3+54x$, then k will be,

(a) $3-3$ (b) 4 (c) -4 (d) $6-6$

14. If $x^3-\frac{1}{x^3}=108+76\sqrt{2}$, then $x-\frac{1}{x}$

(a) $3+\sqrt{2}$ (b) $3+2\sqrt{2}$

(c) $3-2\sqrt{2}$ (d) $\sqrt{3}+2\sqrt{2}$

15. If $3x+2y=13$ and $xy=6$, then $27x^3+8y^3$ will be equal to

(a) 1859 (b) 729 (c) 793 (d) 891

16. If $x+y=4$, $xy=4$, then $2x^3+y^3$ will be equal to

(a) 15 (b) 17 (c) 16 (d) 24

17. If $a+b=6$, and $ab=8$, then value of (a^2+b^2-ab) will be

(a) 6 (b) 8 (c) 12 (d) 16

18. The value of $(x-1)(x^2+1+x)(x^6+x^3+1)$, will be equal to

(a) x^9+1 (b) x^6+1
(c) x^6 (d) x^9-1

19. If $a+b+c=15$ and $a^2+b^2+c^2=83$ then the value of $(ab+bc+ca)$ will be equal to

(a) 71 (b) 74 (c) 72 (d) 116

20. If $a+b+c=15$, and $a^2+b^2+c^2=83$, then the value of $a^3+b^3+c^3-3abc$ will be

(a) 105 (b) 90 (c) 108 (d) 180

21. If $\frac{x}{y}+\frac{y}{x}=-1$ then $x^3-y^3=$

(a) –8 (b) 0
(c) 1 (d) –1

22. $30^3+20^3-50^3=$

(a) 11500 (b) – 90000
(c) 0 (d) 9000

23. If $a+b+c=6$ and $a^3+b^3+c^3=6\left(3+\frac{abc}{2}\right)$ then $ab+bc+ca$ will have the value equal to
(a) 16 (b) −11 (c) 11 (d) 12

24. The volume of a cuboid is $3x^2-27$, its possible dimensions are
(a) $3, x^2, -27x$ (b) $3, x-3, x+3$
(c) $3, x^2, 27n$ (d) 3, 3, 3

25. If $a^3+b^2+c^2=ab+bc+ca$, then the value of $(a+b+c)$ will be
(a) 0 (b) 1 (c) −1 (d) 2

26. If $a^2+b^2+c^2=ab+bc+ca$, then
(a) $a=b=c$ (b) $a+b+c$
(c) $a+b=c$ (d) $b+c=a$

27. If $a+b+c\neq 0$, then the value of $a^3+b^3+c^3-3abc$ will
(a) Never posses a zero value,
(b) Always posses a zero value
(c) Zero value possession at $a=b+c$
(d) Posses a zero value, iff $a=b=c$.

28. If $(a+b+c)$ has a positive value then the sign of value of $(a^3+b^3+c^3-3abc)$ will be
(a) Non-negative,
(b) Positive,
(c) Negative,
(d) Will be always zero.

29. The factors of $1-2ab-(a^2+b^2)$ will be,
(a) $(1+a+b)(1-a+b)$
(b) $(1-a+b)(1+a+b)$
(c) $(1+a+b)(1+a-b)$
(d) $(1+a+b)(1-a-b)$

30. (x^2-x+1) is a factor of :
(a) x^2+x+1 (b) x^4-7^2+2
(c) x^4+x^2+1 (d) x^4+2x^2+1

31. Factors of $x^2+6\sqrt{2}x+10$ will be
(a) $\left(x+5\sqrt{2}\right),\left(x+\sqrt{2}\right)$
(b) $\left(x-5\sqrt{2}\right),\left(x-\sqrt{2}\right)$
(c) $\left(x-5\sqrt{2}\right),\left(x+\sqrt{2}\right)$
(d) $\left(x-5\sqrt{2}\right),\left(x+\sqrt{3}\right)$

32. If a rectangle has its area as $2x^2+3\sqrt{5}x+5$, and length $=x+\sqrt{5}$, what will be its breadth?
(a) $2x+\sqrt{5}$ (b) $2x-\sqrt{5}$
(c) $x+\sqrt{3}$ (d) $x-\sqrt{5}$

33. The expression $(a-b)^3+(b-c)^3+(c-a)^3$ can be factorised as
(a) $(a-7)(b-c)(c-a)$
(b) $-3(a-b)(b-c)(c-a)$
(c) $3(a-b)(b-c)(c-a)$
(d) $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

34. If $x^3-3x^2+3x-7=(x+1)(ax^2+bx+c)$, then $a+b-c=$
(a) 8 (b) −4 (c) −10 (d) 4

35. If $(x-a)^3+(x-b)^3+(x-c)^3-3(x-a)(x-b)(x-c)=0$, then $7+b+c=$
(a) 0 (b) $3x$ (c) $2x$ (d) $4x$

36. The value of
$$\frac{(0.013)^3+(0.007)^3}{(0.013)^2-(0.013\times 0.007)+(0.007)^2}$$
(a) 0.0091 (b) 0.00181
(c) 0.02 (d) 0.008

37. If $x=2$ and $x=0$ are roots of $f(x)=ax^2+bx$, then a and b are
(a) 0, 0 (b) cannot be calculated
(c) − 2, −1 (d) −2

38. If the polynomials ax^3+4x^2+3x-4 and x^3-4x+a equal to
(a) 2 (b) −2
(c) −1 (d) 1

39. If $(3x-1)^7=a_6x^7+a_6x^6+a_5x^5 \ldots\ldots + a_1x + a_0$ then, $a_7+a_6+a_5 \ldots\ldots + a_0=$
(a) 1 (b) 20
(c) 64 (d) 128

40. If $x^{140}+2x^{151}+k$ is divisible by $(x+1)$ then the value of k is
(a) 1 (b) −2 (c) 2 (d) − 3

41. If $x-1$ is a factor of $4x^3+3x^2-4x+k$, then $k=$

(a) 3 (b) -2 (c) -3 (d) 1

42. If ax^3+bx^3+x-6 has $x+2$ as a factor and leaves 4 as remainder when divided by $(x-2)$, then (a, b) will be

(a) (2, 0) (b) (0, 2) (c) (0, −2) (d) (−2, 0)

43. If $x-3$ is a factor of $ax^2+18=0$, then $a=$

(a) 3 (b) -3 (c) -2 (d) 2

44. The value of x, which is be added to the expression $x^4+2x^3-2x^2+x-1$ to make it completely (exactly) divisible by x^2+2x-3 is

(a) $x+2$ (b) $x-2$
(c) $x-4$ (d) $x+3$

45. If (x^2+x+1) is a factor of $3x^3+8x^2+8x+3+5k$, then $k=$

(a) $-\frac{2}{5}$ (b) $-\frac{5}{2}$ (c) $\frac{2}{5}$ (d) 0

Answer Key

1. (c)	2. (a)	3. (c)	4. (b)	5. (c)	6. (a)	7. (a)	8. (d)	9. (c)	10 (a)
11. (d)	12. (a)	13. (a)	14. (b)	15. (c)	16. (d)	17. (c)	18. (d)	19. (a)	20. (c)
21. (b)	22.(b)	23. (c)	24. (b)	25. (a)	26. (a)	27. (c)	28. (a)	29. (d)	30. (c)
31. (a)	32. (a)	33. (c)	34. (d)	35. (b)	36. (c)	37. (b)	38. (c)	39. (d)	40. (a)
41. (c)	42. (b)	43. (c)	44. (a)	45. (c)					

Hints and Solutions

1. **(c)** Given $x+\frac{1}{x}=3$

$$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2=(3)^2-2=9-2=7$$

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2=x^4+\frac{1}{x^4}+2=49$$

$$\Rightarrow x^4+\frac{1}{x^4}=49-2=47$$

2. **(a)** If $x+y=12$ and $xy=35$ then

$(x+y)^2=(12)^2$

Now $x^2+y^2=(x+y)^2-2xy$

$\Rightarrow\quad x^2+y^2=144-2(xy)=144-2(35)$

$=144-70=74$

3. **(d)** If $a=b=c$, then

$(a+b+c)=(a+a+a)=3a$

$\therefore\ (a+b+c)^2-xa^2=(3a)^2-xa^2=0$

$\Rightarrow\quad xa^2=9a^2$

$\Rightarrow\quad x=9$

4. **(b)** Here $991\times 1009=(1000-9)\times(1000+9)$

$=(1000)^2-(9)$

$=(10)^6-81=999919$

5. **(c)** $a^2+b^2+c^2-ab-bc-ca$ can be written as

$$\frac{(a-b)^2+(b-c)^2+(c-a)^2}{2}$$

$\therefore$ The numerator contains the terms having even powers.

$\therefore$ The numerator will be always zero or positive, i.e., non-negative.

6. **(a)** $(x-y+z)^2$ will contain 6 terms in its simplied form.

7. **(a)** Given $a^2+4b^2+9c^2+6ac+4ab+12bc$

$=(a)^2+(2b)^2+(3c)^2+2(3c)(a)+2(a)(2b)+2(3c)(2b)$

$=(a+2b+3c)^2$

$\therefore$ Square root of the above expression will be $(a + 2b + 3c)$.

8. **(d)** Here $(a+b+c)^2 = (a^2+b^2+c^2) + 2(ab+bc+ca)$

$\Rightarrow \quad (a+b+c)^2 = (16) + 2(10) = 16 + 20 = 36$

$\Rightarrow \quad (a+b+c) = \sqrt{36} = \pm 6$

9. **(c)** $9a^2 + 4b^2 + 16c^2 + 12ab - 24ac - 16bc$

$= (3a)^2 + (2b)^2 + (-4c)^2 + 2(3a)(2b) + 2(3a)(-4c) + 2(3a)(-4c)$

$= (3a + 2b - 4c)^2$

Putting $a = 2$, $b = 1$, $c = -2$, we have required expression as

$[3(2) + 2(1) - 4(-2)]^2$

$= [6 + 2 + 8]^2 = (16)^2 = 256$

10. **(a)** $x^4 + \frac{1}{x^4} = 47$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = \left(x^4 + \frac{1}{x^4}\right) + 2$

$= (47 + 2) = 49$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = 7$

$\therefore \quad \left(x + \frac{1}{x}\right)^2 = \left(x^2 + \frac{1}{x^2}\right) + 2$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 7 + 2 = 9 \Rightarrow \left(x + \frac{1}{x}\right) = 3,$

$\therefore \quad \left(x + \frac{1}{x}\right)^3 = \left(x^3 + \frac{1}{x^3}\right) + 3\left(x + \frac{1}{x}\right)$

$\Rightarrow \quad x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$

$\Rightarrow x^3 + \frac{1}{x^3} = (3)^3 - 3(3) = 27 - 9 = 18$

11. **(d)** $x + \frac{1}{x} = -3$

$\therefore \quad \left(x^3 + \frac{1}{x^3}\right) = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$

$= (-3)^3 - 3(-3)$

$= -27 + 9 = -18$

12. **(a)** We have $\frac{27}{x^3} - \frac{8}{x^6} - \frac{54}{x^4} + \frac{36}{x^5}$

$= \left(\frac{3}{x}\right)^3 + \left(\frac{-2}{x^2}\right)^3 + 3 \times \left(\frac{3}{x}\right)^2 \times \left(\frac{-2}{x^2}\right) + 3 \times \left(\frac{3}{x}\right) \times \left(\frac{-2}{x^2}\right)^2$

$= \left(\frac{3}{x} - \frac{2}{x^2}\right)^3$

$\therefore$ Cube root will be $\left(\frac{3}{x} - \frac{2}{x^2}\right)$.

13. **(a)** $(a+b)^3 + (a-b)^3 = 2a^3 + 6ab^2$

$\Rightarrow (x+k)^3 + (x-k)^3 = 2x^3 + 6 \times x \times k^2$

$\Rightarrow \quad k^2 = 9$

$\Rightarrow \quad k = 3, -3$

14. **(b)** $\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$

Let $\left(x - \frac{1}{x}\right) = A$, then,

$\Rightarrow A^3 + 3A = 108 + 76\sqrt{2}$

$\Rightarrow \quad A = 3 + 2\sqrt{2}$

15. **(c)** Given $3x + 2y = 13$, $xy = 6$

$\because \quad 3x + 2y = 13$

$\therefore (3x + 2y)^3 = (13)^3$

$\Rightarrow 27x^3 + 8y^3 + 3 \times 3x \times 2y\,(3x + 2y) = (169) \times 13$

$\Rightarrow 27x^3 + 8y^3 + 18xy\,(3x + 2y) = (169) \times 13$

$\Rightarrow 27x^3 + 8y^3 + 18 \times 6 \times (13) = (169) \times 13$

$\Rightarrow 27x^3 + 8y^3 = 61 \times 13 = 793$

16. **(d)** Given $x + y = 4$, $xy = 4$.

$\Rightarrow \quad (x+y)^2 = (x-y)^2 + 4xy$

$\Rightarrow \quad (4)^2 = (x-y)^2 + 4 \times 4$

$\Rightarrow \quad (x-y)^2 = 16 - 16 = 0$

$\Rightarrow \quad x = y$

$\therefore$ if $\quad x + y = 4$

$\Rightarrow \quad y = x = 2$

Hence $2x^3 + y^3 = 3x^3 = 3(2)^3 = 24$

17. **(c)** Here $a+b=6, ab=8$

$\therefore \quad (a+b)^2=(6)^2$

$\Rightarrow a^2+b^2+2ab=36$

$\Rightarrow a^2+b^2=36-2(ab)=36-(18)=20$

$\therefore (a^2+b^2-ab)=20-8=12$

18. **(d)** We have $(x-1)(x^2+x+1)=(x^3-1)$

Now,

Let $x^3=p$, then $x^6=p^2$

$\because \quad (x^3-1)(x^6+x^3+1)$

$=(p-1)(p^2+p+1)=(p^3-1)$

$=(x^3)^3-1$

$=x^9-1$

19. **(a)** Here $(a+b+c)^2=(15)^2$

$\Rightarrow (a^2+b^2+c^2)=(15)^2-2(ab+bc+ca)$

$\Rightarrow \frac{(83)-225}{2}=-(ab+bc+ca)$

$\Rightarrow \quad -71=-(ab+bc+ca)$ ...(i)

$\Rightarrow \quad ab+bc+ca=71$ ...(i)

20. **(c)** $a^3+b^3+c^3-3abc=(a+b+c)$
$(a^2+b^2+c^2-ab-bc-ca)$

$=(15)(83-71)$ [see sol. 19 from eq (i)]

$=15(12)$

$=180$

21. **(b)** Given $\frac{x^2+y^2}{xy}=-1$

$\Rightarrow \quad x^2+y^2=-xy$

$\Rightarrow \quad x^2+y^2+xy=0$ ...(i)

$\therefore x^3-y^3=(x-y)(x^2+y^2+xy)$

$=(x-y)(0)=0$ [using (i)]

22. **(b)** Let $a=30$, $b=20$ $c=-50$

$\because \quad a+b+c=0$

$\therefore \quad a^3+b^3+c^3=3abc$

$=3(30)(20)(-50)$

$=-90{,}000$

23. **(c)** Given $a+b+c=6$, and,

$a^3+b^3+c^3=18+3abc$

$\Rightarrow a^3+b^3+c^3-3abc=18$

$\Rightarrow a^3+b^3+c^3-3abc=18$

$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow a^2+b^2+c^2-ab-bc-ca=3$ ...(i)

Now,

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow (6)^2=a^2+b^2+c^2+2(ab+bc+ca)$ ...(ii)

Subtracting eq (i) from eq (ii), we get

$(36-3)=3(ab+bc+ca)$

$\Rightarrow ab+bc+ca=\frac{33}{3}=11$

24. **(b)** Here $3x^2-27=3(x^2-9)$

$=3[(x)^2-(3)^2]$

$=3(x+3)(x-3)$

25. **(a)** $\because a^3+b^3+c^3-3abc$

$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

If $a\neq b\neq c$, then,

For $a^3+b^3+c^3=3abc$, $(a+b+c)$ Must posses zero value.

26. **(a)** Given $a^2+b^2+c^2-ab-bc-ca=0$

$\Rightarrow \quad -\left[(a-b)^2+(b-c)^2+(c-a)^2\right]=0$

$\Rightarrow \quad (a-b)^2+(b-c)^2+(c-a)^2=0$

For the resultant summation of 3 even power braces to be zero, the numbers in each brace should be equal to zero

$\therefore \quad a=b=c$

27. **(d)** We have $a^3+b^3+c^3-3abc$

$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

If $(a+b+c)\neq 0$ then $a^3+b^3+c^3-3abc$ will posses 0 value, iff $a^2+b^2+c^2-ab-bc-ca=0$, and this will only happen,

when $a=b=c$ {problem no –26}

28. **(a)** We know $a^3+b^3+c^3-3abc$

$=\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

The sign of $(a^3+b^3+c^3-3abc)$ will only depend on the sign of $(a+b+c)$, if it is positive then the resulting sign will be positive, if a,b,c are not respectively equal and the value will be zero if $a = b = c$.

$\therefore$ The expression will have a non– negative value.

29. **(d)** Here $1 - 2ab - a^2 - b^2$
$= 1 - (a^2 + b^2 + 2ab)$
$= 1 - (a + b)^2 = (1)^2 - (a + b)^2$
$= (1 - a - b)(1 + a + b)$

30. **(c)** $(x^4 + x^2 + 1)$ has two factors, i.e., $(x^2 + x + 1)$ and $(x^2 - x + 1)$

31. **(a)** Here $x^2 + 6\sqrt{2}x + 10$
$= (x)^2 + 2\times 3\sqrt{2}\times x + \left(3\sqrt{2}\right)^2 - \left(3\sqrt{2}\right)^2 + 10$
$= \left(x + 3\sqrt{2}\right)^2 + 10 - 18$
$= \left(x + 3\sqrt{2}\right)^2 - \left(2\sqrt{2}\right)^2$
$= \left(x + 3\sqrt{2} + 2\sqrt{2}\right)\left(x + 3\sqrt{2} - 2\sqrt{2}\right)$
$= \left(x + 5\sqrt{2}\right)\left(x + \sqrt{2}\right)$

32. **(a)** Here
$2x^2 + 3\sqrt{5}x + 5 = 2x^2 + 2\sqrt{5}x + \sqrt{5}x + 5$
$= 2x\left(x + \sqrt{5}\right) + \sqrt{5}\left(x + \sqrt{5}\right)$
$= \left(2x + \sqrt{5}\right)\left(x + \sqrt{5}\right)$
$\therefore$ breadth $= \dfrac{\text{Area}}{\text{length}} = \left(2x + \sqrt{5}\right)$

33. (c) Let $a - b = A,\ b - c = B,\ c - a = C$
Then, $A + B + C = (a - b) + (b - c) + (c - a) = 0$
$\therefore A^3 + B^3 + C^3 = 3ABC$
$\Rightarrow 3(a - b)(b - c)(c - a)$

34. **(d)** Given
$x^3 - 3x^2 + 3x - 7 = (x + 1)\,(ax^2 + bc + c)$
Comparing coefficients from RHS and LHS.
(i) $x^3,\ 1 = a \Rightarrow a = 1$
(ii) $x^2,\ -3 = a + b$
$\Rightarrow a + b = -3 \Rightarrow b = -3 - 1 = -4$
(iii) Constant term,
$-7 = c$
$\therefore a + b - c = 1 - 4 + 7 = 4$

35. **(b)** Let $(x - a) = p,\ (x - b) = q,\ (x - c) = r$,
Then
$p^3 + q^3 + r^3 - 3abc = 0$ {given}
$\because\ p \neq q \neq r$
$\therefore\ p + q + r = 0$
$\Rightarrow\ (x - a) + (x - b) + (x - c) = 0$
$\Rightarrow\ a + b + c = 3x$

36. **(c)** Let $0.013 = a,\ 0.007 = b$
Then, $\dfrac{a^3 + b^3}{a^2 - ab + b^2}$, is the required expression.
$\therefore\ \dfrac{(a + b)a^2 + b^2 - ab}{a^2 + b^2 - ab} = (a + b)$
$\therefore$ Required value $= 0.013 + 0.007$
$= 0.020$

37. **(b)** Given $f(x) = ax^2 + bx$
$= x(ax + b)$
$x = 0$, will be a factor or not
Now,
For $x = 2, f(2) = 2(2a + b) = 0$
So, 'a' and 'b' cannot be calculated as we have 2 unknowns and only one equation.

38. **(c)** $f(3)$ will be remained in both cases.
$\therefore a(3)^3 + 4(3)^3 + 3(3) - 4 = (3)^3 - 4(3) + a$
$\Rightarrow 27a + 36 + 9 - 4 = 27 - 12 + a$
$\Rightarrow\ 26a = 15 - 5 - 36$
$\Rightarrow\ 26a = -26$
$\Rightarrow\ a = -1$

39. **(d)** Putting $x = 1$ in the expression, we have
$(3x - 1)^7 = a_7 + a_6 + a_5 \ldots\ldots + a_1 + a_0$
$= (2)^7 = a_7 + a_6 + a_5 \ldots\ldots + a_1 + a_0$
$\Rightarrow a_7 + a_6 + a_5 \ldots\ldots + a_1 + a_0 = 2^7 = 128$

40. **(a)** $(x - (-1))$ will be a factor of $x^{140} + 2x^{151} + k$, then $f(-1) = 0$
$\Rightarrow\ (-1)^{140} + 2(-1)^{151} + k = 0$
$\Rightarrow\ 1 - 2 + k = 0$
$\Rightarrow\ k = 1$

41. **(c)** If $x = 1$ is a factor of $f(x)$, then $f(1) = 0$

$\Rightarrow \quad = f(1) = 4(1)^3 + 3(1)^2 - 4(1) + k = 0$

$= 4 + 3 - 4 + k = 0$

$\Rightarrow \quad k = -3$

42. **(b)** Here $f(-2) = 0$

{ as $x + 2$ is a factor of $f(x)$}

$\therefore \quad a(-2)^3 + b(-2)^2 + (-2) - 6 = 0$

$\Rightarrow \quad -8a + 4b - 8 = 0$

$\Rightarrow -2a + b(-2)^2 + (-2) = 0 \quad$ ...(i)

Remainder when $f(x)$ is divided by $x - 2$, is $f(2)$

$a(2)^3 + b(2)^2 + (-2) - 6 = 4$

$\Rightarrow \quad 8a + 4b = 8 \quad$ ...(ii)

Adding eq (i) and eq (ii), we get

$b = 2\ a = 0$

$\therefore$ (0, 2) will be the solution.

43. **(c)** $a - (-3)$ is a factor of $f(x)$

$\therefore \quad f(-3) = 0$

$\Rightarrow \quad a(-3)^2 + 18 = 0$

$\Rightarrow \quad a = -2$

44. **(b) Method (i)**

$x^2 + 2x - 3 = 0$

$\Rightarrow \quad x^2 + 3x - x - 3 = 0$

$\Rightarrow \quad (x + 3)(x - 1) = 0$

$\Rightarrow \quad x = -3, 1$

To make the expression completely divisible by $(x^2 + 2x - 3)$, it should have factors -3 and 1

Condition (i) : Considering -3 as a factor

$x^4 + 2x^3 - 2x^2 + x - 1 + x = f(x)$

$\Rightarrow \quad f(-3) = 0$

$\Rightarrow (-3)^4 + 2(-3)^3 - 2(-3)^2 + (-3) - 1 + x = 0$

$\Rightarrow \quad 81 - 54 - 18 - 4 + x = 0$

$\Rightarrow \quad 27 - 22 + 2 = 0$

$\Rightarrow \quad x = -5$

Condition (ii) : Considering 1 as a factor

$1^4 + 2(1)^3 - 2(1)^2 + 1 - 1 + x = f(1) = 0$

$\Rightarrow \quad 1 + 2 - 2 + 1 - 1 + x = 0$

$\Rightarrow \quad x = -1$

Both the conditions of x satisfies the equation,

$\therefore$ $(x - 2)$ will be the correct choice.

Method (ii) : Division Method

$$
\begin{array}{r|l}
 & x^2 + 1 \\
x^2 + 2x - 3 & x^4 + 2x^3 - 2x^2 + x - 1 \\
 & x^4 + 2x^3 - 3x^2 \\
 & -\quad -\quad + \\
\hline
 & x^2 + x - 1 \\
 & x^2 + 2x - 3 \\
 & -\quad -\quad + \\
\hline
 & -x + 2
\end{array}
$$

$\because$ Remainder is $(-x + 2)$ i.e., $(-x + 2)$, $(-x + 2)$ will be subtracted from

$x^4 + 2x^3 - 2x^2 + x - 1$ to make it completely divisible by

$x^2 + 2x - 3$,

$\because$ Added quantity $= (x - 2)$

45. **(c)**

$$
\begin{array}{r|l}
 & 3x + 5 \\
x^2 + x + 1 & 3x^3 + 8x^2 + 8x + 3 \\
 & 3x^3 + 3x^2 + 3x \\
 & -\quad -\quad - \\
\hline
 & 5x^2 + 5x + 3 \\
 & 5x^2 + 5x + 5 \\
 & -\quad -\quad - \\
\hline
 & -2
\end{array}
$$

If -2 is subtracted of 2 is added to the dividend, then it will become exactly divisible by $(x^2 + x + 1)$.

$\therefore$ Added quantity $= 5k = 2$

$\Rightarrow \quad k = \dfrac{2}{5}$

3. Co-ordinate Geometry

Learning Objective:

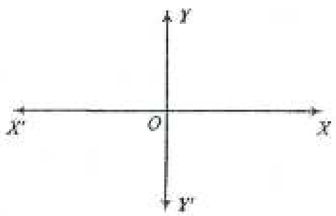
In this chapter, we shall learn about:

*Cartesian Co-ordinate Axes
*Quadrants
*Cartesian Co-ordinates of a Point
*Convention of Signs

Cartesian Co-ordinate Axes

Let $X'OX$ and $Y'OY$ be two mutually perpendicular lines through a point O in the plane of a graph paper. The line $X'OX$ is called the x-axis and the line $Y'OY$ is called the y-axis.

The point of intersection of $X'OX$ and $Y'OY$, i.e, O is called the origin.



The axes are together called coordinate axes.

Quadrants

The coordinate axes (XOX' and YOY') divide the plane into four parts called quadrants.

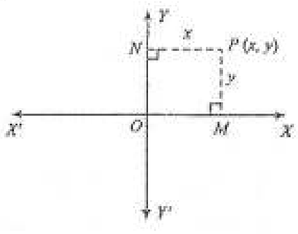
Ist	quadrant $\rightarrow$	XOY
IInd	quadrant $\rightarrow$	$X'OY$
IIIrd	quadrant $\rightarrow$	$X'OY'$
IVth	quadrant $\rightarrow$	XOY'

Cartesian Co-ordinates of a Point

For a given point P, the distance y is called ordinate and the distance x is called the abscissa.

The coordinates of M are $(x, 0)$ and the coordinates of N are $(0, y)$.

The coordinates of origin are $(0, 0)$.



Convention of Signs

Sign convention for ordinates for quadrants are :

Ist	Quadrant → (+)	IInd	Quadrant → (–)
IIIrd	Quadrant → (–)	IVth	Quadrant → (+)

Sign convention for abscissa for quadrants are :

Ist	Quadrant → (+)	IInd	Quadrant → (+)
IIIrd	Quadrant → (–)	IVth	Quadrant → (–)

Therefore, the combined sign convention is (+, +), (+, –) (–, –) (+, –) for 1st, 2nd, 3rd and 4th quadrant respectively.

Example 1: The point (– 3, –5) will lie in which quadrant ?

Solution: Sign convention of point is (–, –)

∴ point will lie in 3rd quadrant.

Example 2: The point (4, – 2) will lie in ....

Solution: Sign convention is (+, –)

∴ Point will lie in the 4th quadrant.

Example 3: The abscissa of any point on y–axis is ....

Solution: Zero

Example 4: The perpendicular distance of (–3, 4) from y-axis will be .... units.

Solution: Perpendicular distance from y-axis = |abscissa| = |–3| = 3 units.

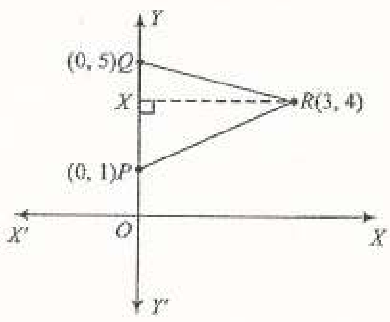
Example 5: The distance of (12,5) from origin is ------- units.

Solution: Distance of origin from (12, 5) = $\sqrt{(12-0)^2+(5-0)^2} = \sqrt{12^2+5^2}$

$$= \sqrt{144+25} = \sqrt{169} = 13 \text{ units.}$$

Example 6: The area of Δ formed by $P(0, 1)$, $Q(0,5)$ and $R(3, 4)$ is .... square units.

Solution: Drop a perpendicular RX on y-axis intersecting y-axis at x.



$$RX = |\text{abscissa}| = |3| = 3 \text{ units}$$

$$PQ = \sqrt{(5-1)^2} = \sqrt{4^2} = 4 \text{ units.}$$

$$\therefore \text{Area } (\Delta PQR) = \frac{1}{2} \times PQ \times RX$$

$$= \frac{1}{2} \times 4 \times 3 = 6 \text{ Square units.}$$

Example 7: The perpendicular distance of point (7, – 5) from x –axis is .... units.

Solution: Distance from x-axis = |ordinate| = |–5| = 5 units.

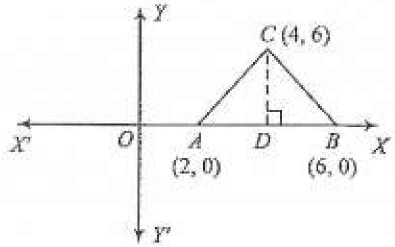
Example 8: Find the area of the triangle formed by $A(2, 0)$, $B(6, 0)$ and $C(4, 6)$.

Solution: CD = Perpendicular distance of C from x-axis

$= |\text{ordinate}| = 6$ units

$AB = \sqrt{(6-2)^2 + 0^2} = 4$ units,

$\therefore \text{Area} = \frac{1}{2} \times AB \times CD = \frac{1}{2} \times 4 \times 6 = 12$ sq units



Multiple Choice Questions

1. The coordinate axes (x and y-axis) divide the plane, in how many quadrants?
   (a) 4 (b) 3 (c) 8 (d) 16
2. The point of intersection of both the axes is called:
   (a) Ordinate (b) Origin
   (c) Quadrant (d) Abscissa
3. The ordinate of any point on x-axis will be
   (a) 1 (b) 0
   (c) –1 (d) unknown
4. If two points P and Q have same abscissae and different ordinates, then points P and Q will definitely lie on
   (a) Line parallel to x-axis
   (b) x-axis
   (c) y-axis
   (d) Line parallel to y-axis
5. If the abscissa of a point is negative. The point will lie in
   (a) I or III quadrant
   (b) II or III quadrant
   (c) I or IV quadrant
   (d) In III or IV quadrant.
6. Minimum distance of point (4, 6) from x-axis will be
   (a) 4 (b) 6 (c) 8 (d) $\sqrt{52}$
7. If a circle is such that x-axis is tangent to it, if the coordinate of centre of circle is (2, 3), then the point of tangency will have ordinate equal to
   (a) 2 (b) 0
   (c) – 3 (d) – 4
8. A square is constructed parallel to the coordinate axis. If the perimeter of square is equal to 24 units and the coordinate of a vertex is (5, 3) then the point on square which is at a distance 6 units from the point (5, 3) can have ordinate :
   (a) 9, –3, 3 (b) 5, 9,3
   (c) 3, –3, 5 (d) 5,3, –9
9. The perpendicular distance of point (–11, –2) from y-axis will be :
   (a) 11 (b) 2
   (c) $\sqrt{125}$ (d) –11
10. Point $P(4 - 3)$ will lie in :
    (a) I quadrant (b) II quadrant
    (c) III quadrant (d) IV quadrant
11. A point on line $y = 3x + 2$ has equal ordinate and abscissa, then the point will lie in
    (a) I quadrant (b) II quadrant
    (c) III quadrant (d) IV quadrant
12. If two points M and N lie on y-axis, and have same absolute value of abscissa but different signs. If the abscissa of point M is K, then the distance between M and N is equal to :
    (a) $|K|$ (b) $|2K| = 2|K|$
    (c) $2K$ (d) $4|K|$
13. If P, Q and R are the vertices of a triangle and coordinates of points are (0,4) (0,0) and (3, 0) respectively then the perimeter of ΔPQR will be :
    (a) 12 units (b) 10 units
    (c) 5 units (d) 13 units

14. Area of ΔPQR in problem 13 will be :

(a) 12 sq. units (b) 6 sq. units
(c) 5 sq. units (d) 8 sq. units

15. $A(2, 3)$, $B(3, 0)$ and $C(14, 13)$ are vertices of triangle ABC. Then, the centroid of triangle will lie in :

(a) Ist quadrant (b) IInd quadrant
(c) III rd quadrant (d) IVth quadrant

16. Point $(-3, -2)$ will lie in :

(a) 1st (b) 2nd
(c) 3rd (d) 4^{th} quadrant

17. The mirror image of point (4, 3) about x-axis will be

(a) (4, –3) (b) (–4, –3)
(c) (–4, –3) (d) (5, –3)

18. The mirror image of point (–4, –2) about x-axis will lie in :

(a) I st quadrant (b) II nd quadrant
(c) III rd quadrant (d) IV th quadrant

19. A point P on line $2x + 3y = 5$, has equal value of both ordinate and abscissa, then the mirror image of point P about y-axis will be :

(a) (1 , –1) (b) (–1, 1)
(c) (–1, –1) (d) (–2, 1)

20. A point 'A' in Ist quadrant has its coordinate (3, 2) is reflected about x-axis. The image of point A about x-axis is point Q, and, then the point Q is reflected about y-axis The coordinates of final point will be :

(a) (–3, –2) (b) (–3, 2)
(c) (3, –2) (d) (–2, –3)

21. The distance between (12, 5) and origin is

(a) 13 (b) 12 (c) 5 (d) 17

22. The area of triangle formed by the points $A(2, 0)$, $B(6, 0)$, $C(4, 6)$ is

(a) 10 sq. units (b) 6 sq. units
(c) 12 sq. units (d) 24 sq. units

23. Equation of y–axis will be

(a) $y = 0$ (b) $y = x$
(c) $x = 0$ (d) $x = 5$

24. A line which makes 60° with x-axis in anticlockwise sense has equation :

(a) $y-\sqrt{3}x=0$ (b) $y+\sqrt{3}x=0$
(c) $\sqrt{3}y+x=0$ (d) $\sqrt{3}y-x=0$

25. A line makes equal angle with x and y-axis and pass through first quadrant has equation :

(a) $x = 2y$ (b) $x = y$
(c) $x + y = 0$ (d) $x + \sqrt{3}y = 0$

26. The area of triangle formed by points $A(3, 0)$, $B(0, -4)$, $C(0, 4)$ will be (in sq. units)

(a) 6 (b) 12 (c) 24 (d) 10

27. A point P is first reflected and has coordinate (3, 5) point P is first reflected about y-axis, and the reflected point is Q. If O is the origin, then the area of ΔPOQ will be (in sq. units)

(a) 6 (b) 9 (c) 12 (d) 15

28. Distance of point (– 24 ,10) from origin will be

(a) 24 (b) 10 (c) 26 (d) 14

29. Distance between points (24, 10) and (–48, 10) will be

(a) 72 (b) 48 (c) 24 (d) 26

30. A rectangle $PQRS$ is constructed having its sides parallel to coordinate axes, where, P (3, 4), Q(6, 4) R(6, 8), S (3, 8) Then the length of diagonal PR will be

(a) 3 (b) 5 (c) 10 (d) 4

31. The difference between ordinates of point $P(3, -6)$ and $Q(-6, 3)$ is

(a) 9 (b) – 9 (c) 6 (d) –3

32. The point of intersection of lines having equations $x + y = 6$ and $x - y = 2$, is

(a) (4, 2) (b) (2, 4)
(c) (3, 3) (d) (1, 5)

Direction (33 to 35):

(a) Both the Assertion (A) and Reason (R) are true and R is correct explanation of A.
or (b) Both A and R are true but R is not a correct explanation of A.
or (c) A is true R is false
or (d) A is false, R is true

33. **A :** If $a \neq b$ then $(a, b) \neq (b, a)$
R : P (3, 3) lies in 2^{nd} quadrant

34. **A :** Point $(x, 0)$ is on y-axis

    R : Point $(0, 3)$ is on y-axis

35. **A :** Point $(0, -4)$ is on y-axis

    R : Every point on y-axis has coordinates of the form $(0, y)$.

36. A trapezium $ABCD$ has its coordinates:

    $A(3, 0), B(8, 0), C(5, 3), D(6, 3)$

    Its area (in sq units) will be

    (a) 6 (b) 10

    (c) 12 (d) 9

37. A rhombus $PQRS$ has side length equal to 5 units, where, $P(0, 0)$, $Q(6, 0)$, $R(3, 4)$ then the coordinates of S will be :

    (a) $(-3, -4)$ (b) $(-3, 4)$

    (c) $(3, -4)$ (d) $(5, 4)$

38. The area of equilateral triangle whose two vertices are (3, 0) and (4 ,0) will be {in sq. units} :

    (a) $\frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{3}}{2}$

    (c) $\sqrt{3}$ (d) $2\sqrt{3}$

39. A circle has its centre (3, 5) has its point of tangency (3, 0). The area of circle will be (in sq. units)

    (a) 25π (b) 9π (c) 16π (d) 4π

40. The length of diagonal of a square whose two vertices are $P(0, -3)$ and $Q(0, 4)$ is (units)

    (a) $7\sqrt{2}$ (b) $3\sqrt{2}$ (c) $4\sqrt{2}$ (d) $5\sqrt{2}$

Answer Key

1. (a)	2. (b)	3. (b)	4. (d)	5. (b)	6. (b)	7. (b)	8. (a)	9. (a)	10 (d)
11. (c)	12. (a)	13. (a)	14. (b)	15. (a)	16. (c)	17. (a)	18. (b)	19. (b)	20. (a)
21. (a)	22. (c)	23. (c)	24. (a)	25. (b)	26. (b)	27. (d)	28. (c)	29. (a)	30. (b)
31. (b)	32. (a)	33. (c)	34. (d)	35. (a)	36. (d)	37. (c)	38. (a)	39. (a)	40. (a)

Hints and Solutions

1. **(a)** x-axis and y-axis divide the plane in 4 quadrants.

2. **(b)** The point of intersection of both the axes is called origin.

3. **(b)** Every point on x-axis is of the form $(x, 0)$

   $\therefore$ ordinate of any point on x-axis = 0

4. **(d)** If P and Q have same abscissa but different ordinates then P and Q have coordinates, as $P(a, c), Q(a, b)$

   $\because$ x-coordinate is constant.

   $\therefore$ P and Q will lie on line parallel to y-axis.

5. **(b)** Sign convention for Quadrants are

   Ist quadrant $\rightarrow (+, +)$,

   IInd quadrant $\rightarrow (-, +)$

   IIIrd quadrant $\rightarrow (-, -)$,

   IVth quadrant $\rightarrow (+, -)$

   $\therefore$ Required point will lie in II, III quadrant.

6. **(b)** Minimum distance of point (a, b) from x-axis =

   Perpendicular distance between point and x-axis =

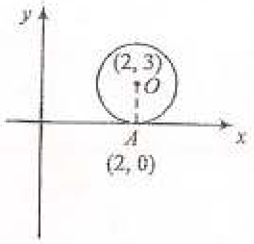
   |y-coordinate (ordinate) of the point|

   $= |6| = 6$

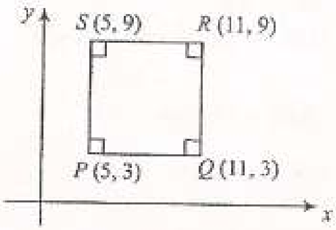
7. **(b)** O is the centre of circle and A is the point of tangency.

   $\because$ Point of tangency lies on x-axis

   $\therefore$ Ordinate of point = 0



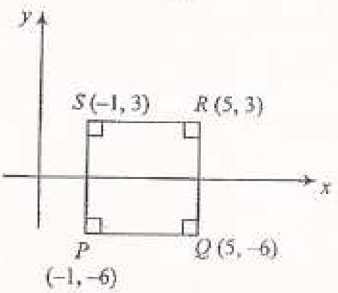
8. **(a)**



Perimeter of square $= 4a = 24 \Rightarrow a = 6$

$\because$ (5, 3) can be any of the vertices

$\therefore$ We will find the ordinates of other three points assuming (5, 3) to be coordinates of *PQR* and *S* respectively.



If, *P* (5, 3) Then *Q* has ordinate = 3

S has coordinates (5, 3 + 6) = (5, 9)

$\therefore$ *R* and *S* have ordinates = 9

If, *Q* (5, 3) Then *P* has ordinate = 3

R has coordinates (5, 9)

$\therefore$ *R* and S have ordinates = 9

Similarly,

If *R* or *S* is conceded as (5,3) then

P and *Q* have ordinates = 3 – 6 = –3

$\therefore$ Possible ordinates are 9, –3 and 3.

9. **(a)** Perpendicular distance from *y*–axis

$$= |\text{abscissa}| = |-11| = 11$$

10. **(d)** Point (4 , 3) have (+, –) sign convention, that belongs to IVth quadrant.

11. **(c)** If abscissa = ordinate, i.e, $x = y$ then using this relation in equation of line, we have

$$x = 3x + 2 \Rightarrow x = -1$$

$\therefore$ Point has coordinates = (–1, –1)

Point has sign convection of (–, –)

$\therefore$ Point will lie in 3[rd] quadrant

12. **(b)** Let the coordinates of point M be (0, *k*)

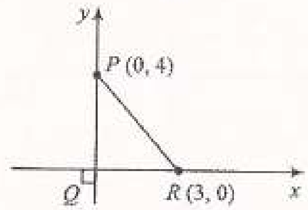
{$\because$ M is on *y*-axis}

$\therefore$ Coordinates of point $N = (0, -K)$

$\therefore$ Distance between point *M* and *N*

$$= |K - (-K)| = 2|K|$$

13. **(a)**



$QP = 4$ units [From the figure]

$QP = 3$ units

$\therefore \quad \angle PQR = 90°$

$\therefore$ In ΔPQR

$PQ^2 + QR^2 = PR^2$ {Pythagoras Theorem}

$\Rightarrow \quad 4^2 + 3^2 = PR^2$

$\Rightarrow PR = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$ units

$\therefore$ Perimeter of $\Delta PQR = PQ + QR + PR$

$= 4 + 3 + 5$

$= 12$ units

14. **(b)** Area of $PQR = \frac{1}{2} \times PQ \times QR$

$$= \frac{1}{2} \times 4 \times 3 = 6 \text{ (units)}^2$$

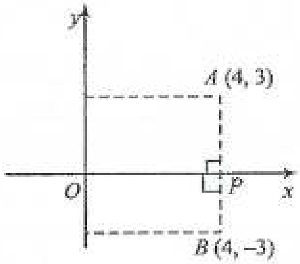
15. **(a)** $\because$ Centroid of any Δ lies within it, and all the coordinates A, B and C are in Ist quadrant (Positive)

$\therefore$ centroid will lie in Ist quadrant.

16. **(c)** $\because$ (–3, –2) has (–, –) sign convention.

$\therefore (-3, -2)/_0$ Belongs to 3[rd] quadrant.

17. **(a)**

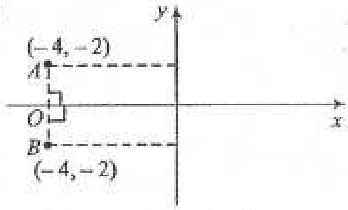


Point A is 3 units above x-axis.

$\therefore$ Its mirror image will be 3 units below x-axis, and the x-coordinate will remain constant

$\therefore$ Coordinates of point $B \equiv (4, -3)$

18. **(b)**



Point A is the reflected point.

Point A will have coordinates, as, abscissa will not change and ordinate will change sign

$\therefore$ Reflected point will lie in 2[nd] quadrant.

19. **(b)** When ordinate = abscissa, then $y = x$

$\therefore \quad 2x + 3x = 5$

$\Rightarrow \quad 5x = 5$

$\Rightarrow \quad x = 1$

$\therefore$ $x = y = 1$ will be point on line having equal abscissa and ordinate

$\therefore$ Point $P = (1, 1)$

$\therefore$ Its image about y-axis will be $(-1, 1)$.

20 **(a)** A has coordinate $\equiv (3, 2)$

Point Q has coordinate $\equiv (3, -2)$

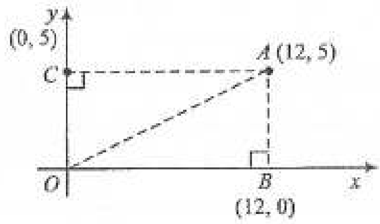
{ordinate will change sign}

Now,

After reflection of point Q about y-axis the ordinate will not vary but abscissa will change sign.

$\therefore$ Coordinates of final point $\equiv (-3, -2)$

21. **(a)**



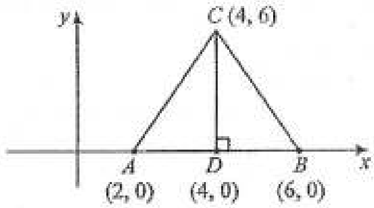
In ΔOAB

$$OA^2 = OB^2 + AB^2$$
$$= (12)^2 + (OC)^2 = (12)^2 + (5)^2$$
$$= 169$$

$\Rightarrow \quad OA = \sqrt{169} = 13$ units

22. **(c)** After plotting figure, it can be clearly seen that ABC is an isosceles triangle in which, $AB = (6 - 2) = 4$ units

$CD = (6 - 0) = 6$ units



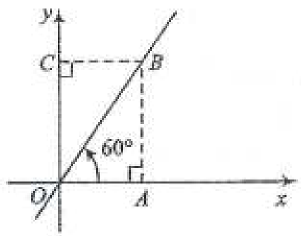
$\therefore$ Ares of $\Delta ABC = \frac{1}{2} \times AB \times CD$

$$= \frac{1}{2} \times 4 \times 6 = 12 \text{ sq. units}$$

23. **(c)** On every point of y-axis, abscissa = 0

$\therefore$ $x = 0$ is the equation of y-axis

24. **(a)** Let a point B on the line



$\therefore$ In ΔOAB,

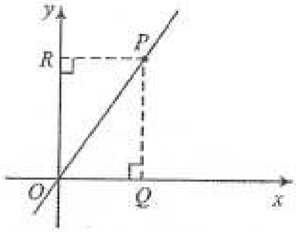
$$\tan 60° = \frac{AB}{OA} = \sqrt{3}$$

$$\Rightarrow \quad \frac{OC}{OA} = \sqrt{3}$$

$\Rightarrow \quad OC = \sqrt{3}\, OA$

$\Rightarrow \quad y = \sqrt{3}x$

25 **(b)** Let a point P on line



$\therefore$ In ΔPOQ,

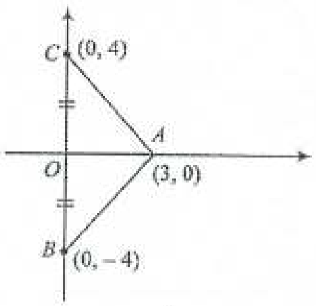
$$\frac{OQ}{PQ} = \tan 45° = 1$$

$\Rightarrow \quad OQ = PQ$

$\Rightarrow \quad OQ = OR$

$\Rightarrow \quad x = y$

26. **(b)** After plotting ΔABC, graphically, it clearly seen that ΔABC is isosceles Δ.

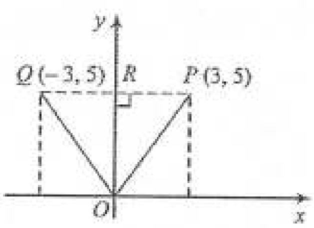


$$\therefore \text{Area of } \Delta ABC = \frac{1}{2} \times AO \times BC$$

$$= \frac{1}{2} \times [3 - 0] \times [4 - (-4)]$$

$$= \frac{1}{2} \times 3 \times 8 = 12 \text{ sq. units}$$

27. **(d)**



Coordinates of point $Q \equiv (-3, 5)$.

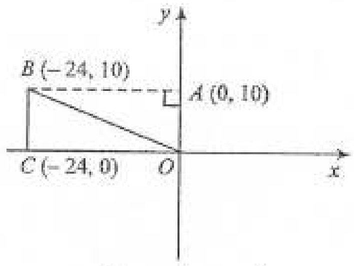
After plotting ΔPOQ on graph, it can be clearly viewed that ΔPOQ is isosceles.

$$\therefore \text{Area of } \Delta POQ = \frac{1}{2} \times OR \times PQ$$

$$= \frac{1}{2} \times 5 \times [3 - (-3)]$$

$$= \frac{1}{2} \times 5 \times 6 = 15 \text{ sq. units}$$

28. **(c)** In ΔOBC



$BC^2 + OC^2 = OB^2$

$\Rightarrow \quad (OA)^2 + (OC)^2 = OB^2$

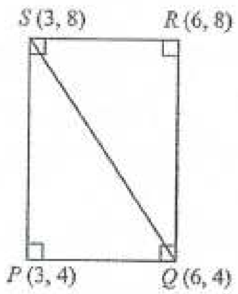
$\Rightarrow \quad (10)^2 + (24)^2 = OB^2$

$\Rightarrow \quad OB = \sqrt{676} = 26$ units

29. **(a)** Distance between (–24, 10) and (48,10) will be equal to the absolute value of difference between the abscissa of the points as, the points have same ordinate.

$\therefore$ Distance $= |48 - (-24)| = 72$ units

30. **(b)** In ΔPSQ



$PQ = (6 - 3) = 3$ units

$PS = (8 - 4) = 4$ units

$PQ^2 + PS^2 = SQ^2$

[Pythagoras Theorem]

$$(3)^2 + (4)^2 = SQ^2$$
$$\Rightarrow \quad SQ = \sqrt{25} = 5 \text{ units}$$

31. **(b)** Ordinates of points are –6 and 3
$\therefore$ Difference $= -6 - 3 = -9$

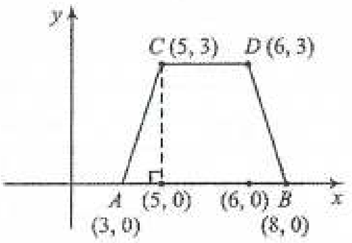
32. **(a)** Let the coordinates of point of intersection be (x_1, y_1)
$$\therefore \quad x_1 + y_1 = 6$$
$$x_1 - y_1 = 2$$
Solving these two equations, we get
$$x_1 = 4, \text{ and }, y_1 = 2$$
$\therefore$ point of intersection $\equiv (4, 2)$

33. **(b)** A is correct and R is also correct because the sign convention is (+, +).

34. **(d)** Any point on y–axis has coordinates of the form $(0, y)$
$\therefore$ A is false.

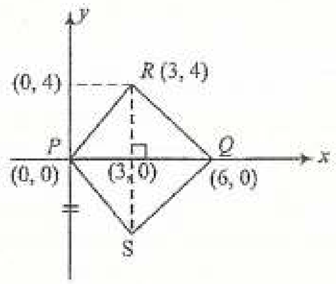
35. **(a)** Both A and R are correct and R is the correct explanation of A.

36. **(d)** $CD \equiv (6 - 5) = 1$ unit
$AB = (8 - 3) = 5$ units
$AC = (3 - 0) = 3$ units



$$\therefore \text{Area of trapezium} = \frac{1}{2} \times (AB + CD) \times AC$$
$$= \frac{1}{2} \times (5 + 1) \times 3$$
$$= \frac{1}{2} \times 6 \times 3 = 9 \text{ sq. units}$$

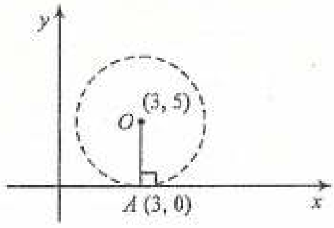
37. **(c)**



Coordinates of point S are the reflection of point S, about x-axis
$\therefore$ Coordinates of point $S \equiv (3, -4)$

38. **(a)** The side length of triangle
$$= (4 - 3) = 1 \text{ unit}$$
$$\therefore \text{Area} = \frac{\sqrt{3}a^2}{4} = \frac{\sqrt{3} \times (1)^2}{4} = \frac{\sqrt{3}}{4} \text{ sq units.}$$

39. **(a)**



Radius of the circle $= (5 - 0) = 5$ units
[$\because$ abscissa of O and A are same]
$$\therefore \text{Area of circle} = \pi(r)^2$$
$$= \pi(5)^2$$
$$= 25\pi \text{ sq. units.}$$

40. **(a)** $\because$ P and Q have same abscissa
$\therefore$ Distance, $PQ = [4 - (-3)] = 7$
$\therefore$ Diagonal length of square $= a\sqrt{2}$
$$= 7\sqrt{2} \text{ units}$$

4. Linear Equations in Two Variables

Learning Objective:

While solving the problems, in most cases first we have to frame an equation.
In this chapter we shall learn about:
*Linear equation
*How to solve linear equation
*Graph of Linear equation

Linear Equation

An equation in which the highest index of the unknown present is one is a linear equation, $x + y = 10$, $2x - y = 5$ are some linear equation.

Linear Equation in Two Variables

An equation of the form $ax + by + c = 0$, where a, b, c are real numbers, $a \neq 0$, $b \neq 0$ and x, y are variables is called a linear equation in two variables. The equation is said to be linear because the degree of polynomial is one.

Example: $x + 3y = 5$, $2x + 5y = 6$, $\frac{2}{3}x + \sqrt{3}y = 9$, etc.

Write the equations in the form of $(ax + by + c)$

Example 1: $x = 5y$

Solution: $x - 5y + 0 = 0$

$a = 1, b = -5, c = 0$

Example 2: $2y + 3 = \sqrt{3}x$

Solution: $-\sqrt{3}x + 2y + 3 = 0$

$a = -\sqrt{3}, b = 2, c = 3$

Example 3: The cost of a notebook is thrice the cost of a pencil. Write a linear equation to represent this statement.

Solution: Let the cost of a pencil be x, and cost of a notebook be y.

According to the question

$$y = 3x$$

$\Rightarrow -3x + y = 0$ is the required equation.

Solution of a Linear Equation

Let $ax + by + c = 0$ is a equation in x and y, where $a \neq 0$, $b \neq 0$, Then, any pair of values of x and y, satisfying the equation is called a solution of $ax + by + c = 0$. A linear equation in two variables has infinitely many solutions.

Example 4: If $x = 2k - 1$ and $y = k$ is a solution of the equation $3x + 2y = 5$, then the value of k will be :

Solution:

$$3(2k - 1) + 2k = 5$$

$$\Rightarrow \quad 6k - 3 + 2k = 5$$

$$\Rightarrow \quad 8k = 8 \Rightarrow k = 1.$$

Example 5: Find the value of a, if $x = -a$ and $y = \frac{5}{2}$ is a solution of the equation $x + 4y - 7 = 0$.

Solution: Given $x + 4y = 7$

$$\Rightarrow \quad -a + 4 \times \frac{5}{2} = 7 \quad \Rightarrow -a + 10 = 7$$

$$\Rightarrow \quad -a = 7 - 10 = -3 \quad \Rightarrow \quad a = 3$$

Example 6: If $x = 6$ and $y = 1$ satisfies the equation $8y + a^2 - ax = 0$, then find the value of a.

Solution: The given equation is $8y + a^2 = ax$

$$\Rightarrow \quad 8 + a^2 = 6a$$

$$\Rightarrow \quad a^2 - 6a + 8 = 0$$

$$\Rightarrow \quad a^2 - 4a - 2a + 8 = 0$$

$$\Rightarrow \quad (a - 4)(a - 2) = 0$$

$$\Rightarrow \quad a = 4 \text{ or } 2.$$

Graph of Linear Equation

To draw a graph of linear equation. write the equation as $ax + by + c =$ and, express, $y = \left(\frac{-c - ax}{b}\right)$, then, give any two values of x and determine the corresponding values of y. Plot the values of the pairs (x, y) and join the points to obtain the graph of linear equation.

Note:

1. The graph of x = constant will lie parallel to y-axis.
2. The graph of y = constant will be parallel to x-axis.
3. The abscissa of every point on y-axis is zero.
4. The ordinate of any point on x-axis is zero.

Example 7: A number is 27 more than the number obtained by reversing its digits. If one of the digits is 3, obtain the other digit.

Solution: Let the units place digit be x, and then tenths place digit be y.

$\therefore$ Number $= 10y + x$, Number after revising digits $= 10x + y$.

$\therefore$ Difference $= (10y + x) - (10x + y)$

$$= 9(y - x) = 27$$

$$\Rightarrow \quad y - x = 3$$

$$\Rightarrow \quad y = x + 3$$

$$\Rightarrow \quad y = x + y = 3 + 3 = 6$$

$\therefore$ Other digit $= 6$.

Example 8: Write 2 solutions of the equation $x + 3y = -2$.

Solution: $y = \left(\frac{-2-x}{3}\right)$,

Now, putting $x = 2, y = \frac{-4}{3}$ and, $x = 0, y = \frac{-2}{3}$

Example 9: Write the equation of line parallel to x-axis and passing through $(-2, -3)$.

Solution: Equation of line parallel to x-axis

$\Rightarrow$ $y = -3$.

$\Rightarrow$ $y + 3 = 0$

Example 10: The area bounded by the graph of equation $5x + 12y = 60$, and the axes.

Solution:

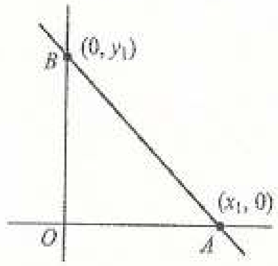
Coordinates of $A \Rightarrow 5x_1 = 60, \Rightarrow x_1 = 12. \Rightarrow A\ (12, 0)$

Coordinates of $B \Rightarrow 12y_1 = 60, \Rightarrow y_1 = 5. \Rightarrow B\ (0, 5)$

$\therefore$ Area of $\Delta OAB = \frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 12 \times 5$

$= 30$ sq. units.



Multiple Choice Questions

1. The cost of a chair is half of the cost of a dining table. The linear equation representation of the above will be :
   (a) $x = 2y$ (b) $3x = 4y$
   (c) $2x + 3y - 2 = 0$ (d) $x = 4y$
2. Which of the following are the solutions of the equation $2x + 3y = 13$?
   (a) (4 , 2) (b) (2, 3)
   (c) (2 , 2) (d) (3, 3)
3. The value of k, if, (3, 2) is a solution of equation $4x + y = k$ is :
   (a) 16 (b) −14 (c) 14 (d) 12
4. If $2x + 16y = 13$ and $x + y = p$, have same set of solution, then the possible value of p is (are) :
   (a) 1 (b) 0
   (c) 2 (d) All of the above
5. If $a^2x + ay = 3$, is satisfied by $x = 1, y = 2$, then the value of a will be :
   (a) −1, 3 (b) 1, − 3 (c) 2, − 3 (d) 3, − 2
6. If $x = k^2$ and $y = k$ are solutions of equation $x - 5y = -6$ then $k =$
   (a) 2, 3 (b) 3, −2 (c) −3, 2 (d) −2, −3
7. The equation $x - y + 1 = 0$ is satisfied by $x = a^2$ and $y = a$ then $a =$
   (a) Can't be determined
   (b) 2
   (c) −1
   (d) −2
8. The solution of equation $x - y + 8 = 0$ is $x = k^3$ and $y = 0$, then $k =$
   (a) 2 (b) −2 (c) −3 (d) $-\frac{1}{2}$
9. If the equation $x + 3y + 4k = 6$ is satisfied by (2, 3) then the value of k is :
   (a) $\frac{5}{4}$ (b) $-\frac{5}{4}$ (c) $\frac{3}{4}$ (d) $-\frac{3}{4}$

10. If the equation $(x + 3y) - (3x + y) + (x - y) = (a - b)$, then which of the following is a solution of the above equation ?
(a) (a, b) (b) (b, a)
(c) $(-b, -a)$ (d) $(b, -a)$

11. If the equation $k(x^3 - y^3) = (x^2 + y^2 + xy)$ and $y = \frac{1}{k}$, then the value of x is
(a) $\frac{1}{2k}$ (b) $-\frac{1}{2k}$ (c) $\frac{2}{k}$ (d) $-\frac{2}{k}$

12. If the equation, $x - y + \left(\sqrt{x} + \sqrt{y}\right) = 10$ and the value of x is 9, then value of y will be,
(a) 4 (b) – 4
(c) 9 (d) 16

13. The equation, $(x + y) + \left(x^{\frac{2}{3}} - y^{\frac{2}{3}} - (xy)^{\frac{1}{3}}\right) = 12$, then, u = ?? if $x = 8$
(a) 2 (b) 1 (c) –8 (d) 27

14. If $(2k - 3k)$ is a solution of the equation $6x + 2y = k - 5$, then $k =$
(a) –1 (b) –2 (c) 1 (d) 2

15. Arun and Kajol together contributed 100 rupees for the Prime Minister Relief fund. If the money donated by Arun is ₹ 80 less than twice the money donated by Kajol then the money donated by Arun is :
(a) ₹ 40 (b) ₹ 60 (c) ₹ 80 (d) ₹ 20

16. A number is 27 more than the number obtained by revising its digits. If one of the digits is 3, then the other digit is
(a) 5 (b) 6 (c) 3 (d) 9

17. If the point (4, 5) lies on the graph $3y = ax + 3$, then $a =$
(a) 2 (b) 3 (c) – 3 (d) 4

18. If the point $A(3, 5)$ and $B(1, 4)$ lie on the graph of line $ax + by - 7 = 0$, then (a, b) will be :
(a) (1, 2) (b) (1,–2) (c) (–1, 2) (d) (–1,–2)

19. If $C = \frac{(F-32)\times 5}{9}$, where C denotes the temperature in Celsius and F denotes the temperature in Fahrenheit, The temperature (in Celsius) at which the numerical value on the both scales is same will be
(a) –30°C (b) –20°C
(c) –40°C (d) –80°C

20. The area bounded by the graph of the equation $\frac{x}{4} + \frac{y}{5} = 1$ the coordinate axes will be
(a) 20 sq. units (b) 10 sq. units
(c) 5 sq. units (d) 15 sq. units

21. The point of intersection of graphs of the equations $3x + 4y = 12$ and $6x + 8y = 48$ is
(a) (3, 4)
(b) (4, 3)
(c) (5, 3)
(d) The graphs will not intersect

22. The point of intersection of $3x + 4y = 15$ and x-axis will be
(a) (0, 5) (b) (5, 0)
(c) (–5, 0) (d) (0, 3)

23. The graph of the equation $15x + 36y = 108$ will cut the y- axis at :
(a) (0, –3) (b) (0, 5)
(c) (0, 6) (d) (0, 3)

24. The distance between the graphs of the equations $x = 3$ and $x = -3$ is
(a) 5 (b) 8
(c) 6 (d) 4

25. The equation $3x + 2y = 8$ has :
(a) Unique solution (b) No solution
(c) Infinite solutions (d) Two solutions

26. The equation of the parallel to x-axes and passing though the point (3, –4) will be :
(a) $y = 3$ (b) $x = 3$
(c) $x = -4$ (d) $y = -4$

27. The graph $4x + 3y = 12$ cuts the coordinate axes at A and B
(a) 7 (b) 4
(c) 12 (d) 24

28. If $(a^2, 3a)$ lies on the graph of the equation $x - 4y + 32 = 0$ then, $a =$

(a) 2, 3 (b) 2, 4
(c) 4, 8 (d) 4, 12

29. The equation $3x = 9$ is pitied on graph paper, then which point lies on the graph?
(a) $(-3, -2)$ (b) $(-3, 9)$
(c) $(-3, 3)$ (d) $(3, 9)$

30. The area of triangle whose vertices are $A(0, 3)$, $B(0, 7)$ and $C(4, 5)$ is
(a) 8 sq. unit (b) 4 sq. unit
(c) 6 sq. unit (d) 9 sq. unit

31. The monthly incomes of A and B are in the ratio 8 : 7 and their expedites are in the ratio 19 : 16 If the savings of both A and B is ₹ 2500, then the month income of A is
(a) ₹ 10500 (b) ₹ 5000
(c) ₹ 10000 (d) ₹ 12000

32. A man's age is 3 times the sum of the ages of his 2 sons after 5 years, His age will be twice the sum of ages of his 2 sons. The age of man (in years) will be :
(a) 30 (b) 40
(c) 45 (d) 49

33. The graph of equation $4x + 3y = 12$, intersects the x-and y axes at A and B respectively. If O is origin then area of Δ is
(a) 12 (b) 24 (c) 6 (d) 9

34. In a ΔABC, $\angle C = 3$, $\angle B = 2(\angle A + \angle B)$, then $\angle C =$
(a) 50° (b) 60°
(c)120° (d) 90°

35. Krishna and Kansh walked on a straight rood. If Kansh took 3 hours more than Krishna to walk 30km. If Kansh doubles his speed, he is ahead of Krishna by $\frac{3}{2}$ less. The speed of walking of Krishna will be :
(a) 5 km/h (b) 7 km/h
(c) 5.5 km/h (d) 7.5 km/h

Answer Key

1. (a)	2. (b)	3. (c)	4. (d)	5. (b)	6. (a)	7. (a)	8. (d)	9. (b)	10. (b)
11. (c)	12. (a)	13. (b)	14. (c)	15. (a)	16. (b)	17. (b)	18. (c)	19. (c)	20. (b)
21. (d)	22. (b)	23. (d)	24. (c)	25. (c)	26. (d)	27. (c)	28. (c)	29. (d)	30. (a)
31. (d)	32. (c)	33. (c)	34. (c)	35. (d)					

Hints and Solutions

1.**(a)** Let the cost of a chair be ₹ y and cost of dining table be ₹ x

According to the question

$$y = \frac{x}{2}$$

$$\Rightarrow \quad x = 2y$$

2. **(b)** The given linear equation is $2x + 3y = 13$

Now substituting the values of x and y from option in equation (i), we see

For (a) $2 \times 4 + 3 \times 2 = 8 + 6 = 14 \neq 13$

$\therefore$ (a) is not correct option.

Again $2 \times 2 + 3 \times 3 = 4 + 9 = 13 = 13$

$\therefore$ (b) is required answer.

3. **(c)** Given $4x + y = k$

$\because$ (3, 2) is a solution of above equation

$\therefore$ (3, 2) will satisfy the above equation

$\therefore \; 4 \times 3 + 2 = 12 + 2 = 14 = k$

4. **(d)** Given equations are

$3x + 16y = 13$, and , $x + y = p$

These equations may have many set of solutions commons for different values of p.

5. **(b)** Here (1, 2) $a^2x + ay - 3 = 0$

then $a^2(1) + a(2) - 3 = 0$

$\Rightarrow a^2 + 2a - 3 = 0$

$\Rightarrow a^2 + 3a - a - 3 = 0$

$\Rightarrow a(a+3) - 1(a+3) = 0$

$\Rightarrow (a+3)(a-1) = 0$

$\Rightarrow a + 3 = 0$, or $a - 1 = 0$

$a = -3$, or $a = 1$

6. **(a)** (k, k) will satisfy $x - 5y + 6 = 0$

$\Rightarrow k^2 - 5k + 6 = 0$

$\Rightarrow k^2 - 3k - 2k + 6 = 0$

$\Rightarrow k(k-3) - 2(k-3) = 0$

$\Rightarrow (k-3)(k-2) = 0$

$\Rightarrow k - 2 = 0$ or, $k - 3 = 0$

$\Rightarrow k = 2$ or 3

7. **(a)** (a^2, a) is a solution of the equation

$x - y + 1 = 0$

$\Rightarrow a^2 - a + 1 = 0$

$\Rightarrow$ The above equation has negative determined.

$\therefore$ value of a cannot be determined

8. **(b)** $(k^3, 0)$ satisfies the equation, $x - y + 8 = 0$

$\Rightarrow k^3 - (0) + 8 = 0$

$\Rightarrow k^3 = -8$

$\Rightarrow k = (-8)^{\frac{1}{3}} = -2$

9. **(b)** (2, 3) satisfies the equation $x + 3y + 4k = 6$, then ,

$2 + 3(3) + 4k = 6$

$\Rightarrow 2 + 9 + 4k = 6$

$\Rightarrow 4k = -5$

$\Rightarrow k = \frac{-5}{4}$

10. **(b)** $x + 3y - 3x - y + x - y = a - b$

$\Rightarrow -x + y = a + b$

$\Rightarrow y - x = a - 6$

$\therefore (x, y)$ is satisfied by (b, a)

11. **(c)** $k(x^3 - y^3) = x^2 + y^2 + xy$

$\Rightarrow k = \frac{(x^2 + y^2 + xy)}{(x-y)(x^2 + xy + y^2)}$

$\Rightarrow x - y = \frac{1}{k} \Rightarrow x = \frac{1}{k} + y = \frac{1}{k} + \frac{1}{k} = \frac{2}{k}$

12. **(a)** $x - y = \left(\sqrt{x}\right)^2 - \left(\sqrt{y}\right)^2$

$= \left(\sqrt{x} + \sqrt{y}\right) \times \left(\sqrt{x} - \sqrt{y}\right)$

$\Rightarrow x - y + \left(\sqrt{x} + \sqrt{y}\right) = \left(\sqrt{x} + \sqrt{y}\right)\left\{\sqrt{x} - \sqrt{y} + 1\right\}$

$= \left(\sqrt{x} + \sqrt{y}\right)\left(\sqrt{x} - \sqrt{y} + 1\right)$

According to the question

$\left(\sqrt{x} + \sqrt{y}\right)\left(\sqrt{x} - \sqrt{y} + 1\right) = 10$

$\Rightarrow \left(\sqrt{9} + \sqrt{y}\right)\left(\sqrt{9} - \sqrt{y} + 1\right) = 10$

$\Rightarrow \left(\sqrt{y} + 3\right)\left(3 - \sqrt{y} + 1\right) = 10$

$\Rightarrow \left(\sqrt{y} + 3\right)\left(4 - \sqrt{y}\right) = 10$

Let $\sqrt{y} = p$

$\Rightarrow (p+3)(4-p) = 10$

$\Rightarrow p = 2$

$\therefore y = p^2 = 4$

13. **(a)** Here $x + y = \left(x^{\frac{1}{3}}\right)^3 + \left(y^{\frac{1}{3}}\right)^3$

$= \left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right)\left(x^{\frac{2}{3}} + y^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}}\right)$

$\Rightarrow$ The equation will be reduced to,

$\left(x^{\frac{1}{3}} + y^{\frac{1}{3}} + 1\right)\left(x^{\frac{2}{3}} + y^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}}\right) = 12$

$\Rightarrow \left(3 + y^{\frac{1}{3}}\right)\left(4 + y^{\frac{2}{3}} - 2y^{\frac{1}{3}}\right) = 12$

$\Rightarrow \quad 12 + y - 2y^{\frac{2}{3}} + 3y^{\frac{2}{3}} + 4y^{\frac{1}{3}} 6y^{\frac{1}{3}} = 12$

$\Rightarrow \quad y + y^{\frac{2}{3}} - 2y^{\frac{1}{3}} = 0$

$\Rightarrow \quad y^{\frac{2}{3}} - y^{\frac{1}{3}} - 2 = 0$

Let ,

$y^{\frac{1}{3}} = k$

$\therefore \quad k^2 + k - 2 = 0$

$\Rightarrow \quad k = 1$

14. **(c)** Here $(2k - 3, k)$ satisfies the equation

$6x + 2y = k - 5$

$\therefore \quad 6(2k - 3) + 2k = k - 5$

$\Rightarrow 12k - 18 + 2k - k + 5 = 0$

$\Rightarrow \quad 13k = 13$

$\Rightarrow \quad k = 1$

15 **(a)** Let the amount donated by Kajol be ₹ x

$\therefore$ Amount donated by Arun = ₹ $(2x-80)$

According to the question

$x + 2x - 80 = 100$

$\Rightarrow \quad 3x = 180$

$\Rightarrow \quad x = 60$

$\therefore$ Money donated by Arun = ₹ $(2 \times 60 - 80)$

= ₹ 40

16. **(b)** Let the unit's place digit be x,

and tens place digit be y,

$\therefore$ Number = $10y + x$

The new number of ten reversing the digits

$= 10x + y$

$\therefore$ Difference = $(10y + x) - (10x + y)$

$= 9y - 9x = 9(y - x)$

According to the question

$9(y - x) = 27$

$\Rightarrow \quad y - x = 3$

$\Rightarrow \quad y - 3 = 3$

$\Rightarrow \quad y = 6$

$\therefore$ If one of the digit is 3, then other is 6.

17. **(b)** Point (4,5) lies on the graph of the equation $3y = ax + 3$

$\therefore \quad 3 \times 5 = 4a + 3$

$\Rightarrow \quad 4a = 12 \Rightarrow a = 3$

18. **(c)** $A(3, 5)$ and $B(1, 4)$ lie on graph of line

$ax + by = 7$

$\therefore \quad 3a + 5b = 7 \quad$ ...(i)

$a + 4b = 7 \quad$ ...(ii)

$\therefore$ From equations (i) and (ii), we get

$b = 2, a = -1$

$\therefore (a, b) \quad (-1, 2)$

19. **(c)** Let the numerical value of temperature be x

$\therefore \quad x = \frac{(x-32)\times 5}{9}$

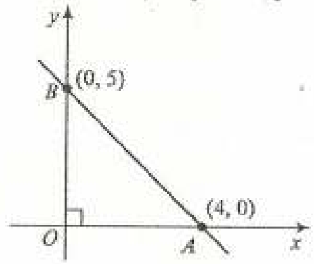
$\Rightarrow \quad 9x = 5x - 160$

$\Rightarrow \quad 4x = -160$

$\Rightarrow \quad x = -40$

$\therefore$ the temperature is equal in both the scales at $-40°C$.

20. **(b)** The given curve intersect the x and y-axes at $A(4, 0)$ and $B(0, 5)$ respectively then



Area of $\Delta OAB = \frac{1}{2}\Delta OAB = \frac{1}{2} \times 4 \times 5$

= 10 sq. units

21. **(d)** Let the point of intersection of lines be (a, b).

$\therefore \quad 3a + 4b = 12$, and

$6a + 8b = 48$

The above two equations have no solutions for (a, b)

$\therefore$ The graph will not intersect.

22. **(b)** $\therefore$ The ordinate of every point on x-axis = 0

∴ The line $3x + 4y = 15$ and the x-axis will intersect where value y of the line becomes zero

∴ $3x = 15$

⇒ $x = 5$

∴ The point of intersection is (5,0)

23. **(d)** At y – axis, ordinate ≠ 0 abscissa = 0

∴ $x = 0$

⇒ $36y = 108$

⇒ $y = 3$

∴ point of intersection = (0,3)

24. **(c)** The distance between the graphs

$= 3 - (-3) = 3 + 3 = 6$ units

25. **(c)** The equation can be written as,

$$y = \frac{8 - 3x}{2}$$

∴ For different values of x, different values of y will exist.

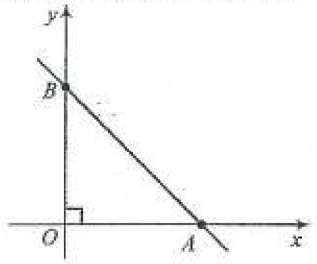
∴ The above equation has many solutions.

26. **(d)** Equation of line parallel to x-axis, will be of the form y = constant.

∴ Desired equation of line

$y = -4$

27. **(c)** The given curve is $4x + 3y = 12$ ...(i)



For point A,

Put $y = 0$ in (i)

∴ $4x = 12$

⇒ $x = 3$

For point B, putting $x = 0$ in (i)

$3y = 12$

⇒ $y = 3$

∴ $A(3, 0)$, $B(0, 4)$ and $O(0, 0)$ are the 3 vertices of ΔOAB.

ΔOAB is a right – angled triangle.

∴ $AB = \sqrt{OA^2 + OB^2}$ [Pythagoras theorem]

$= \sqrt{3^2 + 4^2}$

$= \sqrt{25} = 5$ units

28. **(c)** Here $(a^2, 3a)$ will satisfy the equation

$x - 4y + 32 = 0$

∴ $a^2 - 4(3a) + 32 = 0$

⇒ $a^2 - 12a + 32 = 0$

⇒ $a^2 - 8a - 4a + 32 = 0$

⇒ $a(a - 8) - 4(a - 8) = 0$

⇒ $(a - 4)(a - 8) = 0$

⇒ $a = 4$ or 8

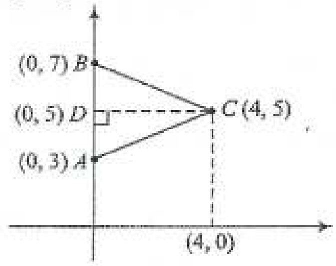
29. **(d)** Given $3x = 9$

⇒ $x = \frac{9}{3} = 3$

∴ Line is parallel to y-axis and passes through $x = 3$

∴ Point (3, 9) will lie on $3x = 9$

30. **(a)** As per question



∴ Area of $\Delta ABC = \frac{1}{2} \times AB \times CD$

$= \frac{1}{2} \times 4 \times 4$

$= 8$ sq. units

31. **(d)** Income of $A = 8x$, Income of $B = 7x$

Expenditure of $A = 19y$

Expenditure of $B = 16y$

According to the question

$8x - 19y = 2500$

⇒ $7x - 16y\ 2500$

⇒ From these two equations,

We have,

$x = 1500, y = 500$

$\therefore$ Income of A = ₹ 8×1500

= ₹ 12000

32. **(c)** Let the sum of ages of two sons be x, and their father's age = y years

According to the question

$$y = 3x$$

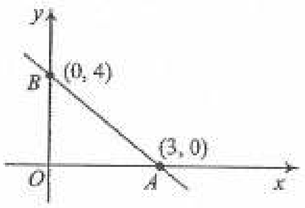
and $y + 5 = 2(x + 10)$

$\therefore$ $3x + 5 = 2x + 20$

$\Rightarrow$ $x = 15$ years

and $y = 45$ years

33. **(c)** Given $\frac{x}{3} + \frac{y}{4} = 1$



$\therefore$ Area of $\Delta OAB = \frac{1}{2} \times 3 \times 4$

= 6 sq. units

34. **(c)** We have $\angle A + \angle B + \angle C = 180°$

According to the question

$$\angle C = 3, \angle B = 2\ (180° - \angle C)$$

$\Rightarrow$ $\angle C = 360° - 2\angle C$

$\Rightarrow$ $3\angle C = 360°$

$\Rightarrow$ $\angle C = 120°$

35. **(d)** Let the speed of Krishna and Kansh be x and y km/h respectively.

According to the question

$$\frac{30}{x} = \frac{30}{y} + 3 \text{ and}$$

$$\frac{30}{2 \times y} = \frac{30}{x} + \frac{3}{2}$$

Let $\frac{1}{x} = X$, and $\frac{1}{y} = Y$.

$\therefore$ On solving $x = 7.5$ km/h.

5. Introduction to Euclid's Geometry

Learning Objective:

In this chapter, we shall learn about:

*Axioms or postulates

*Euclid's geometry

*Important terms related to geometry

Statements

A sentence which can be judged to be true or false is called a statement.

Examples:

(i) The sum of all the angles of a triangle is $180°$.

(ii) $x - 30 > 80$, is a sentence, but not a statement.

Axioms or Postulates

The basic facts which are taken for granted, without proof, are called axioms. These are obvious universal truths.

Examples: (i) Halves of equals are equal

(ii) The whole is grates than each of its pouts

(iii) The sun rises in east.

(iv) A line contains infinitely many points.

(v) Two points determine a unique line.

Theorems

There are statements which are proved, using definitions, axioms and previously proved statements.

Examples: (i) The sum of all angles of quadrilateral is $360°$.

(ii) The sum of all angles around a point is $360°$.

Corollary

A statement whose truth can easily be deduced from a theorem is called its corollary.

Euclid's five postulates

Postulate 1: A straight line may be drawn from any one point to any other point.

Postulate 2: A terminated line can be produced indefinitely.

Postulate 3: A circle can be drawn with any centre and any radius.

Postulate 4: All right angles are equal to one other.

Postulate 5: For every line L and for every point P not lying on L, there exists a unique line M passing through P and parallel to L.

Terms Related to Geometry

Point

A point is a dimensionless thing and an exact location. It is represented by a dot.

• Point

Line segment

The straight path between two points A and B is called the line segment $\overline{AB}$.

Ray

A line segment AB when extended indefinitely in one direction is the ray $\overrightarrow{AB}$.

Two rays, i.e., $\overrightarrow{AB}$ and $\overrightarrow{BA}$ can be drawn from a given line segment AB.

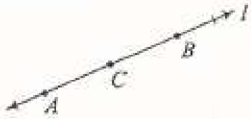
Line

A line segment AB, when extended indefinitely in both the directions is called the line $\overleftrightarrow{AB}$.

Collinear points

Three or more than three points are said to be collinear if there is a line containing all the points.

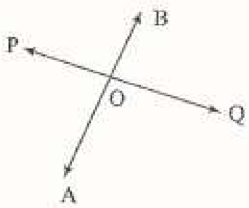
Example: Points A, B and C are collinear.



Intersecting lines

Two lines having a common point are called intersecting lines.

Example: Lines PQ and AB, have a common point O. Therefore, AB and PQ are intersecting lines.



Parallel lines

If two lines, are indefinitely extended and have no point in common, then the line are said to be parallel.

Plane

A plane is a surface such that every point of the line joining any two points on the surface completely lie on it.

Concurrent lines

Three or more lines which are intersected at the same common point are said to be concurrent.

Incidence and Parallel Axioms on Lines

Incidence Axioms

(a) A line contains infinitely many points.

(b) Through a given point, infinitely many lines can be drawn.

(c) One and only one line can pass through two given points.

Parallel Axioms on Lines

(a) Two distinct lines cannot have more than one point in common.

(b) One and only one line can be drawn parallel to a given line AB and passing through a unique point P.

(c) If $AB \parallel PQ$ and $PQ \parallel l$, then $AB \parallel l$.

Example 1: If lines AB, AC, AD and AE are parallel to line l show that points A, B, C, D and E are collinear.

Solution: **Proof :**

$\because$ AB, AC, AD and AE all are parallel to l, and pass through A. Therefore, one and only one line pass through A, then, AB, AC, AD and AE determine a single line.

$\therefore$ Points A, B, C, D and E are collinear.

Multiple Choice Questions

1. Select the incorrect statement.
   (a) An axiom is a statement that is taken for granted without proof.
   (b) A sentence which can be judged to be true or false is called statement.
   (c) A statement whose truth can easily be deduced from a theorem is called postulate
   (d) A statement that requires a proof is called theorem.
2. Which of the following statement is axiom :
   (a) Halves of equals are equal
   (b) The sum of angles of a triangle is 180°
   (c) The sum of angles of a quadrilateral is 360°
   (d) The sun rises from the west.
3. Which of the following statements is correct?
   (a) A line contains definite number of points
   (b) Through a point 2 lines can be drawn only
   (c) If there are 2 fixed points A and B then there will be two lines AB between them
   (d) A terminated line can be produced infinitely.
4. Which of the following statement is wrong?
   (a) A circle can be drawn with any centre and any radius.
   (b) All right angles are equal to one another.
   (c) Things which are heifers of the same thing are equal to one another.
   (d) The part of a thing is greater than whole.
5. Euclid divided his books in how many chapters?
   (a) 11 (b) 12 (c) 13 (d) 10
6. The number of planes passing through three non-collinear points is :
   (a) 1 (b) 2 (c) 3 (d) 4
7. Which of the following does not need a proof?
   (a) Axiom (b) Theorem
   (c) Statement (d) Definition
8. How many lines can pass through 2 given points ?
   (a) 1 (b) 2
   (c) 3 (d) Infinite
9. How many parallel lines can be drawn parallel to a given line L and passing through a fired point P ?
   (a) 0 (b) Infinite
   (c) 2 (d) 1

10. If 4 lines pass through a point these are said to be
    (a) Unique lines (b) Parallel line
    (c) Concurrent lines (d) Perpendicular.
11. $\overline{AB}$ has how many end points ?
    (a) Alone (b) 2 (c) 3 (d) Infinite
12. Two circles are said to be congruent, if and only if :
    (a) They have equal radius
    (b) They are intersections,
    (c) They are having equal length of common chord
    (d) They are non-intersecting.
13. If $AB \parallel CD$ and $CD \parallel PQ$ then AB and PQ are
    (a) Parallel to each other
    (b) Perpendiculars to each other
    (c) Intersecting but not perpendicular
    (d) Collinear points (A, B, and Q)
14. Two lines can intersect in how many points (maximum) ?
    (a) 2 (b) 3 (c) 4 (d) 1
15. For determination of a plane, how many unique line are required (minimum) ?
    (a) 2 (b) 3 (c) 4 (d) 1
16. A point has
    (a) 1 determination (b) 2 determination
    (c) 3 determination (d) 0 determination
17. A surface has
    (a) 2 determination (b) 3 determination
    (c) 1 determination (d) 0 determination
18. A surface has
    (a) Definite end points
    (b) Indefinite end points
    (c) 1 end point
    (d) 2 end points
19. If two planes intersect each other, then the minimum point of intersection will be
    (a) 2 (b) 3 (c) 1 (d) 0
20. If an angle is such that, its complementary angle is 20°, the n angle is
    (a) 50° (b) 70° (c) 20° (d) 160°
21. The measure of an angle if the five times its complement is 12° less than twice its supplement
    (a) 32° (b) 36° (c) 34° (d) 42°
22. Boundaries of surfaces and solids are :
    (a) Curved, solids
    (b) Curved, surfaces
    (c) Linear, point
    (d) Dimensionless curved respectively
23. If line $AB \perp CD$ and $CD \perp PQ$, then AB and PQ are
    (a) Perpendicular (b) Parallel
    (c) Intersecting
    (d) Skew if the points A, B, C, D. P and Q are in same plane.
24. The numbers of lines that can be drawn through 4 distinct points in a plane if none three points are collinear
    (a) 12 (b) 6 (c) 8 (d) 4
25. In problem – 24 number of lines that can be drawn if 3 of the 4 points are collinear will be
    (a) 4 (b) 6
    (c) 8 (d) 12
26. A ray has :
    (a) 1 end point (b) 2 end point
    (c) 3 end point (d) no end point
27. Every line segment has:
    (a) 1 (b) 2
    (c) 3 (d) infinite, mid points
28. Two triangles are congruent if
    (a) All respective angles are same
    (b) Respective sides are same
    (c) Both (a) and (b)
    (d) None of these
29. A pyramid has, definitely
    (a) Triangular base (b) Square base
    (c) Triangular Face (d) Square Face
30. If a line segment has its midpoint as point P, then point P, should lie
    (a) Outside the line
    (b) On the line
    (c) On the plane of line
    (d) Not

Answer Key

1. (c)	2. (a)	3. (d)	4. (d)	5. (c)	6. (a)	7. (a)	8. (a)	9. (d)	10 (c)
11. (a)	12. (a)	13. (a)	14. (d)	15. (a)	16. (d)	17. (a)	18. (b)	19. (c)	20. (b)
21. (c)	22. (b)	23. (b)	24. (b)	25. (a)	26. (a)	27. (a)	28. (c)	29. (c)	30. (b)

Hints and Solutions

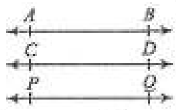
1. **(c)** Statement (c) is incorrect, because it basically a corollary from whose truth can be easily deduced using a theorem.
2. **(a)** The first statement is an axiom and does not require a proof.
3. **(d)** A terminated line can be produced infinitely.
4. **(d)** All the other statements ie (a) (b) and (c) are Euclid's postulates .
5. **(c)** 13.
6. **(a)** Line are formed using 3 non–collinear points.

   ∴ 3 non-collinear points will form 1 plane.
7. **(a)** Axiom does not require a proof.
8. **(a)** Through 2 given points, 1 and only one line can be drawn.
9. **(d)** Only one line can be drawn parallel to a given line and passing through a fixed point.
10. **(c)** Concurrent lines.
11. **(a)** $\overleftrightarrow{AB}$ is a line.

    ∴ It has no end point.
12. **(a)** Two circles are congruent, iff they have equal radii.
13. **(a)** $\because AB \parallel CD$, and

    $CD \parallel PQ$

    $\therefore AB \parallel QP$



14. **(d)** Two lines can intersect in one and only one point.
15. **(a)** Minimum 2 lines are required for determination of a plane.
16. **(d)** Point is dimensionless, ie having no dimension.
17. **(a)** A surface has 2-dimensions i.e., a surface is 2D figure.
18. **(c)** A surface has curved boundary and curve contains infinite points.
19. **(c)** Two planes can intersect each other in minimum 1 point.
20. **(b)** Let the measure of angle be x°

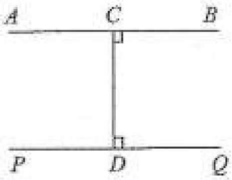
    Then, its complementary angle will be

    $$(90^\circ - x^\circ) = 20^\circ$$

    $$\Rightarrow \quad x = 70^\circ$$
21. **(c)** Let the measure of angle be x° then, its complementary angle will be $90^\circ - x^\circ$ supplement $= 180^\circ - x^\circ$.

    $$\therefore \quad 5(90^\circ - x^\circ) = 2(180^\circ - x^\circ) - 12^\circ$$

    $$\Rightarrow \quad x = 34^\circ$$
22. **(b)** Boundary of surface is curved and boundary of solid is surface.
23. Parallel.



{$\because$ Sum of interior angles $= 90^\circ + 90^\circ = 180^\circ$}

24. **(b)** No. of lines $= \dfrac{n(n-1)}{2} = \dfrac{4 \times 3}{2} = 6$
25. **(a)** No. of lines $= \dfrac{n(n-1)}{2} - \dfrac{m(m-1)}{2} + 1$

$$= \frac{4(4-1)}{2} - \frac{3(3-1)}{2} + 1$$

$$= 6 - 3 + 1 = 4$$

26. **(a)** A ray has 1 end point.
27. **(a)** Every line segment has an unique mid-point.
28. **(c)** Both (a) and (b) are correct.
29. **(c)** A pyramid can have base of any shape but the face is triangular.
30. **(b)** A midpoint of a line should line on the line.

6. Lines and Angles

Learning Objective:

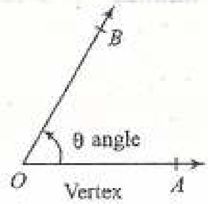
In this chapter, we will learn about:

*Angles, Types of Angles

*Properties of angles and lines

*Angles Made by a Transversal with Two Lines

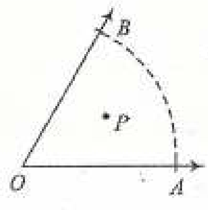
Angle



Two rays OA and OB having a common end point O form angle AOB, written as $\angle AOB$. Basically, it is the inclination between two rays, at a point of intersection.

The rays which meet to form the angle are called arms of the angle and the point of intersection of rays, i.e., O is called vertex.

Interior of an angle



The interior of $\angle AOB$ is set of all points in its plane which lie on the same side of OA as B and also on the same side of OB as A.

Example: Point P is interior of angle $\angle AOB$.

Exterior of an angle

The exterior of an angle $\angle AOB$ is the set of all points, which do not lie on the angle or its interior.

Measure of an angle

The exterior of turning of the line OA to OB is called the measure of $\angle AOB$. An angle is measured in degrees, radians, minutes and seconds.

Angle of 360°

If a ray starting from its original position OA, rotates about O in the anticlockwise direction and after making a complete revolution it comes rotated through 360°.

The 360^{th} part of an angle of 360° is equal to 1°.

$1° = 60$ minutes, written as $60'$.

$1' = 60$ seconds, written as $60''$.

Types of Angles

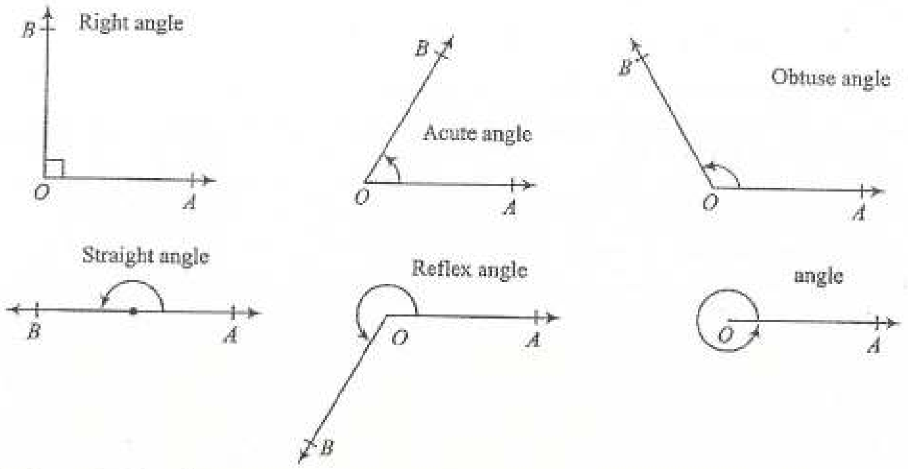
(a) **Right Angle:** An angle whose measure is 90° is called a right angle.

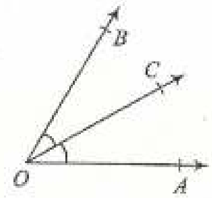
(b) **Acute Angle:** An angle whose measure is less than 90° is called an acute angle.

(c) **Obtuse Angle:** An angle whose measure is more than 90° but less than 180° is called an obtuse angle.

(d) **Straight Angle:** An angle whose measure is equal to 180° is called a straight angle.

(e) **Reflex Angle:** An angle whose measure is greater than straight angle but less than a complete angle, i.e., 360° is called a reflex angle.





In the adjoining figure, the measure of angle $\angle AOC$ is equal to the measure of angle $\angle AOC$ is equal to the measure of angle $\angle COB$. Such pair of angles are called equal angles, and the ray OC which divides the $\angle AOB$ in two equal parts is called angle of bisector.

Complementary angles

Two angles are said to be complementary, if the sum of their measures is 90°.

Two complementary angles are called the complement of each other.

Example: Angles measuring 32° and 58° are complementary angles.

Supplementary angles

Two angles are said to be supplementary, if the sum of their measures is 180°.

Two supplementary angles are called the supplement of each other.

Example: Angles measuring 135° and 45° are supplementary angles.

Example 1: Find the measure of the angles which are in the ratio 3:6 and their sum is equal to $\left(\frac{3}{4}\right)$th of a straight angle.

Solution: Sum of angles $= \frac{3}{4} \times 180° = 135°$

Let the angles be $3x°$ and $6x°$ respectively.

$\therefore \quad 6x° + 3x° = 135°$

$\Rightarrow \quad 9x° = 135°$

$\Rightarrow \quad x = 15°$

$\therefore$ Angles are $\quad 3x = 15° \times 3 = 45°$

and $\quad 6x = 15 \times 6 = 90°$

Example 2: Find the measure of an angle, if seven times its complement is 10° less than three times its supplement.

Solution: Let the measure of angle be x degrees.

$\therefore$ Its complement $= (90 - x)$ degrees

Its supplement = (180 − x) degrees

$\therefore \quad 7\times(90^\circ - x) = 3\times(180^\circ - x) - 10^\circ$

$\Rightarrow \quad 630^\circ - 7x = 540^\circ - 3x - 10^\circ$

$\Rightarrow \quad 4x = 630^\circ - 530^\circ = 100^\circ$

$\Rightarrow \quad x = 25^\circ$

Example 3: Find the angle which is four times of its complement

Solution: Let the measure of angle be x degrees.

$\therefore$ Its complement = (90 − x) degrees.

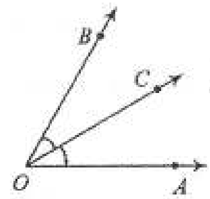
$\therefore \quad x = 4\times(90^\circ - x)$

$\Rightarrow \quad x = 360^\circ - 4x$

$\Rightarrow \quad 5x = 360^\circ$

$\Rightarrow \quad x = 72^\circ$

Adjacent angles

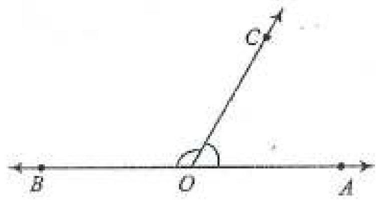


Two angles are called adjacent if they have common vertex 0, they have a common arm and they have uncommon arms on either side of the common arm.

Example: $\angle AOC$ and $\angle BOC$ have common vertex, and they have common arm OC.

$\therefore \angle AOC$ and $\angle BOC$ are adjacent angles.

Linear Pair of Angles



Two adjacent angles are said to form a linear pair of angles, if their non–common arms are opposite rays.

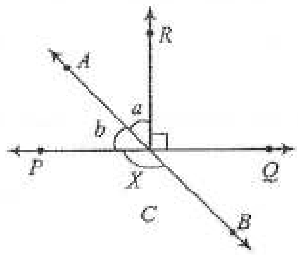
Example: $\angle AOC$ and $\angle BOC$ form a linear pair when a linear pair is formed, then

Sum of adjacent angles is equal to 180°.

$\therefore \angle AOC + \angle BOC = 180^\circ$

$\Rightarrow$ The sum of all the angles round a point is equal to 360°.

Example 4: Lines PQ and AB intersect at X. If $\angle RXQ = 90^\circ$ and $a : b = 2:3$, find c.



Solution: angles $\angle AXP$, $\angle AXR$, and $\angle RXQ$ form straight angle together

$\therefore \quad \angle A + \angle B + 90^\circ = 180^\circ$

$\Rightarrow \quad \angle a + \angle b = 90^\circ \quad$ ...(i)

Let the measures of $\angle a$ and $\angle b$ be $2x^\circ$ and $3x^\circ$ respectively

$\Rightarrow \quad 2x^\circ + 3x^\circ = 90^\circ$

$\Rightarrow \quad 5x^\circ = 90^\circ$

$x = 18^\circ$

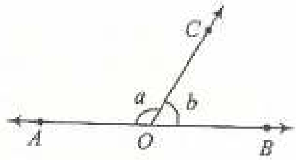
$\therefore \quad \angle a = 36^\circ, \angle b = 54^\circ$

$\therefore \angle c = 180^\circ - (54^\circ) = 126^\circ$ [$\because \angle c$ and $\angle b$ form a linear pair]

Example 5: If $\angle AOC$ and $\angle BOC$ form a linear pair, and $a - 2b = 30°$, then find a and b.

Solution: According to question $a + b = 180°$ ...(i)

and $a - 2b = 30°$ ...(ii)



Subtracting eq (ii) from eq (i), we have

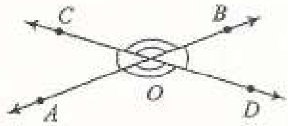
$$b - (-2b) = 180° - 30° \Rightarrow 3b = 150°$$

$\Rightarrow$ $b = 50°$ and $\therefore a = 180° - b = 180° - 50° = 130°$

Vertically Opposite Angles

Two angles are called a point of vertically opposite angles, if their arms form two pairs of opposite rays.

If two lines intersect, then the vertically opposite angles are equal.

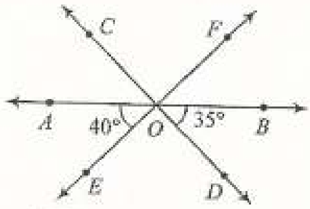


Example: $\angle COB$ and $\angle AOD$ are vertically opposite angles, and $\angle AOC$ and $\angle BOD$ are vertically opposite angles

$\therefore$ $\angle COB = \angle AOD$ and $\angle AOC = \angle BOD$

Example 6: Lines AB,CD and EF intersect at O. Find the measures of $\angle AOC$, $\angle FOC$, $\angle DOE$ and $\angle FOB$.

Solution:



$\angle FOB = \angle AOE = 40°$ [vertically opposite $\angle$S]

$\angle AOC = \angle BOD = 35°$ [vertically opposite $\angle$S]

$\angle FOC = 180° - (\angle AOC + \angle FOB)$

$= 180° - (35° + 40°)$

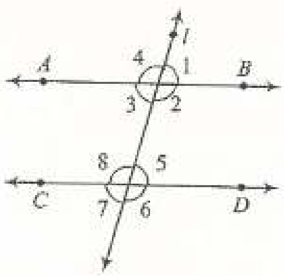
$= 180° - 75°$

$= 105°$

$\angle DOE = \angle FOC = 105°$ [vertically opposite $\angle$S]

Transversal

A line which intersects two or more given lines at distinct points is called a transversal of the given lines.



Angles Made by a Transversal with Two Lines

Corresponding angles

Two angles on the same side of a transversal are called corresponding angles if both lie either above the two lines or below the two lines.

In the above figure, the pair of corresponding angles are:

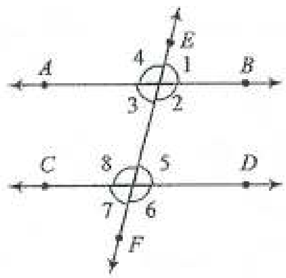
$\angle 1$ and $\angle 5$, $\angle 4$ and $\angle 8$,$\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$.

Alternate Interior angles

The following pairs of angles are called the pairs of alternate interior angles

(i) $\angle 3$ and $\angle 5$ (ii) $\angle 2$ and $\angle 8$.

Parallel lines and transversal



If the lines AB and CD are considered as parallel lines and EF is the transversal then,

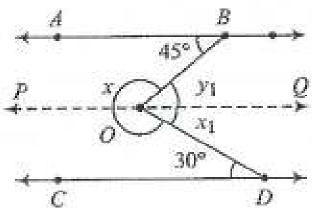
(i) Each pair of corresponding angles are equal, i.e.,

$\angle 1 = \angle 5$, $\angle 4 = \angle 8$, $\angle 2 = \angle 6$ and $\angle 3 = \angle 7$.

(ii) Each pair of consecutive interior angles are supplementary i.e.,

$\angle 2 + \angle 5, = \angle 3 + \angle 8 = 180°$

Example 7: In the figure, $AB \parallel CD$, Determine x.



Solution: Draw a line PQ, such that $PQ \parallel AB \parallel CD$, and PQ passes through O.

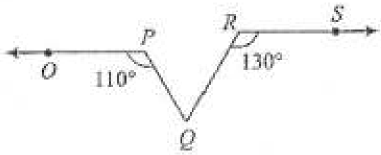
$x_1 = 30°$ [Alternate interior$\angle$].

$Y_1 = 45°$ [Alternate interior$\angle$].

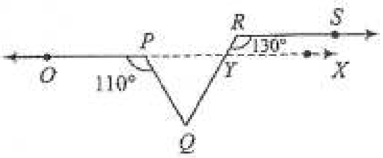
$\therefore \quad x = 360° - (x_1 + y_1)$

$= 360° - 75° = 285°$

Example 8: If $OP \parallel RS$. Determine $\angle PQR$.



Solution: Construct a line $P \times \parallel OP \parallel RS$.



$\angle QRS = \angle QYX = 130°$ [corresponding $\angle$], and

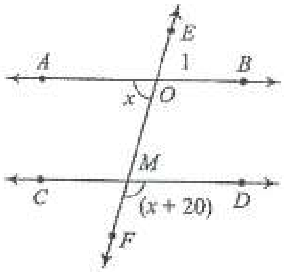
$\angle OPQ + \angle YPQ = 180°$ [Linear pair]

$\Rightarrow \angle YPQ = 180° - \angle OPQ = 180° - 110° = 70°$

$\therefore \angle PYQ = 180° - \angle QYX = 180° - 130° = 50°$

$\therefore \quad \angle Q = 180° - (70° + 50°) = 60°$

Example 9: If $AB \parallel CD$, find x.



Solution: $\angle BOD + \angle AOC = 180°$ [Linear pair]

$\Rightarrow \angle BOD + x = 180°$

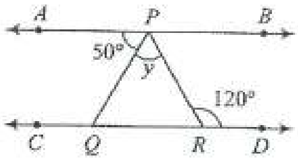
$\Rightarrow \angle BOD = 180° - x$

$\angle BOD = \angle DMF$ [corresponding $\angle s$]

$\Rightarrow 180° - x = x + 20°$

$\Rightarrow 2x = 160° \Rightarrow x = 80°$

Example 10: If $AB \parallel CD$, find y.



Solution: $\angle RPB + \angle PRD = 180°$ [interior $\angle s$]

$\Rightarrow \angle RPB = 180° - \angle PRD = 180° - 120° = 60°$

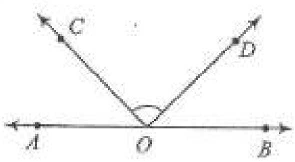
$\angle APQ + y + \angle RPB = 180°$

$\Rightarrow y = 180° - (\angle RPB + \angle APQ)$

$= 180° - (60° + 50°) = 70°$

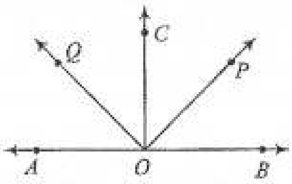
Multiple Choice Questions

1. What is the measure of an angle which is equal to 5 times its supplement?
   (a) 150° (b) 120°
   (c) 90° (d) 135°
2. Two complementary angles are such that two times the measure of one is equal to three times the measure of the other. The measure of the larger angle is :
   (a) 60° (b) 45°
   (c) 54° (d) 36°
3. Two complementary angles are in the ratio 2 : 7. The measure of smaller angle is :
   (a) 70° (b) 45°
   (c) 20° (d) 40°
4. In the figure OA and OB are opposite rays $\angle AOC + \angle BOD = 63°$. The measure of angle $\angle COD$ is;



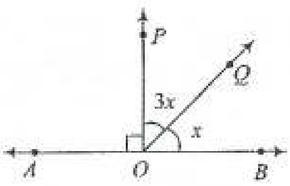
(a) 63° (b) 127°
(c) 117° (d) 27°

5. OP bisects $\angle BOC$ and OQ, $\angle AOC$. Find the measure of $\angle POQ$.



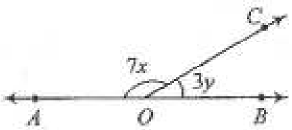
(a) 85° (b) 45° (c) 90° (d) 135°

6. Determine the value of x from the given figure;



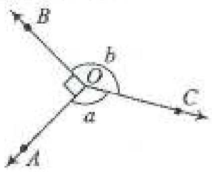
(a) 45° (b) 225° (c) 25.5° (d) 25°

7. If $y - x = 10°$, then $y =$



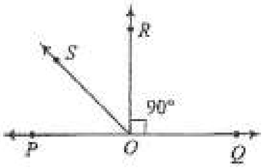
(a) 25° (b) 20° (c) 15° (d) 10°

8. $b = a + 20°$, then $a =$



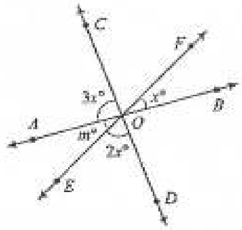
(a) 145° (b) 125° (c) 130° (d) 135°

9. POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR, then $\angle POS$ is equal to :



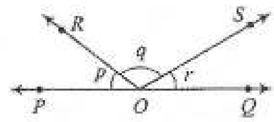
(a) $\angle ROS - \angle QOS$ (b) $\angle QOS - 2\angle ROS$
(c) $\angle QOS + 2\angle ROS$ (d) $2\angle ROS - \angle QOS$

10. The value of m is



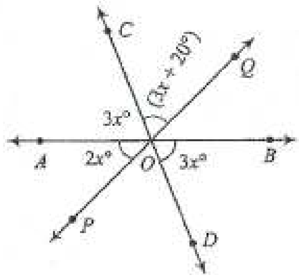
(a) 60° (b) 30° (c) 45° (d) 20°

11. If $\frac{q}{p} = 5$, $\frac{r}{p} = 3$, then $r + p =$



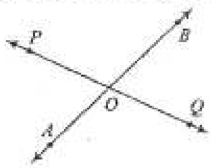
(a) 80° (b) 120° (c) 160° (d) 100°

12. Find x from the figure.



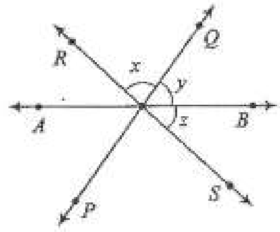
(a) 20° (b)25° (c)10° (d)15°

13. In the adjoining figure, $\angle AOQ : \angle AOP = 5 : 7$, then measure of $\angle BOQ$ is :



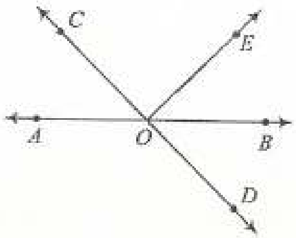
(a) 75° (b)105° (c)60° (d)120°

14. $x = 3y = \frac{6}{7}z$, then, find the value of y.



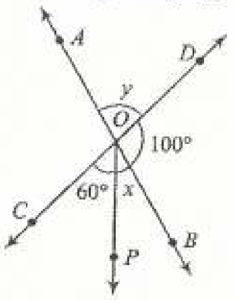
(a) 36° (b) 24° (c) 72° (d) 84°

15. In the adjoining figure, $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, then measure of reflex $\angle BOE$ is



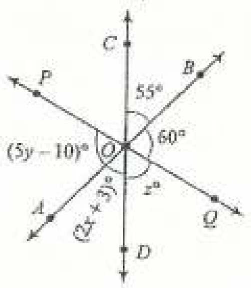
(a) 320° (b) 330°
(c) 290° (d) 250°

16. Find x from the adjoining figure :



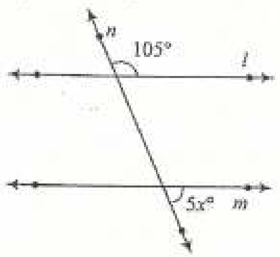
(a) 30° (b)20° (c) 40° (d)80°

17. Find the value of $x - y + z$



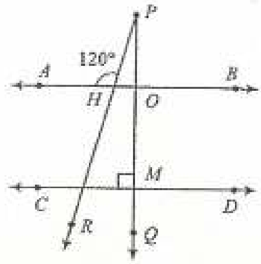
(a) 77° (b) 85° (c) 127° (d) 137°

18. In the figure, $l \parallel m$, Find k.



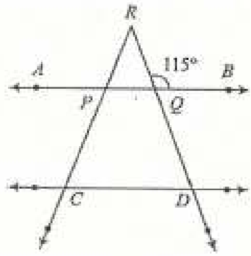
(a) 21° (b) 15° (c) 25° (d) 23°

19. In the adjoining figure, $AB \parallel CD$ and, $PQ \perp AB$, find the measure of $\angle PCM$.



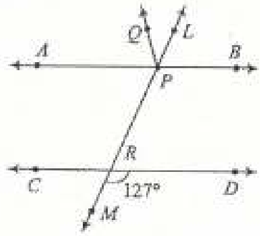
(a) 120° (b) 60° (c) 30° (d) 90°

20. $AB \parallel CD$, and $\angle RQB = 115°$, and $\angle PRQ = 30°$. The measure of $\angle APC$ is :



(a) 115° (b) 45° (c) 85° (d) 30°

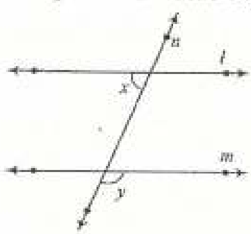
21. PQ trisects $\angle APL$, then the measure of $\angle LPQ$ is :



(a) $\left(42\frac{1}{3}\right)°$ (b) $\left(48\frac{2}{3}\right)°$

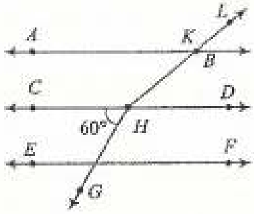
(c) $\left(47\frac{2}{3}\right)°$ (d) $\left(19\frac{1}{3}\right)°$

22. If $x : y = 2:3$, then the value of y is equal to :



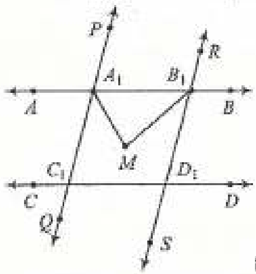
(a) 72° (b) 36° (c) 108° (d) 144°

23. $AB \parallel CD \parallel EF$ and $GH \parallel KL$. The measure of $\angle HKL$ is



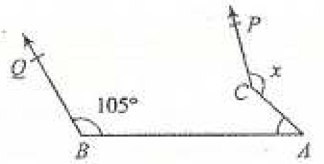
(a) 85° (b) 135° (c) 215° (d) 145°

24. $AB \parallel CD$ and $PQ \parallel RS$, then the measure of $\angle A_1MB$, is (Here $\therefore$ A_1M and B_1M are the bisectors of $\angle MA_1B_1$ and $\angle MB_1A_1$ respectively)



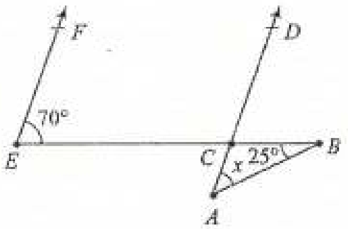
(a) 70° (b) 85° (c) 90° (d) 12°

25. Find x from the given figure ($CP \parallel DQ$) :



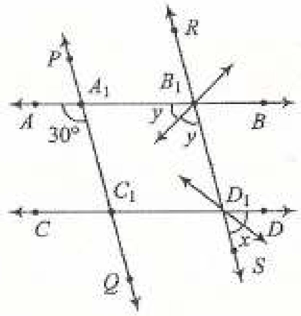
(a) 105° (b)130° (c)125° (d)175°

26. $AD \parallel EF$, Then $x =$



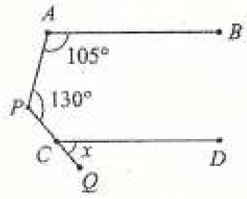
(a) 25° (b)85° (c)45° (d)35°

27. $AB \parallel CD$ and $PQ \parallel RS$, then $x - y =$



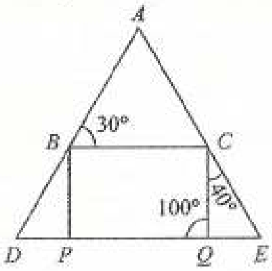
(a) 30° (b) 60°
(c) 75° (d) 90°

28. Find x :



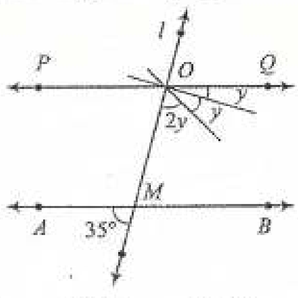
(a) 75° (b) 45° (c) 55° (d) 50°

29. Find $\angle DAE$, if $BC \parallel DE$ and $BP \parallel CQ$:



(a) 60° (b) 75° (c) 90° (d)100°

30. The value of y, if $AB \parallel PQ$ is



(a) 9° (b) 29° (c) 27° (d) 7°

Answer Key

1. (a)	2. (c)	3. (c)	4. (c)	5. (c)	6. (b)	7. (a)	8. (b)	9. (b)	10. (b)
11. (a)	12. (a)	13. (b)	14. (b)	15. (b)	16. (b)	17. (a)	18. (b)	19. (b)	20. (c)
21. (a)	22. (c)	23. (d)	24. (c)	25. (b)	26. (b)	27. (c)	28. (c)	29. (c)	30. (b)

Hints and Solutions

1. **(a)** Let the measure of angle be x°.

$\therefore \quad X^\circ = 5(180^\circ - x)$

$\Rightarrow \quad 6x = 5 \times 180^\circ$

$\Rightarrow \quad x = 150^\circ$

2. **(c)** Let the measure of angle be x.

$\therefore$ Its complement $= (90^\circ - x)$

$\therefore \quad 2x = 3(90^\circ - x)$

$\Rightarrow \quad 5x = 3 \times 90^\circ$

$\Rightarrow \quad x = 54^\circ$

and $\quad (90^\circ - x) = 36^\circ$

$\therefore$ Measure of larger angle $= 54^\circ$

3. **(c)** Let the measure of angles be $2x^\circ$ and $7x^\circ$ respectively.

$\therefore \quad 2x^\circ + 7x^\circ = 90^\circ$

$\Rightarrow \quad 9x = 90^\circ$

$\Rightarrow \quad x = 10^\circ$

$\therefore$ Measure of smaller angle

$= 2x = 2\times10^\circ = 20^\circ$

4. **(c)** $\because$ OA and OB are opposite rays.

$\therefore$ $\angle AOB$ is a straight angle.

$\Rightarrow \quad \angle AOB = 180^\circ$

$\Rightarrow \quad (\angle AOC + \angle BOD) + \angle COD = 180^\circ$

$\Rightarrow \quad 63^\circ + \angle COD = 180^\circ$

$\Rightarrow \quad \angle COD = 117^\circ$

5. **(c)** $\because$ AOB is a straight line

$\therefore \angle AOB = 180^\circ$

$\angle AOC + \angle BOC = 180^\circ$

$$\Rightarrow \quad \frac{\angle AOC}{2} + \frac{\angle BOC}{2} = \frac{180^\circ}{2} = 90^\circ$$

$\Rightarrow \quad \angle COQ + \angle POC = 90^\circ$

$$\left[\because \angle COQ = \frac{\angle AOC}{2} \text{ and } \frac{\angle BOC}{2} = \angle POC\right]$$

$\Rightarrow \quad \angle POQ = 90^\circ$

6. **(b)** $\quad \angle AOB = 180^\circ$

$\Rightarrow \quad \angle AOP + \angle POB = 180^\circ$

$\Rightarrow \angle AOP + \angle POQ + \angle BOQ = 180^\circ$

$\Rightarrow 90^\circ + 3x + x = 180^\circ$

$\Rightarrow \quad 4x = 90^\circ$

$$\Rightarrow \quad x = \frac{90^\circ}{4} = 22.5^\circ$$

7. **(a)** $\angle AOC + \angle BOC = 180^\circ$

$\Rightarrow \quad 7x + 3y = 180^\circ \quad$ ...(i), and

$y - x = 10^\circ$

$\Rightarrow \quad x = y - 10^\circ \quad$ ...(ii)

Using (ii) and (i)

$7(y - 10^\circ) + 3y = 180^\circ$

$\Rightarrow \quad 10y = 250^\circ$

$\Rightarrow \quad y = 25^\circ$

8. **(b)** $\because$ Sum of angles around a point $= 360^\circ$

$\angle AOB + \angle BOC = 360^\circ$

$\Rightarrow \quad 90^\circ + a + b = 360^\circ$

$\Rightarrow \quad a + b = 270^\circ \quad$ ...(i) and

$b = a + 20^\circ \quad$ ...(ii)

Using (ii) in (i)

$a + (a + 20^\circ) = 270^\circ$

$\Rightarrow \quad 2a = 250^\circ$

$\Rightarrow \quad a = 125^\circ$

9. **(b)** Let the measure of $\angle POS$ be x°

$\because \quad \angle POQ = 180^\circ$

$\Rightarrow \quad \angle POS + \angle ROS + \angle QOR = 180^\circ$

$\Rightarrow \quad \angle POS + \angle ROS + 90^\circ = 180^\circ$

$\Rightarrow \quad \angle ROS = (90° - x).$

$\angle QOS = 90° + (90° - x) = 180° - x$

$\therefore \quad \angle POS = x = \angle QOS - 2\angle ROS.$

10. **(b)** $\angle m = \angle x$ [Vertically opposite $\angle$s]

$\because \angle AOB = 180°$

$\Rightarrow \angle BOF + \angle COF + \angle AOC = 180°$

$\Rightarrow \angle BOF + \angle DOE + \angle AOC = 180°$

$\Rightarrow x° + 2x° + 3x° = 180°$

$\Rightarrow \quad 6x° = 180°$

$\Rightarrow \quad x = 30°$

$\therefore \quad m = 30°$

11. **(a)** $q = 5p$, $r = 3p$ and

$\because \angle POQ = 180°$

$\Rightarrow p + q + r = 180°$

$\Rightarrow p + 5p + 3p = 180°$

$\Rightarrow 9p = 180° \Rightarrow p = 20°$

$r = 3p = 3 \times 20° = 60°$

$\therefore r + p = 60° + 20° = 80°$

12. **(a)** Here $\angle COQ = \angle POD$

[vertically opposite $\angle$s]

$\because \quad \angle AOB = 180°$ (AOB is a straight line)

$\Rightarrow \angle POA + \angle POD + \angle BOD = 180°$

$\Rightarrow 2x° + 3x° + 20° + 3x° = 180°$

$\Rightarrow \quad 8x = 160°$

$\Rightarrow \quad x = 20°$

13. **(b)** $\because \angle POQ = 180°$ [PQ is a straight line]

$\Rightarrow \angle AOQ + \angle AOP = 180°$

$\Rightarrow \quad 5k + 7k = 180°$

$\Rightarrow \quad 12k = 180°$

$\Rightarrow \quad k = 15°$

$\therefore \angle BOQ = \angle AOP = 7 \times 15° = 105°$

[vertically opposite $\angle$s]

14. **(b)** $x = 3y$, $z = \dfrac{21}{6} y = \dfrac{7}{2}y.$

$\therefore \quad x + y + z = 180°$

$\Rightarrow 3y + y + \dfrac{7}{2} y = 180°$

$\Rightarrow \quad 4y + \dfrac{7}{2}y = 180°$

$\Rightarrow \quad 15y = 180° \times 2$

$\Rightarrow \quad y = 24°$

15. **(b)** $\angle BOD = \angle AOC = 40°$

[vertically opposite $\angle s$]

$\because \angle SOB$ is a straight angle

$\therefore \angle AOB = 180°$

$\Rightarrow \angle AOC + \angle COE + \angle BOE = 180°$

$\Rightarrow \angle COE = 180° - (\angle AOC + \angle BOC)$

$= 180° - 70° = 110°$

$\Rightarrow \angle AOC + \angle BOE = 70°$

$\angle AOE = 70° - 40° = 30°$

$\therefore$ reflex $(\angle BOC) = 360° - 30 = 330°$

16. **(b)** $\angle AOD = \angle BOC$ [vertically opposite $\angle$s]

$\Rightarrow \quad y = 60° + x$ ...(i) and ,

$\because \quad \angle DOC = 180°$

$\Rightarrow 60° + x + 100° = 180°$

$\Rightarrow \quad x = 20°$

17. **(a)** $\because \angle COD = 180°$

$\Rightarrow \angle BOC + \angle BOQ + \angle DOQ = 180°$

$\Rightarrow 55° + 60° + z = 180°$

$\Rightarrow \quad z = 65°$

Similarly $\angle BOC = \angle AOD$

$\Rightarrow \quad 2x + 3° = 55°$

$\Rightarrow \quad x = 26°$

and,

$\angle AOP = \angle BOQ$ [vertically opposite $\angle$s]

$\Rightarrow \quad 5y - 10° = 60°$

$\Rightarrow \quad y = \dfrac{70°}{5} = 14°$

$\therefore x - y + z = 26° - 14° + 65° = 77°$

18. **(b)** It is clear from the figure that,

$105° + 5x = 180°$

$\Rightarrow \quad 5x = 75°$

$\Rightarrow \quad x = 15°$

19. **(b)** $\angle ANP + \angle PNO = 180°$ [Straight angle]

$\Rightarrow \angle PNO = 180° - 120° = 60°$

$\therefore \angle PNO = \angle PCM = 60°$ [corresponding $\angle S$]

20. **(c)** Here $\angle RQB + \angle RQP = 180°$

($\because AB$ is a straight line)

$\Rightarrow \quad \angle RQP = 180° - 115° = 65°$

Now $\angle PRQ = 30°$

$\because \angle PRQ, \angle RQP$ and $\angle APQ$ are the $\angle S$ of Δ

$\therefore \angle PRQ + \angle RQP + \angle APQ = 180°$

$\Rightarrow \angle APQ = 180° - 65° - 30° = 85°$

$\angle APC = \angle APQ$ [Vertically opposite $\angle S$]

$\therefore \angle APC = 85°$

21. **(a)** $\because AB \parallel CD$

$\therefore \angle DRM = \angle BPR = 127°$

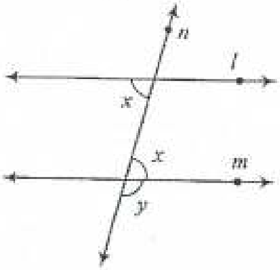
(corresponding $\angle S$)

$\angle BPR = \angle BPR = 127°$

[Vertically opposite $\angle S$]

$$\therefore \angle LPQ = \frac{\angle APL}{3} = \frac{127°}{3} = \left(42\frac{1}{3}\right)^0$$

22. **(c)** $\because X : Y = 2 : 3$



$\therefore$ Let the angles x and y be $2k$ and $3k$ respectively.

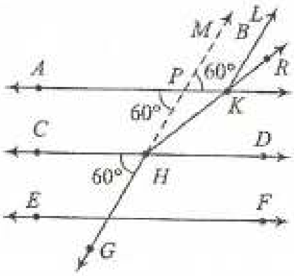
$\therefore \quad 2k + 3k = 180°$

[Sum of $\angle S$ in the interior of transversal]

$\Rightarrow \quad 5k = 180° \Rightarrow k = 36°$

$\therefore \quad y = 3k = 3 \times 36° = 108°$

23. **(d)** Extending GH to M, we have ,



$\angle CHG = \angle APH = 60°$ [Corresponding $\angle S$]

$\angle APH = \angle MPD = 60°$

[Vertically opposite $\angle S$]

$\angle APH = \angle MPD$, then,

$\angle MPD + \angle LKP = 180°$ [Sum of interior$\angle S$]

$\Rightarrow \angle LKP = 180° - 60° = 120°$, also

$\angle KHD = \angle PKH = 25°$ (Alternate $\angle S$)

$\therefore \angle HKL = \angle LKP + \angle PKH$

$= 120° + 25° = 145°$

24. **(c)** $\angle PA_1 B_1 = \angle RB_1A_1 = 180°$

[Sum of interior $\angle S$]

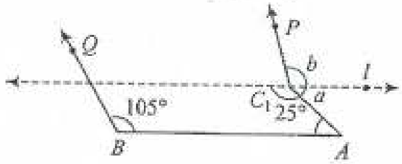
Also,

$$\angle MA_1B_1 = \angle MB_1A_1 = \frac{180°}{2} = 90°$$

($\because MA_1$ and MB_1 are angle bisectors)

$\therefore \angle A_1MB_1 = 180° - 90° = 90°$

25. **(b)** Construct a line $l \parallel AB$,



$\angle a = 25°$ [Alternate $\angle S$]

$\angle c = 105°$

$\therefore \angle b = 105°$ [Vertically opposite $\angle S$]

$\therefore x = a + b = 25° + 105° = 130°$

26. **(c)** $\because AD \parallel EF$

$\therefore \angle EFC = \angle ACE = 70°$

(Alternate opposite $\angle S$)

Also,

$\angle ECA + \angle BCA = 180°$

$\Rightarrow \angle BCA = 180° - 70° = 110°$

Now

In ΔBCA,

$\angle BCA + \angle BAC + \angle ABC = 180°$

$\Rightarrow \quad 110° + x + 25° = 580°$

$\Rightarrow \quad x = 180° - 135° = 45°$

27. **(c)** $\angle DD_1S = \angle B_1D_1C_1 = 2x$

[Vertically opposite $\angle S$]

$\angle B_1D_1C_1 + \angle A_1 B_1D_1 = 180°$ [Interior $\angle S$]

$\Rightarrow \quad 2x + 2y = 180°$

$\Rightarrow \quad x + y = 90° \quad$ ...(i)

Also,

$\angle AA_1C_1 + \angle A_1B_1D_1$ [Corresponding $\angle$S]

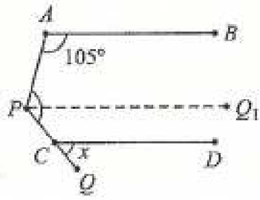
$\Rightarrow \quad 30° = 2y$

$\Rightarrow \quad y = 15° \quad$ ...(ii)

Using (ii) in (i), we get

$x = 90° - y = 90° - 15° = 75°$

28. **(c)** Construct a line $PQ \parallel AB \parallel CD$.



$\angle BAP + \angle APQ_1 = 180°$

[Sum of interior$\angle$S]

$\angle APQ_1 = 180° - 105° = 75°$

$\angle APC + \angle APQ_1 + \angle QPQ_1$

$\Rightarrow \quad 130° = 75° + \angle QPQ_1$

$\Rightarrow \quad \angle QPQ_1 = 55°$

Also

$\angle QPQ_1 + \angle PCD = 180°$

$\Rightarrow \quad \angle PCD = 180° - 55° = 125°$

$\because \angle PCQ$ is a straight angle.

$\therefore \quad \angle PCD + x = 180°$

$\Rightarrow \angle x = 180° - 125° = 55°$

29. **(c)** $\angle CQP + \angle BCQ = 180°$

[Sum of interior$\angle$S]

$\Rightarrow \quad \angle BCQ = 180° - 100° = 80°$

$\because \angle ACE$ is a straight angle.

$\therefore \quad \angle ACE = 180°$

$\Rightarrow \angle ACB + \angle BCQ + \angle QCE = 180°$

$\Rightarrow \quad \angle ACB + 80° + 40° = 180°$

$\Rightarrow \quad \angle ACB = 60°$

Now,

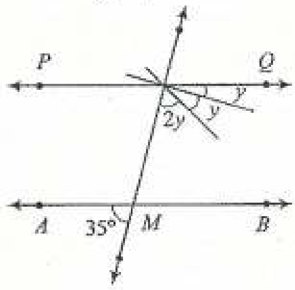
in $\angle ABC$,

$\angle ACB + \angle BAC + \angle ACB = 180°$

$\Rightarrow \quad 30° + \angle DAE + 60° = 180°$

$\Rightarrow \quad \angle DAE = 180° - 90° = 90°$

30. **(b)** From the figure,



$(2y + y + y) + 35° = 180°$ (Interior $\angle$S)

$\Rightarrow \quad 5y = 180° - 35°$

$\Rightarrow \quad y = 36° - 7$

$= 29°$

7. Triangles

Learing Objective:

In this chapter, we will learn about:

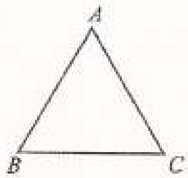
*Triangles, Types of Triangles

*Conference of Triangles

*Rules of Conference

*Properties of Triangles and Inequalities in a Triange

Triangle



A plane figure bounded by three lines in a plane is called a triangle.

A triangle has three vertices, three sides and three angles.

Here, ABC is a triangle denoted as ΔABC. Its sides are AB, BC, CA, angles are $\angle A$, $\angle B$, $\angle C$ and vertices are A, B and C.

Types of Triangles

On the basis of sides, triangles are classified as:

- **Equilateral Triangle:** A triangle whose all sides are equal to one another is called an equilateral triangle.
- **Scalene Triangle:** A triangle whose none of the side are equal to the other is called a scalene triangle.
- **Isosceles Triangle:** A triangle whose two sides equal in length is called an isosceles triangle.

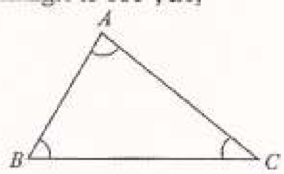
On the basis of angles, triangles are classified as:

- **Acute angled Triangle:** A triangle whose all angles are acute triangle.
- **Right Triangle:** A triangle with one angle a right angle is called a right triangle or right angled triangle.
- **Obtuse Triangle :** A triangle with one angle an obtuse is known as an obtuse triangle.

Theorems on Triangles

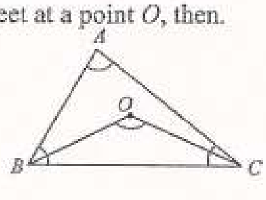
1. **Angle sum property of a triangle:** The sum of the three angles of a triangle is 180°, i.e,

$$\angle A + \angle B + \angle C = 180°$$



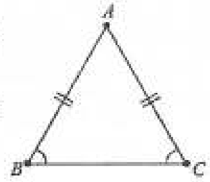
2. If the bisectors of angles $\angle ABC$ and $\angle ACB$ of a triangle ABC meet at a point O, then.

$$\angle BOC = 90° + \frac{1}{2}\angle A$$



3. If two parallel lines are intersected by transversal, then bisectors of the two pairs of interior angles enclose a rectangle.

   If $AB \parallel CD$, and EF is a transversal, then $PQRS$ is a rectangle, given, PQ, PS, RS and QR are angle bisectors.



4. Angles opposite to equal sides of a triangle are equal, i.e, if in ΔABC, $AB = AC$, then $\angle B = \angle C$
5. For an equilateral Δ, Theorem (4) will result in

$$3\theta = 180°$$

$\Rightarrow \theta = 60°$, where, θ denotes angle of an equilateral Δ.

Example 1: In a ΔABC, $\angle A = 70°$, $\angle C = 60°$, Find the measure of $\angle B$.

Solution: We have, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle B = 180° - (\angle A + \angle C) = 180° - (70° + 60°) = 50$

Example 2: A, B, C are the three angles of a triangle. If

$$A - B = 15° , B - \angle C = 30° \text{ Find } \angle C.$$

Solution: Given $\angle A = \angle B + 15°$ ...(i)

and $\angle B = \angle C + 30°$ ...(i) and,

We know $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad (\angle B + 15°) + \angle B + \angle C = 180°$

$\Rightarrow (\angle C + 30° + 15°) + \angle C + 30° + \angle C = 180°$

$\Rightarrow \quad 3\angle C + 75° = 180°$

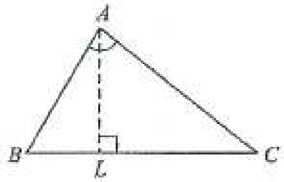
$\Rightarrow \quad 3\angle C = 105°$

$\Rightarrow \quad C = 35°$

Example 3: A triangle ABC is right angled at A and $AL \perp BC$.

Prove that $\angle BAL = \angle ACB$.

Solution: In ΔBAL



$$\angle BAL + \angle BLA + \angle ABL = 180°$$

$\Rightarrow \quad \angle BAL = 180° - \angle ABL - 90°$

$\Rightarrow \quad \angle BAL = 90° - \angle ABL$ ...(i)

In $\angle ALC$,

$\Rightarrow \quad \angle ALC + \angle ACL + \angle LAC = 180°$

$\Rightarrow \quad 90° + \angle ACB + \angle LAC = 180°$

$\Rightarrow \quad \angle ACB = 90° - \angle LAC$ ...(ii)

Also, $\angle LAC = 90° - \angle BAL$ ...(iii)

Using (iii) in (ii)

$$\angle ACB = 90° - (90° - \angle BAL)$$

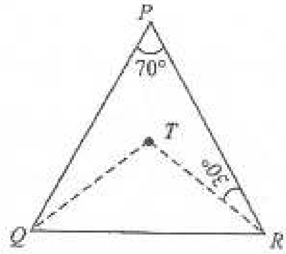
$\Rightarrow \quad \angle ACB = \angle BAL$ Proved.

Example 4: TQ and TR are angle bisectors of $\angle Q$ and $\angle R$ respectively. The measure of $\angle QTR$ will be

Solution: Here $\angle P + \angle Q + \angle R = 180°$

$\Rightarrow \quad 70° + \angle Q + 2 \times 20° = 180°$

$\Rightarrow \quad \angle Q = 70°$



In $\angle QTR$,

$$\angle QTR + \angle TQR + \angle QRT = 180°$$

$$\Rightarrow \quad \angle QTR + \frac{\angle Q}{2} + \frac{\angle R}{2} = 180°$$

$$\Rightarrow \quad \angle QTR + \frac{70°}{2} + \frac{40°}{2} = 180°$$

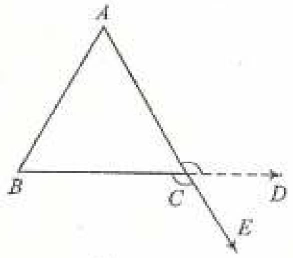
$$\Rightarrow \quad \angle QTR = 180° - 20° - 35° = 125°$$

Exterior Angles of a Triangle

Exterior Angles

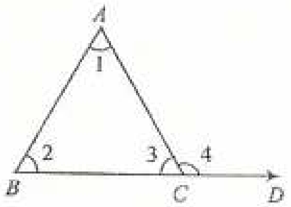
If the side BC of a triangle ABC is produced to form ray BD, Then $\angle ACD$ is called exterior angle of ΔABC and is denoted by ext. $\angle ACD$ etc.

$\angle ACD =$ ext. $\angle BCE$ [Vertically opposite $\angle$S]



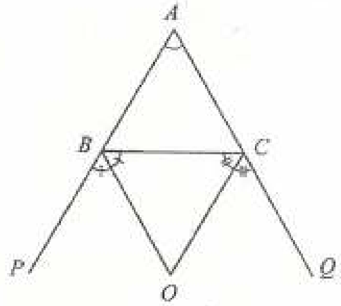
Theorem 1: If a side of a triangle is *protrude*, the exterior angle so formed is equal to the sum of two interior opposite angles.

$$\angle 4 = \angle 1 + \angle 2.$$



Theorem 2: The sides AB and AC of a ΔABC are produced to P and Q respectively. If the bisectors of $\angle PBC$ and $\angle QCB$ intersect at O, then

$$\angle BOC = 90° - \frac{1}{2}\angle A$$



Example 5: Find x from the given figure :

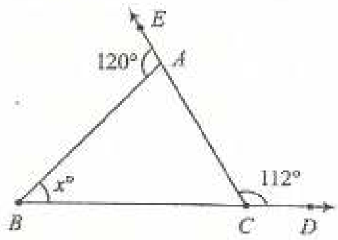
Solution: In ΔABC,

$\angle A + x = 112°$ (Vertical opposite angles) ...(i)

$\angle ACB + x = 120°$ (Vertical opposite angles) ...(ii) and

$\angle A + \angle ACB + x = 180°$ ...(iii)

Adding eq (i) and (ii) and using (iii),



$$\angle A + \angle ACB + 2x = 232^\circ$$

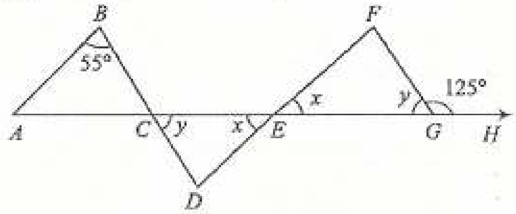
$$(\angle A + \angle ACB + x) + x = 232^\circ$$

$\Rightarrow$ $180^\circ + x = 232^\circ$

$\Rightarrow$ $x = 232^\circ - 180^\circ$

$\Rightarrow$ $x = 52^\circ$

Example 6: Find x and y, given $AB \parallel DC$ and $BD \parallel FG$



Solution: $\angle y + \angle FGH = 180^\circ$ (Linear pair)

$\Rightarrow$ $\angle y = 180^\circ - 125^\circ = 55^\circ$

$\angle FEG = \angle DEC$ (Vertically opposite $\angle$S)

$y = \angle DCE = \angle FGE$ (Alternate $\angle$S)

$\angle BAC = \angle DEC = x$ (Alternate $\angle$S)

Also,

$\angle BAC = \angle DEC = y$ (Vertically opposite $\angle$S)

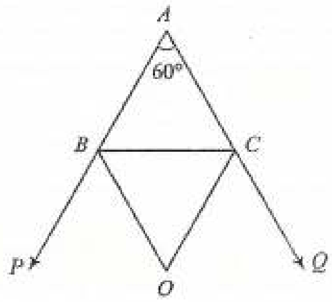
$\therefore$ In ΔABC,

$$55^\circ + x + y = 180^\circ$$

$\Rightarrow$ $x + y = 125^\circ$

$\Rightarrow$ $x = 125^\circ - y = 125^\circ - 55^\circ = 70^\circ$

Example 7: In the adjoining figure, BO and CO are angle bisectors of ext $\angle B$ and ext $\angle C$ respectively. Find the measure of $\angle BOC$.



Solution: Let $\angle B = x$ and $\angle C = y$ then

$$60^\circ + x + y = 180^\circ$$

$\Rightarrow$ $x + y = 120^\circ$ ...(i)

$\therefore$ ext. x + ext. $y = (180^\circ - x) + (180^\circ - y)$

$= 360^\circ - (x + y) = 360^\circ - 120^\circ = 240^\circ$

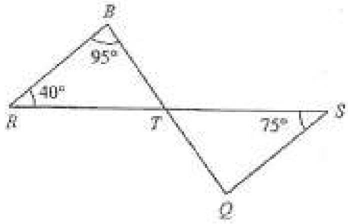
$\therefore$ In ΔBOC,

$$\angle BOC + \angle BCO + \angle BCO = 180^\circ$$

$\Rightarrow$ $\angle BOC + \left(\dfrac{ext.x + ext.y}{2}\right) = 180^\circ$

$\Rightarrow$ $\angle BOC + \dfrac{240^\circ}{2} = 180^\circ \Rightarrow \angle BOC = 60^\circ$

Example 8: Find $\angle SQT$ from the given figure.



Solution: $\angle RTQ = 95° + 40° = 135°$

$\Rightarrow \angle SQT + \angle TSQ = 135°$

$\Rightarrow \quad \angle SQT = 135° - \angle TSQ$

$= 135° - 75° = 60°$

Congruent Triangles

Two line segments are congruent, iff, their lengths are equal.

Two angles are congruent if their measures are equal.

Congruence of Triangles

Two triangles are congruent if and only if one of them can be made to superimpose on the other.

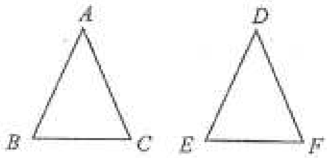
Two triangles are congruent if and only if one of there exists a correspondence between their vertices such the corresponding sides and the corresponding angles of the two triangles are congruent.

If ΔABC is congruent to ΔDEF and the correspondence $ABC \leftrightarrow DEF$ makes the six pairs of corresponding parts of the two triangles then we write,

$\Delta ABC \cong \Delta DEF$, if and only if

$AB = DE$, $EF = BC$ and $AC = DF$, and

$\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$.



Every triangle is congruent to itself.

Criteria for Congruence of Triangles

Side-Angle-Side (SAS) Congruence criterion

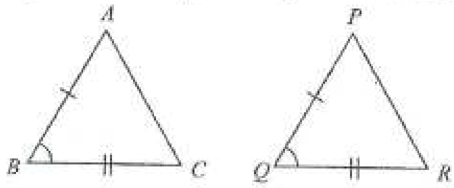
Two triangles are congruent if two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle.

If $AB = PQ$,

$BC = QR$ and the included angle, i.e,

$\angle ABC = \angle PQR$, then

$\Delta ABC \cong DPQR$ [By S-A-S congruency]

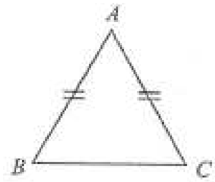


Theorem:

Angles opposite to equal sides are equal, i.e., if

$AB = AC$, then,

$\angle B = \angle C$



Example 9: Line segments AB and CD intersect at O in such a way that $AO = OD$ and $OB = OC$. Prove that $AC = BD$ but AC may not be parallel to BD.

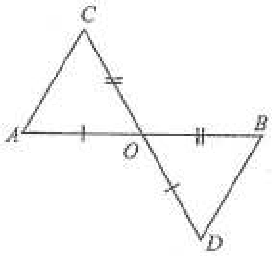
Solution: In Δs AOC and BOC,

$OC = OD$ (Given)

$\angle AOC = \angle BOD$ (Vertically opposite $\angle S$)

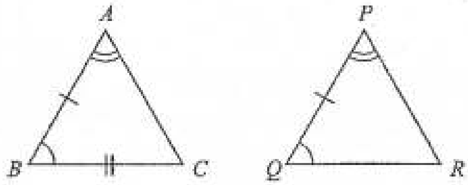
$\therefore \quad \Delta AOC \cong \Delta BOD$ (By S –A–S Congruency)

$\therefore \quad AC = BD$ (C-P-C-T)



Angle-Side-Angle (ASA) Congruence Criterion

Two triangles are congruent if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle.

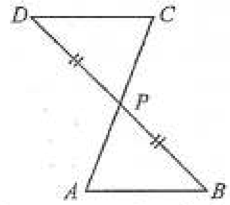


If $AB = PQ$, $\angle ABC = \angle PQR$, and

$\angle QPR = \angle BAC$

$\therefore$ $\Delta ABC \cong \Delta PQR$ (By A–S–A Congruency)

Example 10: If $AB \| DC$ and P is the mid-point of BD, prove that P is also the mid-point of AC.

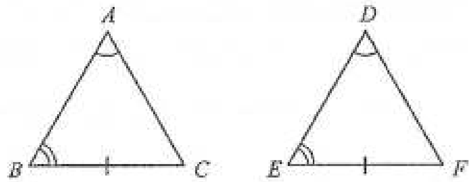


Solution: In ΔDCP and BAP

	$\angle DPC = \angle BPA$	(Vertically opposite $\angle$S)
and	$\angle CDP = \angle PBA$	(Alternate $\angle$S)
also	$DP = BP$	(Given)
$\therefore$	$\Delta DCP \cong \Delta BAP$	(By A-S-A congruency)
$\therefore$	$AP = CP$	(By corresponding parts of congruent triangles)

Angle-Angle-Side (AAS) Criterion of Congruence

If any two angles and a non–included side of one triangle are equal to the corresponding angles and side of another triangle then the two triangles are congruent.



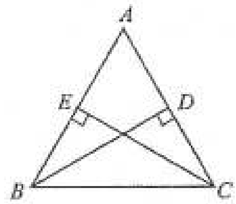
If $\angle ABC = \angle DEC$,

$\angle ABC = \angle EDF$, and ,

$BC = EF$,

$\therefore$ $\angle ABC \cong \angle DEF$ (by AAS congruency)

Example 11: If ΔABC is an isosceles triangle with $AB = AC$. Prove that the perpendiculars from the vertices B and C to their opposite sides are equal.

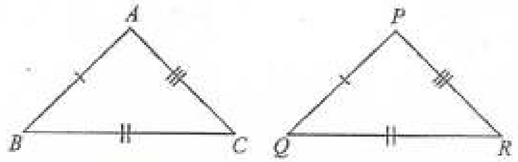


Solution: In Δs ABD and ACE,

	$AB = AC$	(given)
	$\angle BAD = \angle CAE$	(common)
	$\angle E = \angle D = 90°$	(Given)
$\therefore$	$\Delta ABD \cong \Delta ACE$	(C.P.C.T.)
$\therefore$	$CE = BD$	**Proved.**

Side-Side-Side (SSS) Congruence Criterion

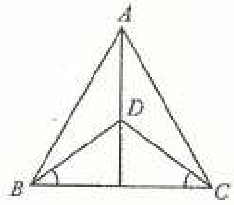
Two triangles are congruent if the three sides of one triangle are equal to the corresponding three sides of the other triangle.



If, $AB = PQ$, $BC = QR$, and $AC = PR$ then

$\Delta ABC \cong \Delta PQR$ (By S-S-S Congruency)

Example 12: If the adjoining figure, $AB = AC$, D is the point in the interior of ΔABC such that $\Delta DBC = \Delta DCB$. Prove that AD bisects $\angle BAC$ of ΔABC.



Solution: $\because$ $AB = AC$

$\therefore$ $\angle ABC = \angle ACB$ (Sides opposite to equal angles are equal)

Similarly

$\because$ $\angle DBC = \angle DCB$

$\therefore$ $BD = DC$ ...(i)

$\therefore$ In Δs ABD and ACD

$AB = AC$ (Given)

$AD = AD$ (Common)

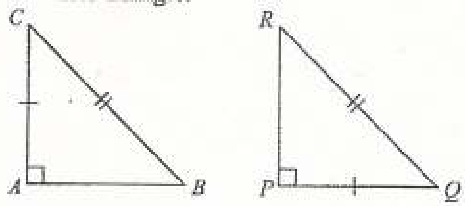
$BD = DC$ (From (i))

$\therefore$ $\angle ABD \cong \angle ACD$

$\therefore$ $\angle BAD = \angle CAD$ (C.P.C.T.)

Right Angle : Hypotenuse-Side (RHS) Congruence Criterion

Two right angles are congruent if the hypotenuse and one side of the triangle are respectively equal to the hypotenuse and one side of the other triangle.



If,

$BC = QR$ (hypotenuse)

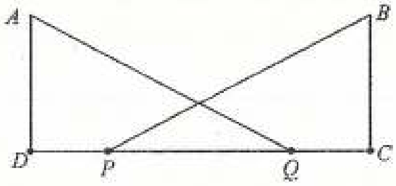
$AC = PQ$ (Side)

$\angle CAB = \angle RPQ = 90°$ (Right angle)

$\therefore$ $\Delta ACB = \Delta PQR$ (By R–H–S congruency)

Example 13: In the adjoining figure, $AD \perp CD$ and $CB \perp CD$. If $AQ = BP$ and $DP = CQ$ prove that

$\angle DAQ = \angle CBP$.



Solution: In ΔS ADQ and BCP

$$\angle ADQ = \angle BCP = 90^\circ$$

$$DP = CQ$$

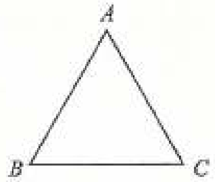
$\Rightarrow$ $DP + PQ = CQ + PQ$

$\Rightarrow$ $DQ = CP$ and $AQ = BP$ (given)

$\therefore$ $\Delta ADQ \cong \Delta BCP$ (By R-H-S congurency)

$\therefore$ $\angle DAQ = \angle CBP$ (C.P.C.T) **Proved.**

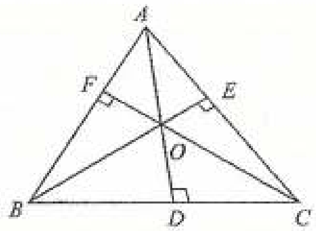
Inequalities in a Triangle



For any ΔABC,

1. The angle opposite to larger side is greater
   if $AC > AB$, then,
   $\angle ABC = \angle ACB$
   The converse is also true.
2. The sum of any two sides of a triangle is greater than the third side
   $AB + AC > BC$, $AB + BC > AC$ and $AC + BC > AB$.
3. The absolute difference between any two sides of a triangle is less than the third side.
   $|AB - BC| < AC$, $|AC - BC| < AB$, $|AB - AC| < BC$.
4. Of the all line segments that can be drawn to a given line, from a point, not lying on it the perpendicular line segment is the shortest.

Example 14: Show that the sum of the three altitudes of a triangle is less than the sum of three sides of the triangle.



Solution: We know that of all the line segments drawn from a given line the perpendicular length will be the shortest one.

$\therefore$ $AD + BC$

$\Rightarrow$ $AB > AD$ and $AC > AD$

$\Rightarrow$ $AB + AC > 2AD$ ...(i)

Similarly

$AB + BC > 2BC$, and ...(ii)

$AC + BC > 2CF$ ...(iii)

Adding (i),(ii)and (iii), we get

$$2(AB + BC + CA) > 2(AD + BE + CF)$$

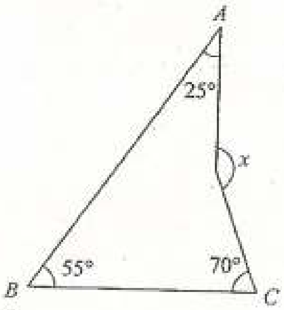
$\Rightarrow$ $AB + BC + CA > AD + BE + CF$ **Proved.**

Multiple Choice Questions

1. If one angle of the triangle is equal to the sum of the other two angles then the triangle is
   (a) Acute angled triangle
   (b) Isosceles/ equilateral triangle
   (c) Obtuse angled triangle
   (d) Right angled triangle

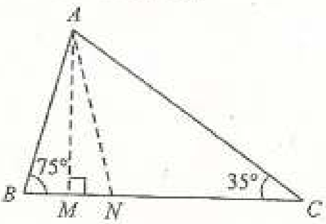
2. An exterior angle of a triangle is $100°$ and the interior opposite angles are in ratio $1:4$. The measure of the smallest angle of the triangle is
   (a) $70°$ (b) $80°$ (c) $20°$ (d) $30°$

3. Find x in the given figure



   (a) $120°$ (b) $135°$
   (c) $150°$ (d) $110°$

4. AN is the bisector of $\angle A$ and $AM \perp BC$. Then measure of $\angle MAN$ is :

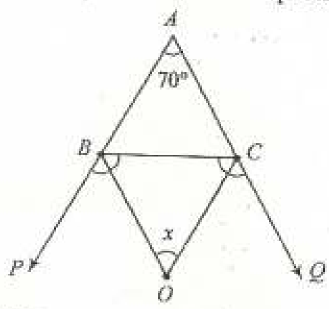


   (a) $35°$ (b) $30°$ (c) $20°$ (d) $25°$

5. If the bisectors of the acute angles of a right triangle meet at O, then the angle between the two bisectors at O is equal to:
   (a) $45°$ (b) $90°$ (c) $135°$ (d) $95°$

6. The sum of all the exterior angles of a triangle is
   (a) $180°$ (b) $360°$ (c) $540°$ (d) $270°$

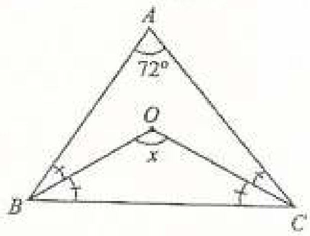
7. Find x if BO and CO are the bisected of exterior angles at B and C respectively.



   (a) $115°$ (b) $125°$
   (c) $65°$ (d) $55°$

8. In an isosceles triangle $AB = AC$. Side AB is extended to P such that $\angle CAP = 108°$. The measure of $\angle ABC$ is :
   (a) $30°$ (b) $126°$
   (c) $108°$ (d) $54°$

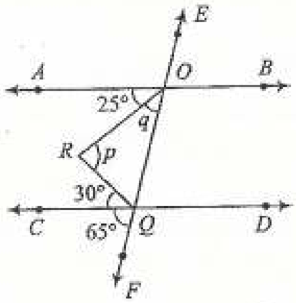
9. The value of x from the adjoining figure will be



   (a) $116°$ (b) $126°$
   (c) $108°$ (d) $132°$

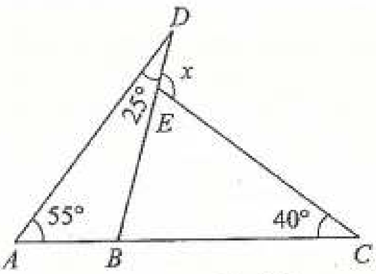
10. Side QR of a triangle PQR is produced both ways and the measures of exterior angles formed are $86°$ and $124°$. The measure of $\angle P$ is :
   (a) $30°$ (b) $40°$
   (c) $60°$ (d) $80°$

11. AB and CD are parallel lines and transversal EF intersects them at P and Q respectively. If $\angle APR = 25°$, $\angle RQC = 30°$ and $\angle CQF = 65°$ then



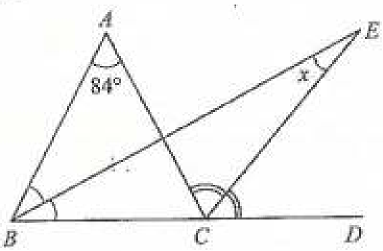
(a) $p = 55°, q = 40°$ (b) $p = 50°, q = 45°$
(c) $p = 35°, q = 60°$ (d) $p = 60°, q = 35°$

12. The value of x in the adjoining figure will be :



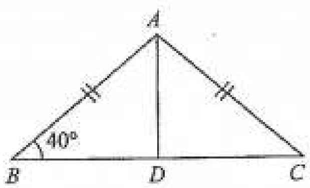
(a) 120° (b) 90°
(c) 65° (d) 80°

13. The value of x from the adjoining figure will be :



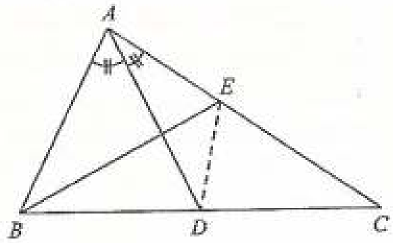
(a) 41° (b) 45°
(c) 42° (d) 48°

14. ABC is an isosceles such that $AB = AC$ and AD is the median to base BC. Then, $\angle BAD =$



(a) 40° (b) 50° (c) 60° (d) 100°

15. ABC is a triangle in which $\angle B = 2\angle C$. D is a point on BC such that AD bisects $\angle BAC$ and $AB = CD$ .BE is the bisector of $\angle B$. The measure of $\angle BAC$ is



(a) 74° (b) 73°
(c) 72° (d) 95°

16. O is any point in the interior of ΔABC, then
   (a) $AB + AC = OB + OC$
   (b) $AB + AC < OB + OC$
   (c) $AB + AC > OB + OC$
   (d) $AB + BC + AC < OA + OB + OC$

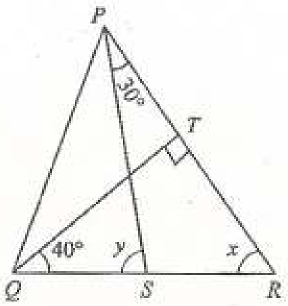
17. $ABCD$ is a quadrilateral having AC as a diagonal, then
   (a) $CD + DC + AB + BC < 2AC$
   (b) $CD + DA + AB + BC = 2AC$
   (c) $CD + DC + AB + BC > 2AC$
   (d) $CD + DA + AB < BC$

18. In ΔPQR, S is any point on the side QR. Then
   (a) $PQ + QR + QP > 2PS$
   (b) $PQ + QR + RP < 2PS$
   (c) $PQ + QR + RP = 2PS$
   (d) $PQ + QR + RP < PS$

19. In ΔABC, $AC > AB$ and AD is the bisector of $\angle A$. Then
   (a) $\angle ADC < 2\angle ADB$
   (b) $\angle ADC < \angle ADB$
   (c) $\angle ADC > \angle ADB$
   (d) $\angle ADC = \angle ADB$

20. In a $\Delta ABC, \angle A = 50°$,$\angle B = 60°$. The longest side of the triangle will be
   (a) AB (b) BC
   (c) CA (d) None of these

21. If $QT \perp PR$, $\angle TQR = 40°$ and $\angle SPR = 30°$. The value of $x + y$ is :



(a) 120° (b) 130°
(c) 110° (d) 100°

22. In a right angled triangle, one acute angle is double the other. If the length of hypotenuse of the triangle be x, then the length of the smallest side is :

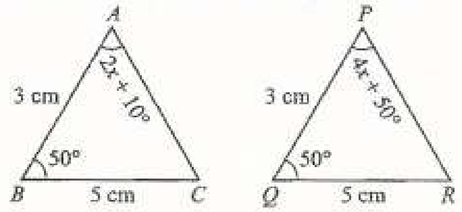
(a) $\frac{x}{3}$ (b) $\frac{x}{2}$

(c) $\frac{x}{4}$ (d) $\frac{2x}{3}$

23. If two isosceles triangles have a common base, then the line joining their vertices will
(a) Bisect them at acute angle
(b) Bisect them at obtuse angle
(c) Bisect them at right angle
(d) NOT

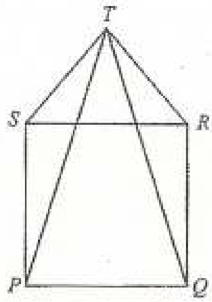
24. If the length of three of the altitudes of a triangle are equal, then the triangle must be a/an
(a) Isosceles triangle (b) Equilateral triangle
(c) Scalene triangle (d) Right triangle

25.



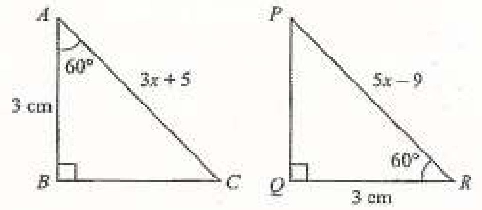
The value of x will be :
(a) 20° (b) 40°
(c) 30° (d) 60°

26. $PQRS$ is a square and SRT is an equilateral triangle. The measure of $\angle TQR$ is :



(a) 25° (b) 55°
(c) 15° (d) 35°

27.

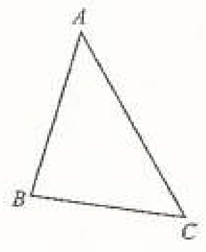


The value of x will be
(a) 8 (b) 7 (c) 6 (d) 5

28. In ΔABC, $AB = AC$, and the bisect are of angles B and C intersect at point O, then the ray AO
(a) will bisect $\angle A$
(b) will not bisect $\angle A$
(c) $AO = CO$
(d) $AO = BO$

29. P is a point equidistant from two lines l and m intersecting at point A, then
(a) $\angle BAP = \angle APC$ (b) $\angle CAP = \angle BPA$
(c) $\angle CAP = \angle BAP$ (d) None of there

30. In the adjoining figure $AB = BC$. If $\angle BAC = 60°$, then, the measure of $\angle ABC$ will be :



(a) 30° (b) 45°
(c) 60° (d) 90

Answer Key

1. (d)	2. (c)	3. (c)	4. (c)	5. (c)	6. (b)	7. (a)	8. (d)	9. (b)	10. (a)
11. (a)	12. (a)	13. (c)	14. (b)	15. (c)	16. (c)	17. (c)	18. (a)	19. (c)	20. (a)
21. (b)	22. (b)	23. (c)	24. (b)	25. (b)	26. (c)	27. (b)	28. (a)	29. (c)	30. (c)

Hints and Solutions

1. **(d)** Let the angles of triangle be x, y and $180° - (x + y)$.

$\therefore \quad 180° - (x + y) = (x + y)$

$\Rightarrow \quad 2(x + y) = 180°$

$\Rightarrow \quad x + y = 90°$

$\therefore \ (180° - (x + y)) = 180° - 90° = 90°$

$\therefore$ The triangle will be right angled triangle

2. **(c)** Let the measure of interior opposite angles be x and $4x$ respectively.

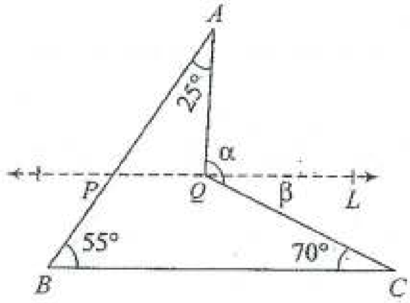
$\therefore$ A/Q,

$x + 4x = 100°$

$\Rightarrow \quad 5x = 100° \Rightarrow x = 20°$

$\therefore$ The measure of the smallest angle $= x = 20°$

3. **(c)**



Constructing a line $PQ \parallel BC$,

$\angle APQ = \angle ABC = 55°$

$\because \angle AQL$ is an exterior angle for ΔAPQ

$\therefore \quad \angle APQ + 25° = \alpha$

$\Rightarrow \quad \alpha = 25° + 55° = 80°$

$\beta = 70°$ (Alternate opposite$\angle$s)

$\therefore x = \alpha + \beta = 70° + 80° = 150°$

4. **(c)** Here $\angle BAC = 180° - (75° + 35°) = 70°$

$$\angle BAN = \angle NAC = \frac{\angle BAC}{2} = \frac{70°}{2} = 35°$$

($\because AN$ is angle bisector of $\angle A$)

Now, in ΔANC,

$\angle ANC + \angle CAN + \angle NAC = 180°$

$\Rightarrow \quad \angle ANC + 35° + 35° = 180°$

$\Rightarrow \quad \angle ANC = 110°$

$\because \angle ANC$ is an exterior angle for ΔAMN

$\because \quad \angle ANC + \angle MAN = 110°$

$\Rightarrow \quad \angle MAN = 110° - \angle AMN$

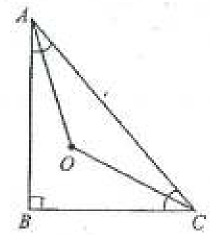
$= 110° - 90° = 20°$

5. **(c)** $\angle AOC = 90° + \frac{1}{2}\angle B$

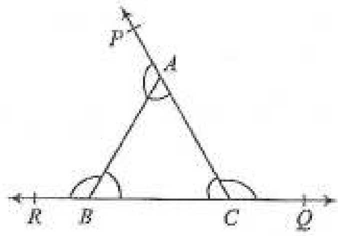
$= 90° + \frac{1}{2} \times 90°$

$= 90° + 45°$

$= 135°$



6. **(b)** In ΔABC



$\angle ABC + \angle ACB + \angle BAC = 180°$

Now,

using exterior angle theorem

$\angle ACB + \angle ABC = \angle BAP$ ...(i)

$\angle ABC + \angle BAC = \angle ACQ$ ...(ii)

$\angle ACB + \angle BAC = \angle ABR$ ...(iii)

Adding Eqns (i), (ii) and (iii), we get

$2(\angle ABC + \angle BAC + \angle ABC)$

$= \angle BAP + \angle ACQ + \angle ABR$

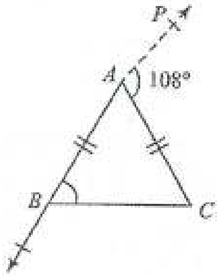
$\Rightarrow$ Sum of all exterior angle $= 2\times180° = 360°$

7. **(a)** $x = 90° - \frac{1}{2}\angle A$

$= 90° - \frac{1}{2} \times 70°$

$= 90° - 35° = 55°$

8. **(d)**



$\because$ $AB = AC$

$\therefore$ $\angle ABC = \angle ACB$

$\because \angle CAP$ is an exterior angle for ΔABC,

$\because$ $\angle CAP = \angle ABC + \angle ACB$ [using (i)]

$\Rightarrow$ $108° = 2\angle ABC$

$\Rightarrow \angle ABC = \frac{108°}{2} = 54°$

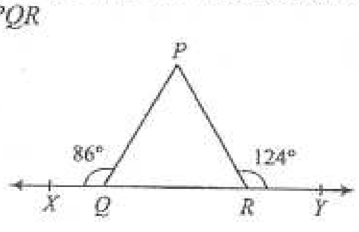
9. **(b)** $x = 90° + \frac{1}{2}\angle A$

$= 90° + \frac{1}{2} \times 72°$

$= 90° + 36°$

$= 126°$

10. **(a)** $\angle PQX$ and $\angle PRY$ are exterior angles for ΔPQR



$\therefore$ $\angle P + \angle PRY = 86°$ ...(i)

$\angle P + \angle PQX = 124°$ ...(ii)

Adding (i) and (ii)

$2\angle P + \angle PRX + \angle PQX = 210°$

$\Rightarrow \angle P + (\angle P + \angle PRX + \angle PQX) = 210°$

$\Rightarrow$ $\angle P + 180° = 210°$

$\Rightarrow$ $\angle P = 30°$

11. **(a)** $\angle APR + \angle CQF = 65°$

(corresponding $\angle s$)

$\angle APR + \angle OPQ = 65°$

$\Rightarrow$ $25° + Q = 65°$

$\Rightarrow$ $Q = 40°$

$\because \angle OQF$ is on exterior angle for $DPOQ$

$\therefore$ $q + \Delta POQ = 65° + 30°$

$\Rightarrow$ $p + q = 65°$

$\Rightarrow$ $p = 95° - 40° = 55°$

12. **(a)** $\because$ $\angle DBC$ is exterior angle for $\angle DAB$

$\therefore$ $\angle ADB + \angle DAB = \angle DBC$

$\Rightarrow$ $\angle DBC = 25° + 55° = 80°$

$\because \angle x$ is an exterior angle for $\angle EBC$

$\therefore$ $\angle EBC + \angle ECB = x$

$\Rightarrow$ $80° + 40° = x$

$\Rightarrow$ $x = 120°$

13. **(c)** $\angle ABC + \angle A = \angle ACD$

$\Rightarrow$ $\angle ACD = \angle ABC + 84°$

$\Rightarrow$ $\frac{\angle ACD}{2} = \frac{\angle ABC}{2} + 42°$

$\Rightarrow$ $\angle ECD = \angle EBC + 42°$ ...(i)

$\because \angle ECD$ is an exterior angle for $\angle EBC$

$\therefore$ $\angle ECD = \angle EBC = x$ ...(ii)

Comparing (i) and (ii), we get

$x = 42°$

14. **(b)** In Δs ABD and ACD,

$AB = AC$ (Given)

$AD = AD$ (Common)

$BD = CD$ ($\because AC$ is median)

$\therefore$ $\Delta ABD \cong \Delta ACD$

[By S-S-S congruence criterion]

$\therefore$ $\angle BAD = \frac{180° - \angle B - \angle C}{2}$

$$= \frac{180° - 40° - 40°}{2} = 50$$

15. **(c)** In Δs ABC and DCE

$$\angle ABE = \angle DCE = \frac{\angle B}{2}$$

$AB = CD$ (given)

$BE = CE$ $(\because \angle ABE = \angle DCE = \frac{\angle B}{2})$

$\therefore \Delta ABE \cong \Delta DCE$ [By S-S-S congruence]

$$\angle BAC = \left(\frac{108°}{3}\right) \times 2 = 36° \times 2 = 72°$$

16. **(c)** $AB + AC > BC$ ...(i)

$OB + OC > BC$ ...(ii)

Using (i) and (ii)

$$AB + AC > OB + OC$$

17. **(c)** In ΔABC

$AB + BC > AC$ ...(i)

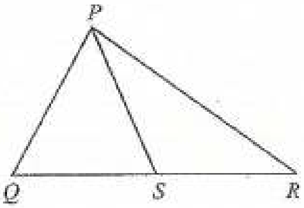
InΔADC,

$AD + DC > AC$ ...(ii)

Using (i) and (ii)

$$CD + AD + AB + BC > 2AC$$

18. **(a)**



In ΔPQS,

$PQ + QS > PS$ ...(i)

In ΔPSR,

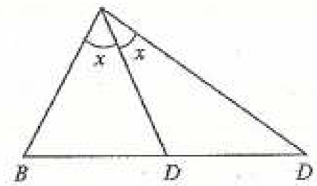
$PR + RS > PS$ ...(ii)

Using (i) and (ii)

$$PQ + PR + (QS + RS) > 2PS$$

$$\Rightarrow PQ + PR + (QR + QR) > 2PS$$

19. **(c)** In ΔABC, A



$\because$ $AC > AB$

$\therefore$ $\angle ABC = \angle ACB$

$\Rightarrow$ $-\angle ABC < -\angle ACB$

$\Rightarrow 180° - \angle ABC < 180° - \angle ACB$

$\Rightarrow 180° - \angle ABC - x < 180° - \angle ACB - x$

$\Rightarrow$ $\angle ADB < \angle ADC.$

20. **(a)** $\angle A = 50°$, $\angle B = 60°$

$\angle C = 180° - (\angle A + \angle B)$

$= 180° - 110°$

$= 70°$

$\because \angle C$ is the largest angle of ΔABC

$\therefore AB$ is the largest side of ΔABC.

21. **(b)** In ΔQTR

$\angle QTR + \angle Q + \angle R = 180°$

$\Rightarrow 90° + 40° + x = 180°$

$\Rightarrow$ $x = 50°$

$\because \angle PSQ$ is an exterior angle for $\angle PRS$

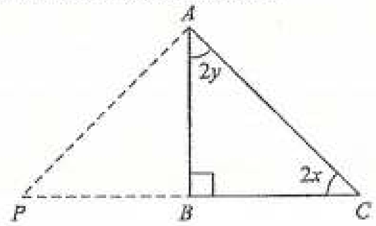
$\therefore$ $\angle PRS + \angle SPR = y$

$\Rightarrow$ $y = 50° + 30° = 80°$

$\therefore$ $x + y = 50° + 80° = 130°$

22. **(b)** $\because \angle BAC$ is the smallest angle

$\therefore BC$ will be the smallest side



Now,

Connecting ΔAPB in such a way that

$PB = BC$

In Δs APB and ACB

$\angle ABC = \angle ABP = 90°$

$PB = BC$ (Common)

$\therefore$ $\Delta ABC \cong \Delta ABP$

(by S-A-S congruence criterion)

$\therefore \quad PA \cong AC$ (CPCT)

$\therefore \quad \angle PAB = \angle BAC = y$ (say)

Now

In ΔAPC

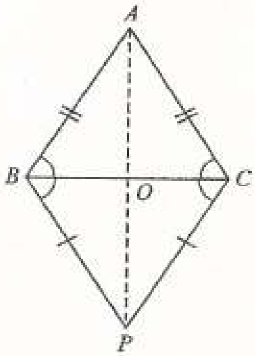
$\because \quad \angle A = \angle C$

$\therefore \quad PC = PA$

$\Rightarrow \quad 2BC = AC$

$\Rightarrow \quad BC = \dfrac{AC}{2} = \dfrac{x}{2}$

23. **(c)**



Given : ΔABC and ΔBCP are isosceles ΔS with common base BC.

In ΔS ABP and ACP

$AB = AC$ (given)

$BP = PC$ (given)

$AP = AP$ (common)

$\therefore \quad \Delta ABP \cong \Delta ACP$ (S-S-S criterion)

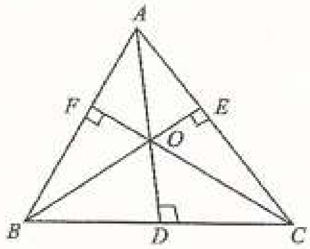
$\therefore \quad BO = OC$ and

$\angle AOB = \angle AOC = 180^\circ$

$\Rightarrow \quad 2\angle AOC = 180^\circ$

$\Rightarrow \quad \angle AOB = 90^\circ$

24.. **(b)** In Δs BEC and CFB



$BC = BC$ (Common)

$BE = CF$ (Given)

$\angle BEC = \angle CFB = 90^\circ$

$\therefore \quad \Delta BEC = \angle CFB = 90^\circ$

(By R-H-S congruence criterion)

$\angle B = \angle C$ (C-P-C-T)

$\therefore \quad AB = AC$ ...(i)

Similarly in Δs ADC and $C + A$

$\Rightarrow \quad \angle A = \angle C$

$\therefore \quad AB = BC$ ...(ii)

Using (i) and (ii)

$AB = BC = AC$ (Δ should be equilateral)

25. **(b)** In Δs ABC and PQR

$AB = PQ = 3\text{cm}$

$BC = QR = 5\text{cm}$

$\angle ABC = \angle PQR = 50^\circ$

$\therefore \quad \angle ABC \cong \angle PQR$ (By S-A-S criterion)

$\angle BAC = \angle QPR$ (C.P.C.T)

$\Rightarrow \quad 2x + 10^\circ = x + 50^\circ$

$\Rightarrow \quad x = 40^\circ$

26. **(c)** In ΔS PTS and QTR

$(TR = TS = SR = PQ = QR = PS)$

$TR = TR$ (given)

$PS = QR$ (given)

$\angle PST = \angle QRT$

$= 90^\circ + 60^\circ = 150^\circ$
(Square) (equilateral Δ)

$\therefore \quad \Delta PTS = \Delta QTR$

(By R-H-S congruence criterion)

$TP = TQ$ (C.P.C.T)

$\therefore \quad \angle TPS = \angle TQR$ (C.P.C.T)

Now,

In ΔTQR

$TR = RQ$

$\therefore \quad \angle RTQ = \angle RQT$, and

$\angle RTQ + \angle RQT + \angle R = 180^\circ$

$\Rightarrow \quad 2\angle RQT + 90^\circ + 60^\circ = 180^\circ$

$\Rightarrow \quad \angle RQT = \dfrac{180^\circ - 150^\circ}{2} = 15^\circ$

27. **(b)** In Δs BCA and QRP

$\angle A = \angle P = 60^\circ$

$AB = QR = 3$ cm

$\angle ABC = \angle PQR = 90^\circ$

$\therefore \quad \Delta ABC \cong \Delta RQP$

(By R-H-S congruence criterion)

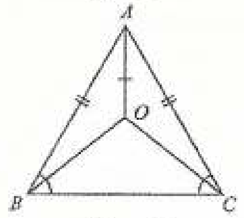
$\therefore \quad AC = RP$ (C.P.C.T)

$\Rightarrow \quad 3x + 5 = 5x - 9$

$\Rightarrow \quad 2x = 14$

$\Rightarrow \quad x = 7$

28. **(a)**



$\because \quad \angle B = \angle C$

$\Rightarrow \frac{\angle B}{2} = \frac{\angle C}{2} \Rightarrow \angle OBC = \angle OCB$

$\Rightarrow \quad OB = OC$ ...(i)

(sides opposite to equal angles are equal)

Now,

In Δs ABO and ACO

$AO = AO$ (common)

$AB = AC$ (given)

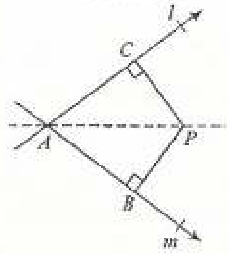
$OB = OC$ (From (i))

$\therefore \quad \Delta ABO \cong \Delta ACO$

(By R-H-S congruence criterion)

$\therefore \quad \angle BAO = \angle CAO = \frac{\angle A}{2}$ (C.P.C.T)

29. **(c)** In Δs ABP and ACP



$PC = PB$ (Given)

$\angle ACP = \angle ABP = 90°$

$AP = AP$ (common)

$\therefore \quad \Delta ABP \cong \Delta ACP$

(By R-H-S congruence criterion)

$\Rightarrow \quad \angle BAP = \angle CAP$ (C.P.C.T)

30. **(c)** $\because \quad AB = AC$

$\therefore \quad \angle BAP = \angle CAP$

(Angles opposite to equal sides are equal)

Now, in ΔABC

$\angle ABC = \angle ACB + \angle A = 180°$

$\Rightarrow \quad 2\angle ACB + 60° = 180°$

$\Rightarrow \quad 2\angle ACB = 120°$

$\Rightarrow \quad \angle ACB = 60°$

8. Quadrilaterals

Learning Objective:

In this chapter, we will learn about:

*Quadrilateral

*Types of Quadrilaterals

*Properties of Quadrilateral

Quadrilateral

The word 'quad' means four and the word ' lethal' means sides so, a plane figure bounded by four lines is called a quadrilateral. A quadrilateral has four sides, four angles and four vertices.

Angle Sum Property of a Quadrilateral

The sum of four angles of a quadrilateral is 360°.

Example 1: The angles of a quadrilateral are respectively 90°,90°,110°. The measure of the 4^{th} angle will be

Solution: Let the measure of angle be x

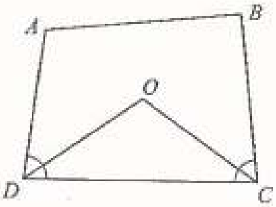
$\therefore$ Applying angle sum property;

$$90° + 90° + 110° + x = 360°$$

$$\Rightarrow \quad x = 70°$$

Example 2: In a quadrilateral $ABCD$, CO and DO are the bisectors of $\angle C$ and $\angle D$ respectively. Prove that $\angle DOC = \frac{1}{2}(\Delta A + \angle B)$.

Solution: $\because ABCD$ is a quadrilateral



$$\therefore \angle A + \angle B + \angle C + \angle D = (180°)2 = 360°$$

$$\Rightarrow \quad \frac{1}{2}(\angle A + \angle B) = 180° - \left(\frac{\angle C}{2} + \frac{\angle D}{2}\right) \quad ...(i)$$

In ΔODC,

$$\angle ODC + \angle OCD = 180° - \angle DOC$$

$$\Rightarrow \quad \angle DOC = 180° - (\angle ODC + \angle OCD)$$

$$= 180° - \left(\frac{\angle C}{2} + \frac{\angle D}{2}\right) \quad ...(ii)$$

Using (i) and (ii)

$$\angle DOC = \frac{1}{2}(\angle A + \angle B)$$

Hence Proved

Types of Quadrilaterals

Trapezium

A quadrilateral having one pair of parallel sides and one pair of non-parallel sides is called a trapezium. When the length of non-parallel sides are equal, then, the trapezium is said to be an isosceles trapezium.

Parallelogram

A quadrilateral is a parallelogram if its both pair of sides are parallel.

In parallelogram the opposite pair of sides are parallel

Hence $AB = CD$ and, $AD = BC$

Rhombus

A parallelogram having all sides equal is called a rhombus.

Rectangle

A parallelogram whose each angle measures 90°, is called a rectangle .

Square

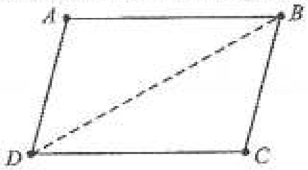
A square is a rectangle whose all sides are equal

Kite

A quadrilateral is a kite if it has row pairs of equal adjacent sides and opposite sides are unequal and non-parallel.

Properties of a Parallelogram

1. A diagonal of a parallelogram divides it into two congruent ΔS.



∴ In ||gm $ABCD$

$$\Delta ABC \cong \Delta CDB$$

(a) $AB = CD$ and $AD = BC$ [Opposite sides of ||gm are equal]

(b) $\angle DAB = \angle BCD$ [Opposite sides of ||gm are equal]

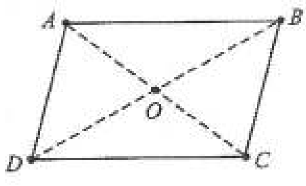
(c) $\angle ABD = \angle BDC$ [Alternate angles are equal]

[∴ Opposite sides of || gm are parallel]

2. The length of diagonals of a ||gm may be equator unequal either,

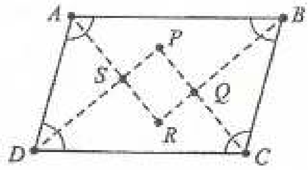
$AC = BD$ [Square, rectangle etc]

Or, $AC \neq BD$ [rhombus, trapezium, etc]



3. The diagonals of a ||gm bisect each other.

Example 3: Prove that the angle bisectors of a ||gm form a rectangle.



Solution: **To Prove :** $PQRS$ is a rectangle.

Proof : In ΔABR and ΔDPC

$\angle ABR + \angle BAR + \angle R = 180°$ and ...(i)

$\angle DPC + \angle DCP + \angle P = 180°$ ...(ii)

From (i)

$$\angle ABR + \angle BAR + \angle R = 180°$$

$$\Rightarrow \quad \left(\frac{\angle B + \angle A}{2}\right) + \angle R = 180°$$

$$\Rightarrow \quad \left(\frac{180°}{2}\right) + \angle R = 180°$$

$$\Rightarrow \quad \angle R = 90°$$

Similarly

$$\angle P = 90°$$

$\because$ Opposite angles are equal and their measure are also equal 90°.

$\therefore$ $PQRS$ is a rectangle. Proved

Main Theorems

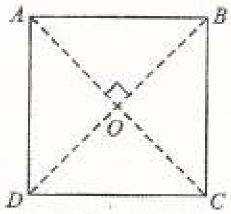
Theorem: A quadrilateral is a parallelogram if its opposite sides are equal.

Theorem: A quadrilateral is a parallelogram if its opposite angles are equal.

Theorem: A quadrilateral is a parallelogram if the diagonals bisect equal other.

Theorem: A quadrilateral is a parallelogram if its all pair of sides are parallel and equal.

Example 4: If the diagonals of a quadrilateral are equal and bisect each other at right angles prove that the quadrilateral must be a square.



Solution: In ΔABO

$$AO = BO \quad (\because AC = BD)$$

$$\therefore \quad \angle OAB = \angle OBA = \frac{180° - \angle AOB}{2} = \frac{90°}{2} = 45° \quad \text{...(i)}$$

Similarly,

$$\angle OAD = \angle ODA = \angle ODC = \angle OCD = \angle OBC = \angle OCB \quad \text{...(ii)}$$

Now

$$\angle A = \angle OAD + \angle OAB$$

$$\angle B = \angle OBA + \angle OBC,\ \angle C = \angle OCB + \angle OCD,\ \angle D = \angle OCB + \angle ODC$$

$$\therefore \quad \angle A = \angle B = \angle C = \angle D = 45° + 45° = 90° \quad \text{...(a)}$$

Also

$$\angle A = \angle B = \angle D = \angle A = 180° \quad \text{(Sum of interior angles is 180°)}$$

$$\because \quad AB \parallel CD \text{ and } BC \parallel AD$$

$$\therefore \quad AB = CD \text{ and } AD = BC \quad \text{...(iii)}$$

Now

In Δs AOD and AOB

$\angle AOD = \angle AOB = 90°$ (given)

$AO = AO$ (Common)

$DO = OB$ (Given)

$\therefore$ $\angle AOD \cong \angle AOB$ (By S-A-S criterion)

$\therefore$ $AD = AB$ ...(iv)

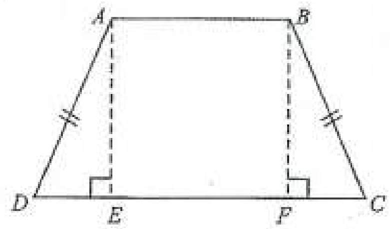
$\therefore$ Using (i), (ii), (iii), (iv) and (a)

$$\angle A = \angle B = \angle C = \angle D = 90° \text{ and } AB = BC = CD = DA$$

$\therefore$ $ABCD$ is a square. Proved

Example 5: If $ABCD$ is a quadrilateral in which $AB \parallel CD$ and $AD = BC$ prove that $\angle A = \angle B$.

Solution: Construction, draw $AE \perp DC$ and $BF \perp DC$.



Now

$\because$ AE and BF are perpendiculars between the same parallels

$\therefore$ $AE = BF$ ...(i)

In ΔS AED and BFC

$AE = BF$ [From (i)]

$AD = BC$ [Given]

$\angle AED = \angle BFC = 90°$

$\therefore$ $\Delta AED \cong \Delta BFC$ (By S -A-S criterion)

$\therefore$ $\angle DAE = \angle CBF$ [C.P.C.T.] ...(ii)

Now

$$\angle A = \angle DAE + \angle EAB = \angle DAE + 90°$$

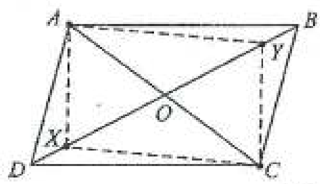
$$\angle B = \angle CBA + \angle FBA = \angle CBF + 90°$$

$\therefore$ $\angle A = \angle B$ (From(ii))

Proved.

Example 6 $ABCD$ is a parallelogram and X and Y are points on the diagonal BD such that $DX = BY$. Prove that $AXCY$ is a parallelogram.

Solution: According to theorem If the diagonals of a quadrilateral bisect each other then , then it will be a $\|^{gm}$



$\because$ $ABCD$ is parallelogram

$\therefore$ $AO = OC$ ...(i) and

$BO = OD$ ...(ii)

$DX = BY$ ...(iii)

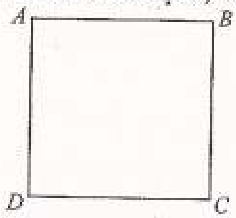
Subtracting (ii) and (iii)

$$OX = OY \qquad ...(iv)$$

$\therefore$ $AXCY$ is a $\|^{gm}$ (From (i) and (iv))

Properties of a Rectangle, Rhombus and Square

1. Since, rectangle, rhombus and square are parallelograms. So all the theorems of $\|^{gm}$ are valid for quadrilaterals. If any one of the angles of a $\|^{gm} = 90°$ then $\|^{gm}$ may be rectangle or square.
2. Each of the four angles of a rectangle and square measure 90° but in rhombus none of the angles will be a right angle.
3. A rhombus and a square have all of their sides equal, i.e, If $ABCD$ is a rhombus or square then



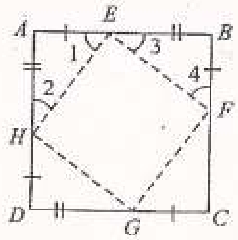
$$AB = BC = CD = DA$$

4. A rectangle or square have equal length of the both diagrams but a rhombus has always unequal length of the diagonals.
5. The diagonals of a rhombus or a square intersect at 90°, but in a rectangle the diagrams will not intersect at right angles.

Example 7: $ABCD$ is a square. E, F, G and H are points on A, B, C, D respectively such that $AE = BF = CG = DH$. Prove that $EFGH$ is a square.

Solution: **To prove:** $EF = FG = GH = EH$ and

$$\angle E = \angle F = \angle G = \angle H = 90°$$



Proof:

$\because$ $ABCD$ is a square.

$\because$ $AB = BC = CD = DA$, and $AE = BF = CG = DH$

$\therefore$ $AB - AE = BC - BF = CD - CG = DA - DH$

$\Rightarrow$ $EB = CF = DG = AH$

$\therefore$ $\Delta AHE \cong \Delta BEF \cong \Delta CFG \cong \Delta HDG$

[By SAS congruence criterion]

$\therefore$ $EFGH$ is a parallelogram (opposite sides are equal)

In the figure,

$2 = \angle 4$ and $\angle 1 = \angle 3$ (c. p. c. t.)

$\therefore$ $\angle 1 + \angle 2 = \angle 3 + \angle 4 = 90°$

$\Rightarrow$ $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 2 \times 90° = 180°$

$\Rightarrow$ $2(\angle 1 + \angle 3) = 180°$

$\Rightarrow$ $\angle 1 + \angle 3 = 90°$ ...(i)

$\because$ $\angle HEF + \angle 1 + \angle 3 = 90° \times 2 = 180°$

$\Rightarrow \quad \angle HEF = 90°$ [Using (i)]

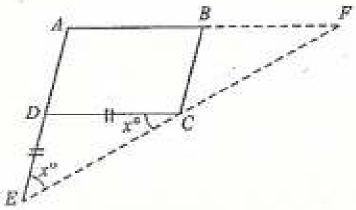
$\because$ $EFGH$ is a $\|^{gm}$ having all the sides equal and one of the angles = 90°

$\therefore$ $EFGH$ must be a square

Hence Proved.

Example 8: $ABCD$ is a $\|^{gm}$. AD is produced to E so that $DE = DC$ and EC produced meets AB produced in F. Prove that $BC = BF$.

Solution: Let $\angle DCE = x°$



For ΔDEC, $\angle ADC$ is an exterior angle

$\therefore \quad \angle ADC = x° + x° = 2x°$

$\therefore \quad \angle DAB = 180° - 2x° \quad [\because AB \parallel BC]$

Now, in $DEAF$,

$$\angle A + \angle E + \angle F = 180°$$

$$\Rightarrow \quad \angle F = 180° - x° - (180° - 2x°) = x°$$

$$\therefore \quad \angle E = \angle F$$

$$\Rightarrow \quad AE = AF$$

$$\Rightarrow \quad AD + DE = AB + BF \quad ...(i)$$

$$\because \quad DE = DC = AB \text{ and } AD = BC$$

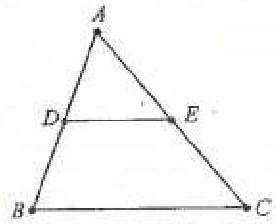
$$\therefore \quad BC = \text{BF}$$

(Using (i)) Proved

Facts about Triangle

Mid-Point Theorem: The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it and vice versa.

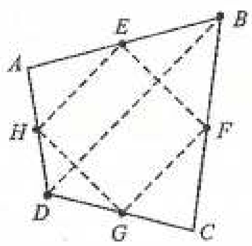
Example: In $\angle ABC$, D and E are the mid-points of AB and AC respectively.



$$DE \parallel BC \text{ and } DE = \frac{1}{2}\ BC$$

Example 9: Prove that the quadrilateral formed by joining the mid-points of any quadrilateral is a $\|^{gm}$.

Solution: E, F, G, H are the mid-points of AB, BC, CD, DA respectively



Now,

in ΔADB

$\because$ E and H are mid-points of AB and AD respectively, then using mid-point theorem,

$$EH = \frac{1}{2}\ BD \text{ and } EH \parallel BD$$

Similarly, in ΔADB

$$GF = \frac{1}{2}\ BD \text{ and } GF \parallel BD$$

$\therefore$ $GF = EH$ and $GF \parallel EH$

$\therefore$ $EFGH$ is a $\|^{gm}$ (One pair of sides is parallel and equal)

Example 10: In the adjoining figure, AD is any line from A to BC intersecting BE in H. P, Q and R are the mid points of AD, AB and BC respectively. Prove that $\angle PQR = 90°$.

Solution: In ΔABH

Q and P are midpoints of AB and AH respectively

$\because$ $QP \parallel BH$ or $QP \parallel BE$ ...(i)

Similarly

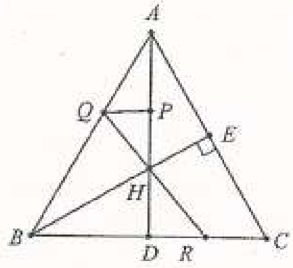
In ΔABC

$QR \parallel AC$ ...(ii)

$\because$ $QP \parallel BE$ and $BE \perp AC$

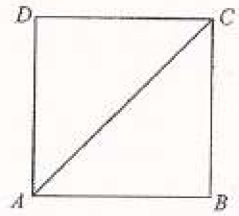
$\therefore$ $QR \perp BE$ or $QR \perp QP$

$\therefore$ $\angle PQR = 90°$ Proved

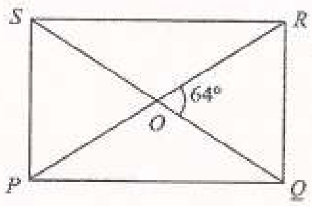


Multiple Choice Questions

1. In a quadrilateral, the angles are in the ratio 1 : 2 : 3 : 4. What is the value of largest angle?
   (a) 108° (b) 144°
   (c) 136° (d) 124°
2. Two opposite angles of a parallelogram are $(3x - 2)°$ and $(50 - x)°$. Find the smallest angle.
   (a) 37° (b) 43°
   (c) 47° (d) 57°
3. The perimeter of a parallelogram is 24 cm. If the longer side measures 8 cm. Then what is the measure of shroter side?
   (a) 4 cm (b) 6 cm
   (c) 2 cm (d) None of There
4. If an angle of a parallelogram is one third of its adjacent angle then what is the measure of smallest angle?
   (a) 135° (b) 45°
   (c) 60° (d) 115°
5. $ABCD$ is a square. What is the value of $\angle ACD$?



   (a) 40° (b) 45°
   (c) 50° (d) 30°
6. The diagonals of a rectangle $PQRS$ meet at O. If $\angle SOR = 64°$ then Find $\angle OAC$?



   (a) 60° (b) 58° (c) 62° (d) 64°
7. The figure formed by joining the mid-points of consecutive sides of a quadrilateral is a
   (a) Parallelogram (b) Trapezium
   (c) Rectangle (d) None of these

8. The angles of a quadrilateral are 98°, 92°,70° respectively. What is the measure of 4[th] angle?

(a) 92° (b) 98°
(c) 100° (d) None of these

9. The angles of a quadrilateral are in the ratio 2:4:5:7. What is the difference between largest and smallest angle?

(a) 80° (b) 100° (c) 60° (d) 90°

10. If the length of each side of rhombus is 15 cm and one of its diagonals is 24 cm what is length of other diagonal?

(a) 16 cm (b) 14 cm (c) 18 cm (d) 12 cm

11. If an angle of a parallelogram is two third of its adjacent angle what is the measure of smallest angle of parallelogram?

(a) 81° (b) 72° (c) 54° (d) 108°

12. In which of the following figures are the diagonals equal?

(a) Rectangle (b) Parallelogram
(c) Rhombus (d) Trapezium

13. If $\angle P$, $\angle Q$, $\angle R$, $\angle S$ of a quadrilateral $PQRS$ taken in order are in the ratio $3:7:6:4$, then $PQRS$ is a

(a) Kite (b) Trapezium
(c) Rhombus (d) Parallelogram

14. The angles of a quadrilateral are in the ratio $3:5:9:13$. What is the sum of largest and smallest angle of quadrilateral?

(a) 168° (b) 192°
(c) 144° (d) None of these

15. The figure formed by joining the mid-points of the adjacent sides of a square is

(a) Parallelogram (b) Rectangle
(c) Rhombus (d) Square

16. $ABCD$ is a parallelogram in which W, X, Y, Z are mid – points of sides AB, BC, CD and DA respectively. AC is the diagonal, then which of the following is correct?

(a) $YZ = AC$ (b) $YZ = \frac{1}{2}$
(c) $YZ = \frac{1}{2}$ (d) None of these

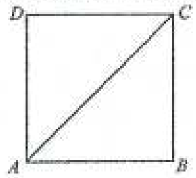
17. D is the mid-point of side AB of a parallelogram $ABCD$. A line through B parallel to PD meets DC at Q and AD produced at R, then which of the following is correct?

(a) $AR = BC$ (b) $AR = \frac{1}{2}BC$
(c) $AR = 2BC$ (d) $AR = 3BC$

18. If consecutive sides of a parallelogram are equal then it is a (none of the angle $\neq 90°$)

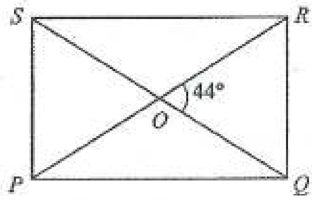
(a) Kite (b) Rectangle
(c) Rhombus (d) Square

19. If $ABCD$ is a square then what is the measure of $\angle DCA$?



(a) 45°
(b) 90°
(c) 55°
(d) None of these

20. The diagonals of a rectangle $PQRS$ meet at O. If $\angle QOR = 44°$ Then what is the measure of $\angle OPS$?

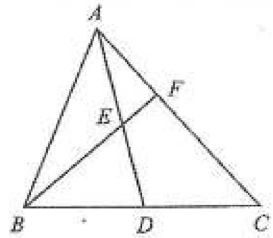


(a) 22° (b) 68° (c) 44° (d) 64°

21. $ABCD$ is a rhombus with $\angle ABC = 56°$. What is the measure of $\angle ACD$?

(a) 42° (b) 62° (c) 52° (d) 48°

22. In ΔABC, AD is the median through A and E is the mid-point of AD. BE produced meets AC in F. then which of the following is correct?



(a) $AF = \frac{1}{2}AC$ (b) $AF = \frac{2}{3}AC$

(c) $AF = \frac{1}{3}AC$ (d) None of these

23. The resulting figure obtained by joining the consecutive midpoint of sides of a rhombus will be a :
(a) Parallelogram having unequal diagonals.
(b) Square
(c) Rectangle
(d) rhombus

24. The resulting figure obtained from joining the consecutive mid points of side of a square is
(a) Rectangle (b) Square
(c) Trapezium (d) Rhombus

25. Select the correct statement
(a) Every rectangle is a square
(b) Every square is a rhombus
(c) Every rhombus is a parallelogram
(d) Every parallelogram is a rhombus

26. Select the incorrect statement
(a) Every square is a rectangle
(b) Every rectangle is a parallelogram
(c) The opposite angles of rhombus are equal
(d) Every kite is parallelogram

27. $ABCD$ is a parallelogram in which $\angle D$ = 120° if the bisectors of $\angle A$ and $\angle B$ meet at P, then
(a) $DC = AD$ (b) $DC = 2AD$
(c) $BC = AP$ (d) $PB = AB$

28. $ABCD$ is a $\|^{gm}$ and X, Y are the mid - points of sides AB and CD respectively, then
(a) $AXCY$ is rectangle
(b) $AXCY$ is square
(c) $AXCY$ is parallelogram
(d) $AXCY$ is rhombus.

29. ABC is an isosceles triangle in which AB = $CP \parallel AB$ and AP is the bisector or exterior $\angle CAD$ of ΔABC then
(a) $\angle PAC = \angle B$
(b) $ABCP$ is a parallelogram
(c) $ABCP$ is a rhombus
(d) $\angle PAC = \angle D$

30. The line segment joining the mid –points of the diagonals of a trapezium is
(a) Parallel to the non-parallel sides
(b) Parallel to the parallel sides and equal ti the sum of the parallel sides .
(c) Parallel to the parallel sides and equal to the difference of the parallel sides .
(d) Parallel to the parallel sides, and , equal to half of the difference of the parallel sides.

Answer Key

1. (b)	2. (a)	3. (a)	4. (b)	5. (b)	6. (b)	7. (a)	8. (c)	9. (b)	10. (c)
11. (b)	12. (a)	13. (b)	14. (b)	15. (d)	16. (b)	17. (c)	18. (c)	19. (a)	20. (b)
21. (b)	22. (c)	23. (c)	24. (b)	25. (c)	26. (d)	27. (b)	28. (c)	29. (b)	30. (d)

Hints and Solutions

1. **(b)** Let the angles of the quadrilateral be $x°$, $2x°$, $3x°$ and $4x°$.
$\because$ Sum of the angles of quadrilateral = 360°
$\Rightarrow x + 2x + 3x + 4x = 360°$
$\Rightarrow \quad 10x = 360°$
$\Rightarrow \quad x = 36°$
$\because$ Measure of largest angle
$= 4 \times x = 1 = 4 \times 36° = 144°$

2. **(a)** $\because$ The opposite angles of a parallelogram are equal.
$\therefore \quad (3x-2)° = (50 - x)°$
$\Rightarrow \quad 4x = 52°$
$\Rightarrow \quad x = 13°$
$\therefore \quad (3x - 2)° = 37°$
$\therefore \quad (50 - x)° = 37°$

3. **(a)** Perimeter of $\|^{gm} = 2(a+b) = 24$ cm

$\Rightarrow \quad a + b = 12$ cm

Given $a = 8$ cm

then $\quad 8 + b = 12$ cm

$\Rightarrow \quad b = 4$ cm

4. **(b)** Let the angle be $x°$

$\therefore$ Adjacent angle $= (180 - x)°$

$\Rightarrow \quad x° = \frac{1}{3}(180 - x)°$

$\Rightarrow \quad 4x = 180°$

$\Rightarrow \quad x = 45°$

5. **(b)** $\because$ $ABCD$ is a square

$\therefore \angle D = 90°$ and $AD = DC = AB = BC$

In ΔADC

$AD = DC$

$\therefore \angle CAD = \angle ACD$, and

$\angle D + \angle ACD + \angle CAD = 180°$

$\Rightarrow \quad 90° + 2\angle ACD = 180°$

$\Rightarrow \quad \angle ACD = \frac{90°}{2} = 45°$

6. **(b)** $\because$ Diagonals of a rectangle bisect each other and are also equal in length.

$\therefore$ In ΔPOS,

$OP = OS$

$\Rightarrow \quad \angle OPS = \angle OSP$

(angles opposite to equal sides are equal)

Also,

$\angle POS + \angle OSP + \angle OPS = 180°$

$\Rightarrow \quad 2\angle OPS = 180° - \angle POS$

$= 180° - 64°$ ($\because \angle POS + \angle QOR$)

{vertically opposite $\angle$s}

$\Rightarrow \quad \angle OPS = \frac{116°}{2} = 58°$

7. **(a)** The figure formed by joining the mid-points of consecutive side of a quadrilateral is a parallelogram.

8. **(c)** Let the measure of 4^{th} angle be $x°$

$\therefore \quad 98° + 92° + 70° + x° = 180° \times 2 = 360°$

$\Rightarrow \quad x° = 100°$

9. **(b)** Let the angles be $2x$, $4x$, $5x$ and $7x$ respectively.

$\therefore$ Difference between largest and smallest angle $= 7x - 2x = 5x$

$\because$ Sum of all angles of a quadrilateral $= 360°$

$\Rightarrow 2x + 4x + 5x + 7x = 360°$

$\Rightarrow \quad 18x = 360°$

$\Rightarrow \quad x = 20°$

$\therefore$ Required difference

$= 7x - 2x = 5x$

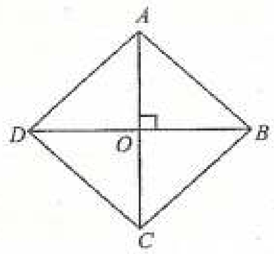
$= 5 \times 20° = 100°$

10. **(c)** $\because$ Diagonals of a rhombus bisect each other at 90°

In ΔAOD

$AD = 15$ cm, $AC = 12$ cm,

$OA = \frac{12 \times 2}{2} = 12$ cm



$\therefore OD = \sqrt{AD^2 - OA^2}$

$= \sqrt{(15)^2 - (12)^2} = 9$ cm

$\therefore BD = 2OD$

$= 2 \times 9 = 18$ cm

11. **(b)** Let the angle be $x°$

$\therefore$ Its adjacent angle $= (180 - x)°$

A/Q,

$x° = \frac{2}{3}(180° - x)°$

$\Rightarrow \quad 3x° = 360° - 2x°$

$\Rightarrow \quad 5x = 360°$

$\Rightarrow \quad x = 72°$

12. **(a)** Rectangle has equal length of diagonals

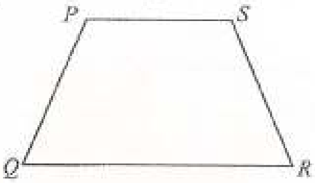
13. **(b)** $\angle P + \angle Q + \angle R + \angle S = 360°$

$\Rightarrow \quad 3x + 7x + 6x + 4x = 360°$

$\Rightarrow \quad 20x = 360°$

$\Rightarrow \quad x = 18°$

∴ Angles are 54° 126,108° 72°



∵ $\angle P + \angle Q = \angle R + \angle S = 180°$

∴ $PQRS$ is a trapezium, because

$\angle R + \angle Q \neq 180°$

(only one pair of sides are equal)

14. **(b)** Let the angles be $3x, 5x, 9x$ and $13x$

∴ Sum of largest and smallest angle

$= 3x + 13x = 16x$

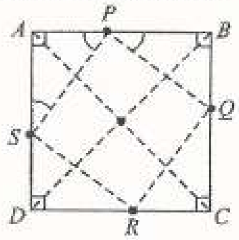
A/Q,

$3x + 5x + 9x + 13x = 360°$

⇒ $30x = 360° \Rightarrow x = 12$

∴ $16x = 16 \times 12° = 192°$

15. **(d)** P, Q, R and S are the mid-points of BA, BC, CD and DA respectively.



∴ $AP = AS = PB = BQ = QC$

$= CR = DR = DS = \frac{AB}{2}$

∴ In ΔAPS

$AP = AS$

$\Rightarrow \angle ASP = \angle APS = \frac{(180° - 90°)}{2} = 45°$

Similarly

$\angle ASP = \angle APS = \angle BPQ = \angle BQP = \angle CQR$

$= \angle CRQ = \angle DSR = \angle DRS = 45°$

Now $\angle P + \angle ASP + \angle APS = 180°$

⇒ $\angle P = 90°$

Similarly,

$\angle P = \angle Q = \angle R = \angle S = 90°$

∴ $PQRS$ is a parallelogram having each of its angles = 90°

Now

using midpoint theorem in ΔABC and ΔACD

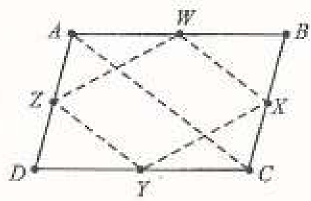
$SR = PQ = \frac{1}{2}AC$, and in $\Delta S\ ABD$ and BDC

$SR = PQ = \frac{1}{2}BD = \frac{1}{2}AC$ [∵ $BD = AC$]

∴ $SP = PQ = QR = RS$

∴ $PQRS$ is a square

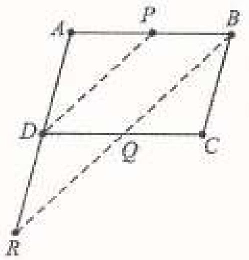
16. **(b)** In ΔACD



$ZY \parallel AC$ and $YZ = \frac{1}{2}$

[Using mid-point theorem]

17. **(c)** In ΔABR



$DP \parallel BR$, and P is the mid-point of side AB

∴ Using mid-point theorem (converse)

Point P is the mid – point of AB and is parallel to BR.

∴ $DP = \frac{1}{2} BR$, and

D will be the mid –point of side AR

∴ $AD = DR = BC = \frac{1}{2} AR$

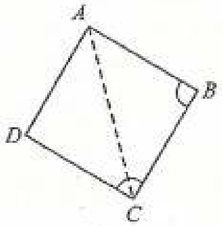
⇒ $AR = 2BC$

18. **(c)** Rhombus is a parallelogram having consecutive sides equal and none of the angles equal a right angle.

19. **(a)** $\angle DAC = \angle DCA = \frac{90°}{2} = 45°$

20. **(b)** $\angle OPS = \frac{180° - 44°}{2} = \frac{136°}{2} = 68°$

21. **(b)** $\angle ABC = 56°$



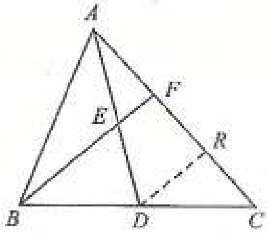
$\therefore \angle ACD = \frac{180° - 56°}{2} = 90° - 28° = 62°$

[$\because$ Adjacent angles sum = 180° and diagonal bisect the angle $\angle A$]

22. **(c)** $\because AD$ is the median of ΔABC

$\therefore \quad BD = DC$

Through D, draw $DR \parallel BF$



Now, in ΔBFC,

$DR \parallel BF$ and D is the mid-point of BC

$\therefore$ R should be the mid-point of FC (according to converse of mid-point theorem)

$\therefore \quad FR = RC$ ...(i)

Similarly, in ΔADR

E is the mid-point of AD and $EF \parallel DR$

$\therefore$ F should be the mid-point of AR

$\therefore \quad FR = AF$ ...(ii)

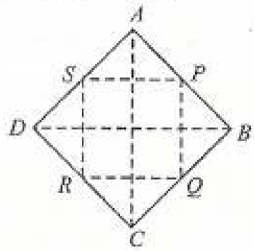
Using (i) and (ii)

$FR = RC = AF$

$\Rightarrow \quad AC = 3AF$

$\Rightarrow \quad AF = \frac{1}{3}AC$

23. **(c)** P, Q, R and S are the midpoints of AB, BC, CD and DA respectively.



Using midpoint theorem in ΔADB and ΔDBC

$SP = RQ = \frac{1}{2}DB$ and $SP \parallel RQ \parallel BD$

Similarly in $\Delta S\ ADC$ and ΔABC,

$PQ = SR = \frac{AC}{2}$ and $PQ \parallel SR \parallel AC$

$\therefore AC \perp BD$ and $SP \parallel BD$

$\therefore \quad \angle S = 90°$,

Also,

$AC \neq BD$, or , $PQ \neq$ PS

$\therefore$ $PQRS$ is a parallelogram having one angle equal to 90° and unequal adjacent sides

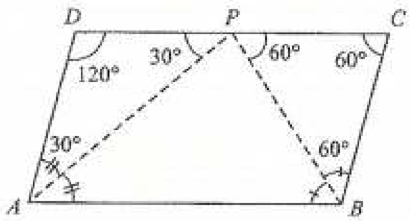
$\therefore$ $PQRS$ is a rectangle.

24. **(b)** The resulting figure will be a square

25. **(c)** Every rhombus is a $\|^{gm}$ because its opposite sides are equal and parallel.

26. **(d)** A Kite is not a $\|^{gm}$.

27. **(b)** In ΔADP



$\angle ADP + \angle DPA + \angle PAD = 180°$

$\Rightarrow 120° + \angle DPA + \frac{60°}{2} = 180°$

$\Rightarrow \quad \angle DPA = 30°$

$\Rightarrow \quad AD = DP$

Similarly,

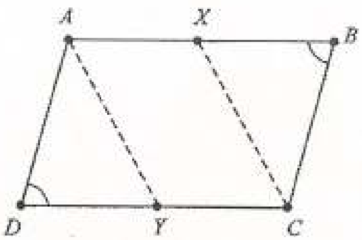
In ΔADP

$\angle CPB = 60°$

$\Rightarrow \quad PC = PB = CB$

$\therefore \quad AD = \frac{1}{2} DC$

28. **(c)** $\because AX = CY = \frac{1}{2} AB = \frac{1}{2} CD$



And also,

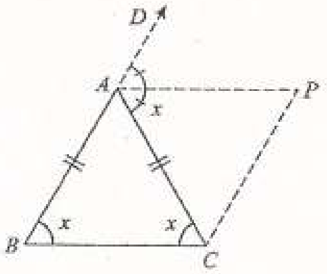
$AB \parallel CD$, or $AX \parallel CY$

$\because \quad AX = CY$ and $AX \parallel CY$

$\therefore AXCY$ is a $\|^{gm}$

($\because$ one pair of sides are equal and parallel)

29. **(b)** Let $\angle B = x°$



$\because \quad AB = AC$

$\therefore \quad \angle B + \angle C = x°$

$\because \angle CAD$ is an exterior angle for ΔABC

$\therefore \quad \angle CAD = \angle B + \angle C = 2x°$

$\therefore \quad \angle PAC = \frac{\angle CAD}{2} = \frac{2x°}{2} = x°$

$\because \quad \angle PAC = \angle ACB$

(Alternate opposite $\angle S$)

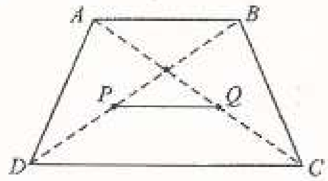
$\because \quad AP \parallel BC$

($\because$Alternate $\angle S$ are equal)

$\because \quad AP \parallel BC$ and $AB \parallel PC$

$\therefore ABCP$ is a $\|^{gm}$.

30. **(d)** P and Q are the mid-points of BD and AC respectively.



Applying mid-point theorem

$PQ \parallel AB \parallel DC$, and

$PQ = \frac{1}{2}(DC - AB)$

9. Areas of Parallelograms and Triangles

Learning Objective:

In this chapter, we learn about:
*Polygomal Regions
*Area Axioms
*Important Formulae
*Important Facts

Polygonal Regions

Triangular Region

The union of a triangle and its interior is called a triangular region. Basically, it is area enclosed by a triangle.

Polygonal Region

The part of plane enclosed by a polygon is called polygonal region.

Area Axioms

(a) Every polygonal region R has an area, measured in square units and denoted by ar (R).

(b) For polygonal regions R_1 and R_2,

(i) $R_1 \cong R_2 \quad \Rightarrow \text{ar}(R_1) = \text{ar}(R_2)$

(ii) $R_1 \leq R_2 \quad \Rightarrow \text{ar}(R_1) \leq \text{ar}(R_2)$

(iii) $\text{ar}(R_1 \cup R_2) \quad \Rightarrow \text{ar}(R_1) + \text{ar}(R_2)$

(iv) $\text{ar}(R_1 \cap R_2) \quad \Rightarrow |\text{ar}(R_1) - \text{ar}(R_2)|$

(c) For a rectangular region $PQRS$ with,

$PQ = p$ units and $QR = q$ units, we have,

ar(rect. $PQRS$) $= pq$ square units.

(d) If a triangle and a parallelogram lie on the same base and between the same parallels, then,

Area of parallelogram = 2 × (ar (triangle)).

Important Formulae

(i) Area of parallelogram = base × corresponding height.

(ii) Area of triangle $= \frac{1}{2} \times$ base × corresponding height.

(iii) Area of rhombus $= \frac{1}{2} \times$ product of the diagonals

(iv) Area of trapezium $= \frac{1}{2} \times$ (sum of parallel sides) × (distance between them)

(v) Area of square $= (\text{side})^2 = \frac{1}{2}\ (\text{diagonal})^2$

Important Facts

(i) A diagonal of a parallelogram divides it into two triangles of equal area.

(ii) Parallelograms on the same base and between the same parallels are equal in area.

(iii) A parallelogram and a rectangle on the same base and between the same parallels are equal in area.

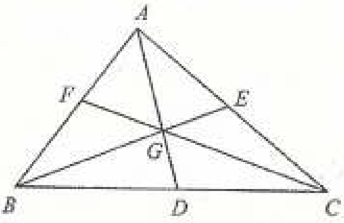
(iv) Triangles between same parallels and on the same base are equal in area.

(v) If a triangle and a parallelogram are on the same base, and between the same parallels, then the area of the triangle is equal to half of the area of the parallelogram.

(vi) The diagonals of a ‖gm divide it into 4 triangles of equal area.

(vii) The median of a triangle divides it into 2 triangles of equal area.

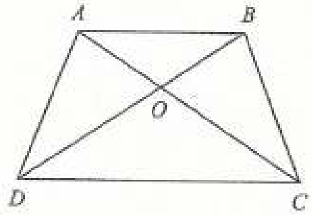
(viii) AD, BE and CF are medians of ΔABC. The medians intersect at a point, G, which is known as centroid of ΔABC. G divides the ΔABC in 3 equal parts.



$\therefore$ $\quad \text{ar}(\Delta ABG) = \text{ar}(\Delta AGC) = \text{ar}(\Delta BGC)$

Example 1: $ABCD$ is a trapezium in which $AB \parallel DC$, and its diagonals AC and BD intersect at O. Prove that $\text{ar}(\Delta AOD) = \text{ar}(\Delta BOC)$.

Proof:



$\because$ ΔABD and ΔABC lie on the same base, i.e., AB and between the same parallels i.e.,

AB and CD, then, $\text{ar}(\Delta ABD) = \text{ar}(\Delta ABC)$ ...(i)

Subtracting $\text{ar}(\Delta AOB)$ from both sides of equation (i).

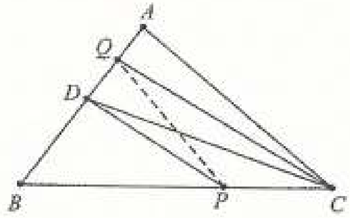
$\therefore$ $\quad \text{ar}(\Delta ABD) - \text{ar}(\Delta AOB) = \text{ar}(\Delta ABC) - \text{ar}(\Delta AOB)$

$\Rightarrow$ $\quad \text{ar}(\Delta AOD) = \text{ar}(\Delta BOC)$ **Proved.**

Example 2: D is the midpoint of side AB of ΔABC and P is any point on BC. If $CQ \parallel PD$ meets AB in Q, prove that,

$$2\text{ar}(\Delta BPQ) = \text{ar}(\Delta ABC).$$

Proof:



Join CD and PQ

$\because$ CD is a median of ΔABC.

$\therefore$ $\quad \text{ar}(\Delta BCD) = \text{ar}(\Delta ADC) = \frac{1}{2}\text{ar}(\Delta ABC).$

$\Rightarrow$ $\quad \text{ar}(\Delta BPD) + \text{ar}(\Delta DPC) = \frac{1}{2}\text{ar}(\Delta ABC)$ ...(i)

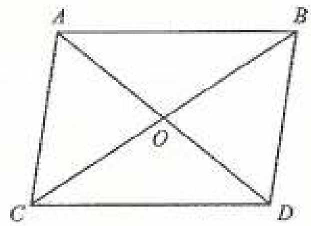
ΔDCP and ΔDPQ are on the same base, i.e., DP and between the same parallels DP and QC.

$\therefore$ $\quad \text{ar}(\Delta DPC) = \text{ar}(\Delta DPQ)$ ...(ii)

Using (i) and (ii), then,

$$ar(\Delta BPD) + ar(\Delta DPQ) = \frac{1}{2}\ ar(\Delta ABC)$$

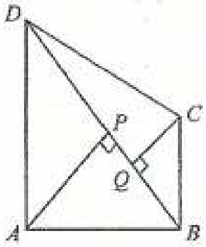
$\Rightarrow$ $$2ar(\Delta BPQ) = ar(\Delta ABC)$$ **Proved.**



Example 3: If area (∥ $ABCD$) = $36cm^2$, then find the area of ΔAOB.

Solution: $ar(\Delta AOB) = \frac{1}{4}\ ar(\|gm\ ABCD) = \frac{1}{4} \times 36\ cm^2 = 9\ cm^2$

Example 4: If $BD = 14cm$, $AP = 8cm$ and $CQ = 6cm$, then find the area of quadrilateral $ABCD$.



Solution: ar(quad. $ABCD$) = $ar(\Delta ADB) + ar(\Delta BDC)$

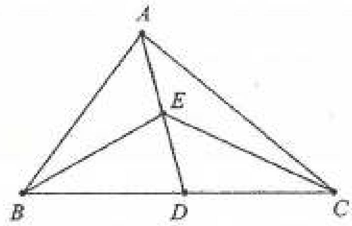
$$= \frac{1}{2} \times AP \times BD + \frac{1}{2} \times CQ \times BD$$

$$= \frac{1}{2} \times BD \times (AP + CQ)$$

$$= \frac{1}{2} \times 14 \times 14 = 98\ cm^2$$

Example 5: The vertex A of ΔABC is joined to a point D on BC. If E is the midpoint of AD, then $ar(\Delta BEC) = x\ ar(\Delta ABC)$. Find x.

Solution: Let the length of altitude from A to BC be p cm, then,



$$ar(\Delta ABC) = \frac{1}{2} \times BC \times p$$

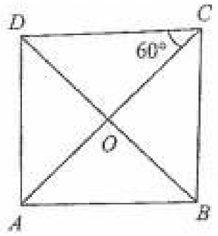
$$ar(\Delta BEC) = \frac{1}{2} \times BC \times \frac{p}{2}$$

$$\therefore\ ar(\Delta BC) = \frac{1}{2}\ ar(\Delta ABC)$$

$$\therefore \quad x = \frac{1}{2}$$

Example 6: $ABCD$ is a rhombus in which $\angle C = 60°$, find $AC : BD$.

Solution: $\because$ Diagonals of a rhombus intersect each other at 90°.



$\therefore \quad \angle DOC = 90°$

Now, in ΔDOC,

$$\angle D + \angle C + \angle DOC = 180°$$

$\Rightarrow \angle D = 180° - 90° - 60° = 30°$

$\because \Delta ADC$ is isosceles triangle, and one angle is 60°.

$\therefore \Delta ADC$ is an equilateral triangle, having,

$$AD = DC = AC = a\text{(say), and,}$$

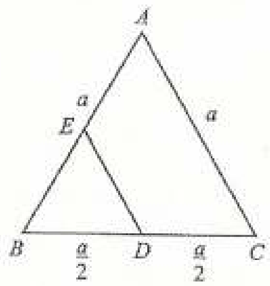
OD is the altitude of equilateral ΔADC.

$$\therefore \quad OD = \frac{\sqrt{3}a}{2}, \Rightarrow BD = 2 \times OD = \sqrt{3}\,a.$$

$$\therefore \quad \frac{AC}{BD} = \frac{a}{\sqrt{3}a} = 1 : \sqrt{3}$$

Example 7: ΔABC and BDE are two equilateral triangles such that D is the midpoint of BC. Then, ar(ΔBDE) is how much if

$$\text{ar}(\Delta ABC) = 48\text{cm}^2\ ?$$



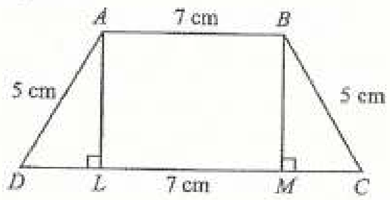
Solution: $\text{ar}(\Delta ABC) = \sqrt{3}\dfrac{a^2}{4} = 48\ \text{cm}^2$

$$\text{ar}(\Delta BDE) = \sqrt{3}\frac{\left(\frac{a}{2}\right)^2}{4} = \frac{\sqrt{3}a^2}{4 \times 4} = \frac{ar(\Delta ABC)}{4}$$

$$= \frac{48}{4}\ \text{cm}^2 = 12\ \text{cm}^2$$

Example 8: Two parallel sides of a trapezium are 7 cm and 13 cm respectively and non-parallel sides measure 5 cm each then the area of trapezium = ... sq. cm.

Solution: $\because$ Trapezium is symmetric.



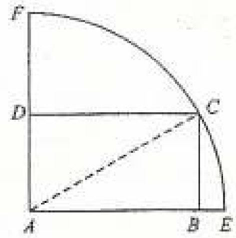
$\therefore DL = MC = \dfrac{13 - 7}{2} = 3\text{cm.}$

In ΔADL,

$$AD^2 = AL^2 + DL^2$$

$$\Rightarrow \quad AL = \sqrt{AD^2 - DL^2} = \sqrt{(5)^2 - (3)^2} = 4\ \text{cm.}$$

$$\text{ar (Trap. } ABCD) = \frac{1}{2} \times 4 \times (13 + 7) = 2 \times 20 = 40\ \text{cm}^2$$



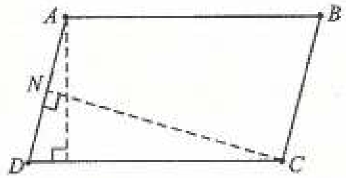
Example 9: $AD = 2\sqrt{5}$ cm, and radius of quadrant of circle is 10 cm. Find the area of rectangle *ABCD*.

Solution: In ΔADC, $AC = 10$ cm, $AD = 2\sqrt{5}$ cm.

Then, $DC = \sqrt{AC^2 - AD^2} = \sqrt{100-80} = \sqrt{80} = 4\sqrt{5}$ cm.

$\therefore$ ar $(ABCD) = 2\sqrt{5}$ cm $\times$ $4\sqrt{5}$ cm $= 40$ cm^2

Example 10: *ABCD* is a ‖gm, in which, $AB = 16$cm, $DEF = 8$cm, and if $AD = 10$cm, then $CN = \ldots$cm.



Solution: ar (‖gm *ABCD*) $= AB \times DM = DC \times DM = AD \times CN$.

$\Rightarrow \quad 16 \times 8 = 10 \times CN$

$\Rightarrow \quad CN = \dfrac{16\times 8}{10} = \dfrac{128}{10}$ cm $= 12.8$ cm

Multiple Choice Questions

1. Tick the incorrect statement :
   (a) If two triangles are congruent, they have equal areas.
   (b) If *AC* is a diagonal of ‖gm *ABCD*, then *AC* divides *ABCD* in two equal areas.
   (c) Parallelograms on the same base and between the same parallels are equal in area.
   (d) Parallelograms on equal bases and between the same parallels are unequal in area.

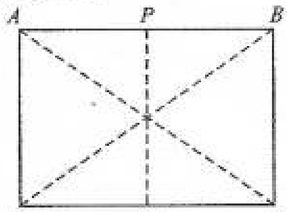
2. If a triangle and a parallelogram are on the same base and between the same parallels, and area of triangle is *A*, then area of ‖gm is :
   (a) $\frac{A}{2}$ (b) $2A$ (c) $3A$ (d) $4A$

3. A parallelogram and a rectangle are constructed on same base and between the same parallels, they have
   (a) Equal area
   (b) Unequal area
   (c) ar(Parallelogram) > ar (Rectangle)
   (d) ar (Rectangle) > ar (Parallelogram)

4. *ABCD* is a parallelogram

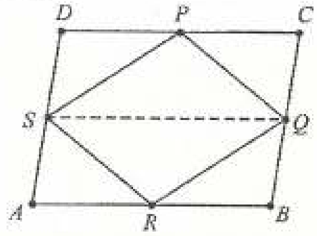
   $AM = 7$ cm, $CN = 8$ cm and $AB = CD = 10$ cm, Find *AD*.
   (a) 6.75 cm (b) 6.25 cm
   (c) 8.75 cm (d) 7.25 cm

5. Point *O* is the point of intersection of *AC* and *BD* of parallelogram *ABCD* and, also, *PQ* ‖ *AD*, then,



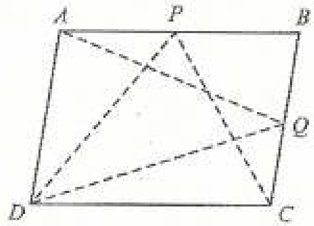
(a) ar(*ABCD*) = 2 × (ar(*PQDA*))
(b) ar (*PQDA*) = ar (*ABCD*)
(c) ar(*POB*) = ar (*AOD*)
(d) ar(*PQDA*) = ar (*AOB*)

6. *P*, *Q*, *R* and *S* are the midpoints of *DC*, *BC*, *AB* and *AD* respectively. area of parallelogram *PQRS* is



(a) $\frac{1}{2}$ ar (*ABCD*) (b) 2 ar (*ABCD*)
(c) $\frac{1}{4}$ ar (*ABCD*) (d) $\frac{2}{3}$ ar (*ABCD*)

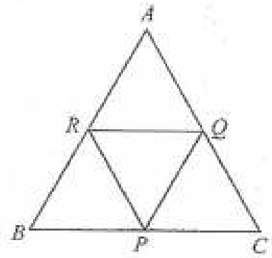
7. *P* and *Q* are the points on *AB* and *BC* respectively of ||gm *ABCD*, then



(a) ar (ΔPDC) = $\frac{1}{2}$ ar (ΔAQD)
(b) ar(ΔPDC) = ar (ΔAQD)
= $\frac{1}{2}$ ar (parallelogram *ABCD*)
(c) ar(ΔPDC) = ar (ΔAQD)
= — ar(parallelogram *ABCD*)
(d) ar (ΔPDC) = $\frac{2}{3}$ ar(ΔAQD)

8. The median of a triangle divides is into two:
(a) Similar Δs (b) Congruent Δs
(c) Isosceles Δs (d) Δs with same areas

9. In ΔABC, *P*, *Q*, *R* are the midpoints of sides *BC*, *CA*, *AB* respectively, then ar(ΔPQR) =



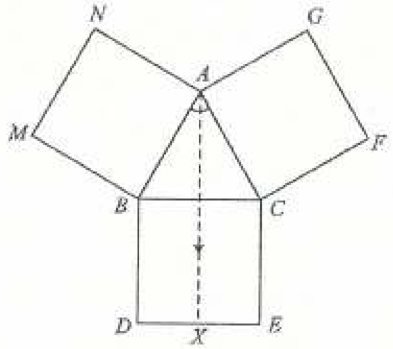
(a) $\frac{1}{2}$ ar(ΔABC) (b) $\frac{1}{4}$ ar(ΔABC)
(c) $\frac{1}{8}$ ar(ΔABC) (d) $\frac{1}{3}$ ar(ΔABC)

10. ar(trapezium *RQBC*) =
(a) $\frac{1}{3}$ ar(ΔABC) (b) $\frac{3}{4}$ ar(ΔABC)
(c) $\frac{1}{4}$ ar(ΔABC) (d) $\frac{1}{2}$ ar(ΔABC)

11. If *P*, *Q*, *R* and *S* are midpoints of *AB*, *BC*, *CD* and *DA* of parallelogram *ABCD* and ar(*ABCD*) = 26 m^2, then ar(*PQRS*) =
(a) 13 m^2
(b) 6.5 m^2
(c) 6.75 m^2
(d) 19.5 m^2

12. If *AD* is median of ΔABC and *P* is a point on *AC* such that ar(ΔADP) : ar(ΔABD) = 2 : 3, then ar (ΔPDC) : ar (ΔABC) is
(a) 1 : 5 (b) 1 : 6
(c) 5 : 1 (d) 3 : 5

13. *ABC* is a right angled Δ at *A*, *BCED*, *ACFG* and *ABMN* are squares on sides *ABC*, *AC*, *AB* respectively. *AX* ⊥ *DE* meets *BC* at *Y* then,



(a) ar $(BCED)$ = ar $(ABMN)$ + ar $(ACFG)$
(b) ar $(CYXE)$ = 2ar (ΔABC)
(c) ar $(BCED)$ = ar$(ABMN)$ + ar(ABC)
(d) ar(ΔABC) = ar$(BCDE)$

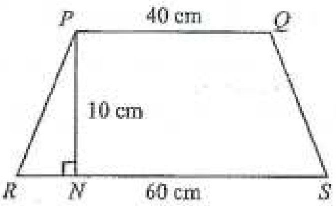
14. $ABCD$ is a trapezium in which $AB \parallel CD$ and $CD = 40$ cm, and $AB = 60$ cm. If X and Y are, respectively, the mid points of AD and BC, then $XY =$

(a) 45 cm (b) 50 cm
(c) 60 cm (d) 55 cm

15. ar(trap. $DCYX$) = K ar$(XYBA)$, then $K =$

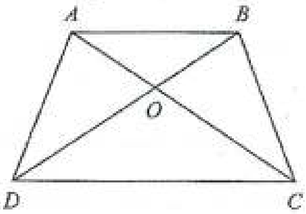
(a) $\frac{9}{11}$ (b) $\frac{10}{11}$ (c) $\frac{1}{11}$ (d) $\frac{3}{11}$

16. area of trapezium, $PQRS$ in the given figure is :



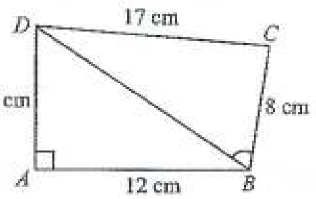
(a) 500 cm^2 (b) 250 cm^2
(c) 125 cm^2 (d) 375 cm^2

17. $ABCD$ is a trapezium in which $AB \parallel DC$, then



(a) ar (ΔAOB) = ar (ΔDOC)
(b) ar $(\Delta AOB) = \frac{3}{4}$ ar (ΔDOC)
(c) ar (ΔAOD) = ar (ΔBOC)
(d) ar $(\Delta AOD) \neq$ ar (ΔBOC)

18. Find the area of quadrilateral $ABCD$,



(a) 114 cm^2 (b) 112 cm^2
(c) 102 cm^2 (d) 97 cm^2

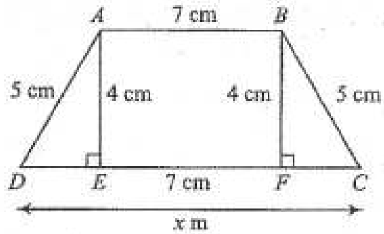
19. $ABCD$ is a rectangle with O as any point in its interior. If ar $(\Delta AOD) = 3$ cm^2, and ar(ΔBOC) = 6cm^2, then ar$(ABCD)$ =

(a) 9 cm^2 (b) 27 cm^2
(c) 18 cm^2 (d) 6 cm^2

20. $ABCD$ is a parallelogram in which BC is produced to E such that $CE = BC$. AE intersects CD at F. If area $\Delta DFB = 3$ cm^2, area of parallelogram $ABCD$ is:

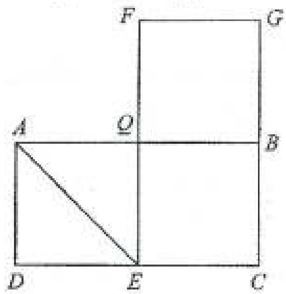
(a) 6 cm^2 (b) 12 cm^2
(c) 18 cm^2 (d) 9 cm^2

21. The area of trapezium is :



(a) 40 m^2 (b) 20 m^2
(c) 30 m^2 (d) 35 m^2

22. $ABCD$ and $FECG$ are parallelograms equal in area. If ar(ΔAQE) = 12cm^2, then ar(parallelogram $FGBQ$) =



(a) 12 cm^2 (b) 24 cm^2
(c) 36 cm^2 (d) 20 cm^2

23. $PQRS$ is a rectangle inscribed in a quadrant of a circle of radius 13 cm. A is any point on PQ. If PS = 5cm, then find ar(ΔARS).

(a) 15 cm^2 (b) 20 cm^2
(c) 25 cm^2 (d) 30 cm^2

24. $ABCD$ is a parallelogram. P is the midpoint

of AB. BD and CP intersect at Q such that $QC : QP = 3 : 1$. If the ar(ΔPBQ) = 10 cm^2, then area of parallelogram $ABCD$ is :

(a) 80 cm^2 (b) 40 cm^2
(c) 160 cm^2 (d) 120 cm^2

25. A rhombus has length of diagonals as 8cm and 6cm. the ratio of area of rhombus and its side length is:

(a) 6 : 12 (b) 24 : 5 (c) 5 : 6 (d) 3 : 5

26. M is a point on base QR of ΔPQR, and N is the midpoint of QR. NX is drawn parallel to MP at X. If ar(ΔPQR) = 12cm^2, then ar(ΔXMR) =

(a) 6 cm^2 (b) 12 cm^2
(c) 9 cm^2 (d) 18 cm^2

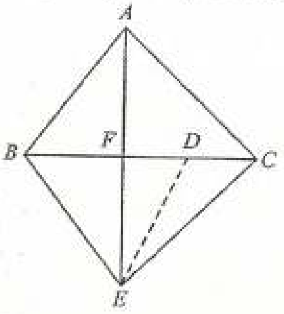
27. The diagonals of parallelogram intersect at O. through O, a line XY is drawn to intersect AD at X and BC at Y, then.

(a) ar($AXYB$) = ar($XYCD$)
(b) ar($ABCD$) = ar($XYCD$)
(c) ar($ABCD$) = ar($AXYB$)
(d) None of these

28. $ABCD$ is a parallelogram, P and Q are the mid - points of BC and CD respectively, then, ar (ΔAPQ) = K (ar ($ABCD$)), K =

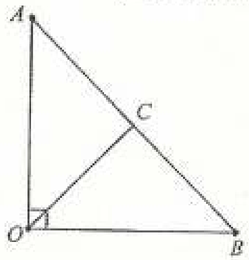
(a) $\frac{1}{8}$ (b) $\frac{1}{4}$
(c) $\frac{3}{8}$ (d) $\frac{1}{2}$

29. ABC and BDE are equilateral triangles, such that D is midpoint of BC. AE intersects BC in F, then ar(ΔBFE) = x ar(ΔEFD), then x =



(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 2 (d) 3

30. $\angle AOB = 90°$, $AC = BC$, OA = 12 cm and OC = 6.5 cm. the area (in cm^2) of ΔAOB is:



(a) 10 (b) 15 (c) 20 (d) 30

Answer Key

1. (d)	2. (b)	3. (a)	4. (c)	5. (a)	6. (a)	7. (b)	8. (d)	9. (b)	10 (b)
11. (a)	12. (b)	13. (a)	14. (b)	15. (a)	16. (a)	17. (c)	18. (a)	19. (c)	20. (b)
21. (a)	22. (b)	23. (d)	24. (c)	25. (b)	26. (a)	27. (a)	28. (c)	29. (c)	30. (d)

Hints and Solutions

1. **(d)** Parallelograms on equal bases and between the same parallels must be equal in area.

2. **(b)** Area of $\Delta = \frac{1}{2} \times b \times h = A$

   Area of parallelogram $= bh = 2A$.

3. **(a)** Base $= b$, distance between parallels $= h$

   Then, Area of rectangle

   $= \text{area of parallelogram} = bh$

4. **(c)** Area of parallelogram

   $= AM \times CD = AD \times CN$

   $\Rightarrow\ 7 \times 10 = AD \times 8$

   $\Rightarrow\ AD = \frac{70}{8} = \frac{35}{4} = 8.75$ cm.

5. **(a)** $\because$ O lies in mid of the parallelogram.

   $\therefore$ PQ divides parallelogram in two parts of equal areas.

   $\therefore$ ar $(PQDA) = \frac{1}{2}$ ar $(ABCD)$

6. **(a)** Joining QS, it can be clearly seen that, $QS \parallel DC \parallel AB$,

   $\therefore$ ar $(PQS) = \frac{1}{2}$ ar$(DCQS)$

   [Area between QS and DC]

   ar $(SRQ) = \frac{1}{2}$ ar $(ASQB)$

   [Area between QS and AB]

   Adding both,

   ar (PQS) + ar $(SRQ) = \frac{1}{2}$ [ar $(DCQS)$ + ar $(ASQB)$]

   ar $(PQRS) = \frac{1}{2}$ ar $(ABCD)$

7. **(b)** $\because$ $ABCD$ and ΔPDC lie on same base, i.e., CD and between the same parallels, i.e., AB and CD.

   $\therefore$ ar $(\Delta PDC) = \frac{1}{2}$ ar$(ABCD)$

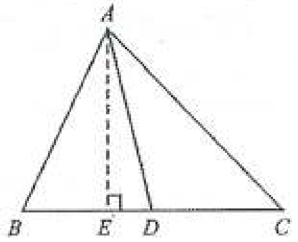
   Similarly,

   ar $(\Delta AQD) = \frac{1}{2}$ ar$(ABCD)$

   $\Rightarrow$ ar (ΔPDC) = ar (ΔAQD)

   $= \frac{1}{2}$ ar (quad. $ABCD$)

8. **(d)** In ΔABC, AD is the median.



   $\therefore$ $BD = CD$,

   Let $AE \perp BC$, then,

   ar$(\Delta ABD) = \frac{1}{2} \times AE \times BD$,

   ar$(\Delta ADC) = \frac{1}{2} \times AE \times DC = \frac{1}{2} \times AE \times BD$

   $\therefore$ ar(ΔABD) = ar(ΔADC)

9. **(b)** Using midpoint theorem,

   $BC = 2\,QR$, $AB = 2\,PQ$, $AC = 2\,PR$.

   Let the area of ΔPQR be A, then, ABC is a Δ resulted by doubling the length of every side of ΔPQR.

   $\therefore$ ar $(\Delta ABC) = 4$ ar (ΔPQR)

   [using Heron's formula]

10. **(b)** Area of trapezium $RQBC$ = ar (ΔRBP) + ar (ΔPQR) + ar (ΔQPC)

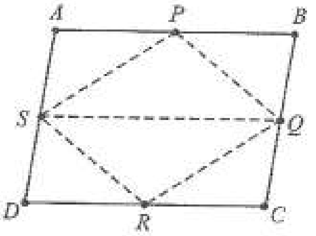
    $= 3 \times$ ar (ΔPQR)

    $= \frac{3}{4}$ ar (ΔABC)

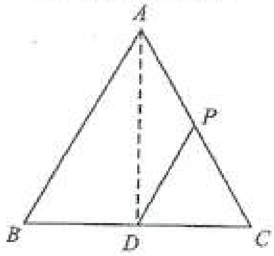
11. **(a)** ar (parallelogram $PQRS$)

    $= \frac{1}{2}$ ar (parallelogram $ABCD$)

    $= \frac{1}{2} \times 26 = 13\ \text{m}^2$



12. **(b)** $\because AD$ median of ΔABC



$\therefore$ ar $(ABD) = \text{ar}(ADC) = \frac{1}{2}\text{ar}(\Delta ABC)$ ...(i)

$$\frac{ar(\Delta ADP)}{ar(\Delta ADC)} = \frac{2}{3}$$

$$\Rightarrow \frac{ar(\Delta PDC)}{ar(\Delta ADC)} = \frac{1}{3} \Rightarrow \frac{2ar(\Delta PDC)}{ar(\Delta ABC)} = \frac{1}{3}$$

$\therefore$ Required ratio $= \frac{1}{3\times 2} = 1:6$

13. **(a)** Let $AB = x$, $AC = y$, then,

$BC = \sqrt{x^2 + y^2}$ (Pythagoras' theorem)

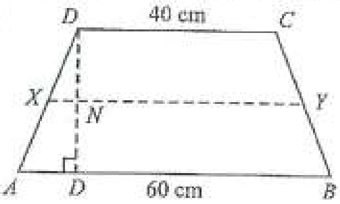
Area $(ABMN) = x^2$

Area $(ACFG) = y^2$

$$\text{ar}(BCED) = \left(\sqrt{x^2+y^2}\right)^2 = x^2 + y^2$$

$\therefore \text{ar}(BCED) = \text{ar}(ABMN) + \text{ar}(ACFG)$

14. **(b)**



$\because$ $XY \parallel AB \parallel DC$

[Let the length of XY be x cm]

$\because$ ar $(DCYX)$ + ar $(XYBA)$ = ar $(ABCD)$

$$\therefore \frac{1}{2}\times(40+x)\times DN + \frac{1}{2}\times(60+x)\times NM$$

$$= \frac{1}{2}\times(40+60)\times DM \quad \text{...(i)}$$

$$\because DN = NM = \frac{DM}{2}$$

$\therefore$ The equation (i) reduces to :

$$(40+x)\times\frac{1}{2} + (60+x)\times\frac{1}{2} = 100$$

$$\Rightarrow \quad 100 + 2x = 200$$

$$\Rightarrow \quad 2x = 100$$

$$\Rightarrow \quad x = 50 \text{ cm.}$$

15. **(a)** According to question

$$\frac{1}{2}\times(40+x)\times\frac{DM}{2}$$

$$= K\frac{1}{2}\times(60+x)\times\frac{DM}{2}$$

$$\Rightarrow \quad (40+50) = K(60+50)$$

$$\Rightarrow \quad K = \frac{9}{11}$$

16. **(a)** Required area

$$\text{ar}(PQRS) = \frac{1}{2}\times(40+60)\times 10$$

$$= \frac{1}{2}\times 100\times 10 = 500 \text{ cm}^2$$

17. **(c)** $\because \Delta ADC$ and ΔBDC lie on same base DC and between the same parallels, i.e., AB and DC.

$\therefore \quad \text{ar}(\Delta ADC) = \text{ar}(\Delta BDC)$

$\text{ar}(AOD) + \text{ar}(DOC) = \text{ar}(BOC) + \text{ar}(DOC)$

$\Rightarrow \quad \text{ar}(AOD) = \text{ar}(BOC)$.

18. **(a)** In ΔADB,

$$DB = \sqrt{AD^2 + AB^2}$$

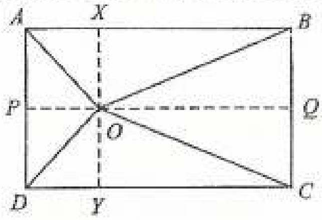
$$= \sqrt{9^2 + 12^2} = 15$$

$$\text{ar}(\Delta ADB) = \frac{1}{2}\times 12\times 9 = 54 \text{ cm}^2$$

$$\text{ar}(\Delta DBC) = \frac{1}{2}\times 15\times 8 = 60 \text{ cm}^2$$

$\therefore$ Total area $= 54 + 60 = 114 \text{ cm}^2$

19. **(c)** Draw $XY \parallel AD \parallel BC$ and $PQ \parallel AB \parallel DC$



Now $ar(\Delta AOB) = \frac{1}{2}\ ar(ABPQ)$

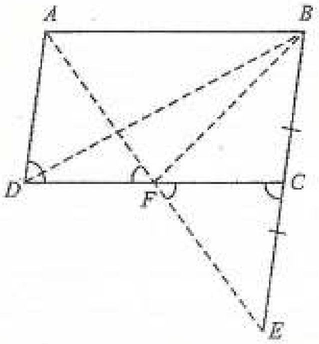
$ar(\Delta DOC) = \frac{1}{2}\ ar(DCQP)$

$\Rightarrow ar(\Delta AOB) + ar(\Delta DOC) = \frac{1}{2} \times ar(ABCD)$

$\Rightarrow ar(\Delta AOD) + ar(\Delta BOC) = \frac{1}{2} \times ar(ABCD)$

$\Rightarrow ar(ABCD) = 2 \times (3 + 6) = 18\ cm^2$

20. **(b)**



$ar(\Delta AFB) = \frac{1}{2}$ ar (parallelogram $ABCD$)

{areas between same parallels and same base}

$\Rightarrow$ ar (parallelogram $ABCD$) $= 2 \times ar(\Delta AFB)$

$= 2 \times ar(\Delta DCB)$ ...(i)

In Δs ADF and ECF.

$\angle AFD = \angle EFC$

{vertically opposite angles}

$\angle ACF = \angle ADF$ {alternate angles}

$BC = CE = AD$

$\therefore$ $\Delta ADF \cong \Delta ECF$ {by AAS congruency}

$\therefore$ $DF = CF$

ar(parallelogram $ABCD$) $= 2 \times ar(\Delta DCB)$

$= 2 \times 2 \times (ar(\Delta DFB))$

$= 2 \times 2 \times 3$

$= 12\ cm^2$

21. **(a)** Here $DE = \sqrt{AD^2 - AE^2}$

$= \sqrt{5^2 - 4^2}$

$= 3$ cm

$\therefore DE = FC = 3$m, and, $EF = 7$m.

$\therefore ar(ABCD) = \frac{1}{2} \times 4 \times (7 + 7 + 3 + 3)$

$= \frac{1}{2} \times 4 \times (20)$

$= 40\ m^2$

22. **(b)** $ar(\Delta AQE) = \frac{1}{2}$ ar (parallelogram $AQED$)

$= \frac{1}{2} \times \frac{1}{2}$ (ar(parallelogram $ABCD$))

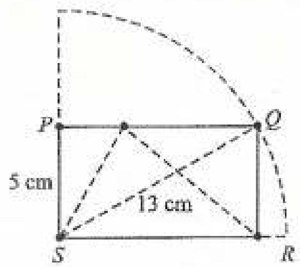
$= \frac{1}{4} \times$ (ar(parallelogram $FECG$))

$12\ cm^2 = \frac{1}{4} \times$ ar(parallelogram $FECG$)

$\therefore ar(FECG) = 48\ cm^2$

$\Rightarrow ar(FGQB) = \frac{1}{2}\ ar(FECG) = \frac{48}{2} = 24\ cm^2$

23. **(d)**



$ar(\Delta ARS) = \frac{1}{2}\ ar(PQRS)$ ....(i)

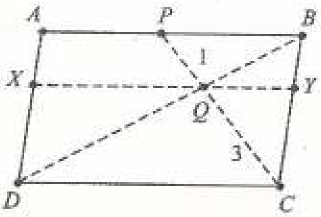
$\Rightarrow$ $QS^2 = PS^2 + PQ^2$

$\Rightarrow$ $PQ = \sqrt{QS^2 - PS^2}$

$= \sqrt{(13)^2 - (5)^2} = 12$ cm.

$\therefore$ ar(ΔARS) $= \frac{1}{2} \times 12 \times 5 = 30$ cm^2

24. **(c)**



ar(ΔPQB) + ar(ΔDQC)

$= \frac{1}{2}$ ar(parallelogram $ABCD$)

$\Rightarrow$ ar (parallelogram $ABCD$)

$= 2$ [ar(ΔPQB) + ar(ΔDQC)]

$= 2$ [10 cm^2 + ar(ΔDQC)]

ar(ΔDQC) = (3 + 3 + 1) × ar (ΔPQB)

$= 7 \times 10 = 70$ cm^2

$\therefore$ ar(parallelogram $ABCD$) $= 2 \times 80$

$= 160$ cm^2

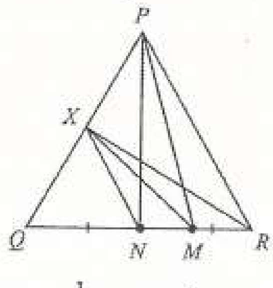
25. **(b)** ar(rhombus) $= \frac{1}{2} \times 8 \times 6 = 24$ cm^2

Length of side $= \sqrt{\left(\frac{8}{2}\right)^2 + \left(\frac{6}{2}\right)^2}$

$= \sqrt{(4)^2 + (3)^2} = 5$

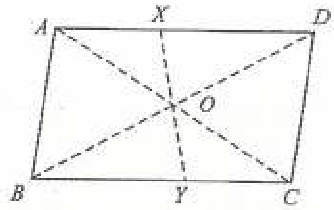
$\therefore$ Ratio = 24 : 5

26. **(a)** $\because$ ar(ΔXMR) $= \frac{1}{2}$ ar(ΔPQR)



$\therefore$ ar(ΔXMR) $= \frac{1}{2} \times 12$cm$^2 = 6$ cm^2

27. **(a)**



Area $AXBY$ is between AX and BY.

Area $XDCY$ is between XD and YC.

Let $AX = x$, and $BY = y$, and length of perpendicular between AD and $BC = p$.

$\therefore$ ar($AXBY$) $= \frac{1}{2} \times (x + y) \times P$

$XD = AD - x$, $YC = AD - y = BC - y$.

$\therefore$ ar($XDCY$) $= \frac{1}{2} \times xy \times P$

$\therefore$ ar($XDCY$) = ar($AXYB$)

28. **(c)** Let $AB = CD = x$, and $BC = AD = y$,

$AM = h_1$, $BN = h_2$

ar (ΔADQ) $= \frac{1}{2} \times AM \times DQ = \frac{1}{2} h_1 \frac{x}{2} = \frac{h_1 x}{4}$

ar (ΔAPB) $= \frac{1}{2} \times BN \times \frac{DC}{2} = \frac{1}{2} h_2 \frac{y}{2}$

$= \frac{h_2 y}{4} = \frac{h_1 x}{4}$

ar (ΔPQC) $= \frac{1}{2} \times QC \times \frac{AM}{2} = \frac{xh_1}{8}$

ar (parallelogram $ABCD$) $= AM \times DC$

$= BN \times AD = h_1 x = h_2 y$

$\therefore$ ar(ΔAQC) $= h_1 x - \left(\frac{h_1 x}{4} + \frac{h_1 x}{4} + \frac{h_1 x}{8}\right)$

$= \frac{3h_1 x}{8} = \frac{3}{8}$ ar($ABCD$)

29. **(c)** Length of altitude from A to $BC = p$ cm.

Length of altitude from E to $BC = q$ cm.

ar(ΔBFE) $= \frac{1}{2} \times BF \times q$, ar($\Delta EFD$)

$= \frac{1}{2} \times FD \times q.$

$$x = \frac{ar(\Delta BFE)}{ar(\Delta EFD)} = \frac{BF}{FD} = \frac{2FD}{FD} = 2$$

[$\because BF = 2FD$]

30. **(d)** $OC = 6.5$ cm,

$\therefore \quad AC = BC = 6.5$

[By similarly of Δs AOC and BOC]

$\therefore AB = AC + BC = 2AC = 2 \times 6.5 = 13$ cm.

$OA = \sqrt{AB^2 - OA^2} = \sqrt{13^2 - 12^2} = 5$ cm.

$\therefore$ ar $(\Delta AOB) = \frac{1}{2} \times 5 \times 12 = 30$ cm^2

10. Circles

Learning Objective:

In this chapter, we will learn about:

*Circle

*Terms Related to Circle

*Central Angle

*Important Theorems

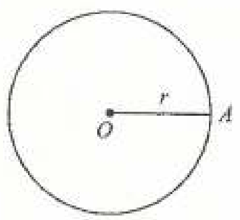
Circle

It is the locus of a point such that its distance from a fixed point is always constant.

Terms Related to Circle

Centre

The fixed point is called centre of the circle.



Radius

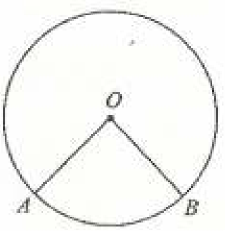
The constant distance is called radius of the circle.

In the above fig, O is the centre and OA is the radius of the circle.

Central Angle

If $C(O, r)$ be any circle, then any angle whose vertex is O is called its central angle.

$\angle AOB$ is a central angle.



Chord

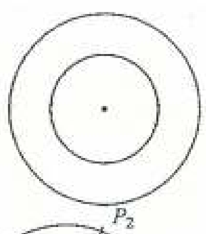
A line segment joining two points on a circle is called chord of a circle.

Diameter

A chord passing through the centre of a circle is called its diameter.

Concentric Circle

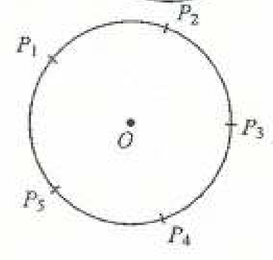
The circles having same centres and having different radii are called concentric circles.



Arc of a Circle

A continuous piece of a circle is called an arc of the circle.

In above fig, P_1P_2, P_2P_3, P_3P_4 etc. are arcs of the circle.



Semi-circle

A diameter of a circle divides it into two equal parts. Each of these parts is called a semi - circle.

Congruent Circle

Two circles are said to be congruent if and only if either of them can be superposed on the other so as to cover it exactly.

Some Theorems

Theorem 1: If two arcs of a circle are congruent then their corresponding chords are equal.

Theorem 2: If two chords of a circle are equal then their corresponding arcs are congruent.

Theorem 3: The perpendicular from the centre of a circle to a chord bisects the chord.

Theorem 4: The line joining the centre of a circle to the mid - point of a chord is perpendicular to the chord.

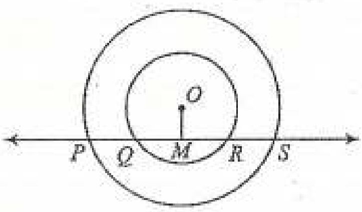
Theorem 5: There is one and only one circle passing through three non - collinear points.

Theorem 6: If two circles intersect in two points then the line through the centre is the perpendicular bisector of the common chord.

Example 1: Two concentric circles with centre O have P, Q, R, S as the points of intersection with the line as shown in fig. If $PS = 12$ cm, $QR = 8$cm, what is the length of PR and QS ?

Solution: Here $OM \perp QR$

$$QM = MR = \frac{1}{2}\,QR = \frac{1}{2} \times 8 = 4 \text{ cm.}$$



and $OM \perp PS$

$\Rightarrow$ $PM = MS = \frac{1}{2}\,PS = \frac{1}{2} \times 12 = 6$ cm.

Now, $PR = PQ + QR$

$PQ = PM - QM = 6 - 4 = 2$ cm.

$PR = 2 + 8 = 10$ cm.

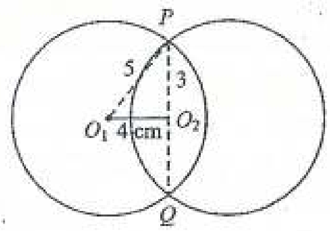
and $QS = QR + RS$

$\therefore$ $RS = MS - MR = 6 - 4 = 2$ cm.

$\Rightarrow$ $QS = 8 + 2 = 10$ cm.

Example 2: Two circles of radii 5 cm and 3 cm intersect at two points and distance between their centres is 4 cm. Find the length of the common chord.

Solution: $O_1P = 5$cm



$$O_2P = 3\text{cm}$$

$$O_1O_2 = 4\text{cm}$$

$$5^2 = 4^2 + 3^2$$

$\Rightarrow$ $O_1P^2 = O_1O_2^{\,2} + O_1P^2$

Hence $\angle O_1O_2P = 90°$

$\therefore$ O_2 is the mid - point of PQ.

then $PQ = 2 \times 3 = 6$ cm.

Important Theorems

Theorem 1: Equal chords of a circle are equidistant from the centre.

Theorem 2: Chords of circle which are equidistant from the centre are equal.

Theorem 3: Equal chords of a congruent circles are equidistant from the corresponding centres.

Theorem 4: Chords of congruent circles which are equidistant from the corresponding centres are equal.

Theorem 5: If the angles subtended by two chords of a circle at the centre are equal then chords are equal.

Theorem 6: Equal chords of congruent circles subtend equal angles at the centre.

Theorem 7: Of any two chords of a circle, the one which is larger is nearer to the centre.

Theorem 8: The angle subtened by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Theorem 9: The angle in a semi - circle is a right angle.

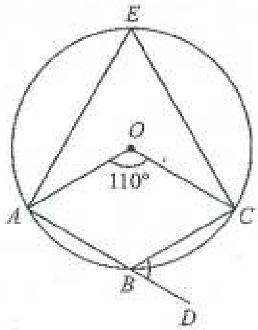
Theorem 10: Angles in the same segment of a circle are equal.

Theorem 11: If a line segment joining two points subtends equal angles of two other points lying on the same side of the line segment then the four points are concylic.

Theorem 12: The sum of either pair of the opposite angles of a cyclic quadrilateral is 180°.

Theorem 13: If a pair of opposite angles of a quadrilateral is supplementary then the quadrilateral is cyclic.

Theorem 14: If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle. .



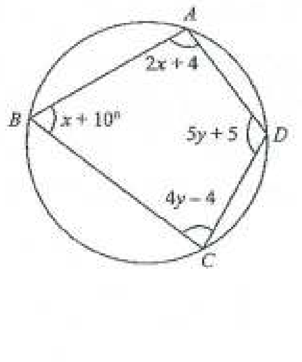
Example 3: O is the centre of the circle. $\angle AOC = 110°$ AB is produced to D. Find $\angle CBD$.

Solution: Here $\angle AEC = \frac{1}{2} \times \angle AOC$

$$= \frac{1}{2} \times 110° = 55°$$

$\therefore$ $\angle CBD = \angle AEC = 55°$

Example 4: Find the values of x and y.



Solution: Here $\angle A = (2x + 4)°$

$\angle B = (x + 10)°$

$\angle C = (4y - 4)°$

$\angle D = (5y + 5)°$

Opposite angles of a cyclic quadrilateral are supplementary.

$$\angle A + \angle C = 180°$$

$\Rightarrow$ $2x + 4 + 4y - 4 = 180°$

$\Rightarrow$ $2x + 4y = 180°$ ....(1)

and $\angle B + \angle D = 180°$

$\Rightarrow$ $x + 10 + 5y + 5 = 180$

$\Rightarrow$ $x + 5y = 165$ ....(2)

From (1) & (2)

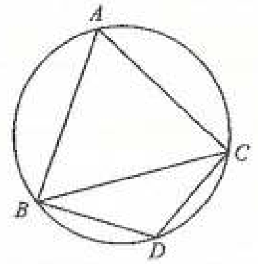
$$2x + 4y = 180$$
$$2x + 10y = 330$$
$$- \quad - \qquad -$$
$$-6y = -150$$

$\Rightarrow$ $y = 25°$

Now from (2) $x + 5y = 165$

$$x = 165 - 5 \times 25$$
$$= 165 - 125 = 40°$$

Example 5: In the given figure equilateral ΔABC is inscribed in a circle and $ABCD$ is a quadrilateral. What is the angle $\angle BDC$?

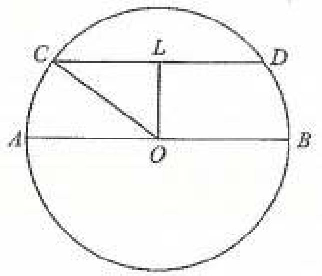


Solution: $\angle BAC = 60°$

In cyclic quadrilateral $ABCD$,

$$\angle BAC + \angle BDC = 180°$$
$$\angle BDC = 180° - \angle BAC$$
$$= 180° - 60° = 120°$$

Example 6: In the given fig. AOB is a diameter of a circle with centre O such that $AB = 34$ cm and CD is a chord of length 30 cm. What is the distance of CD from AB?



Solution: $OC = \text{radius} = \dfrac{34}{2} = 17$ cm

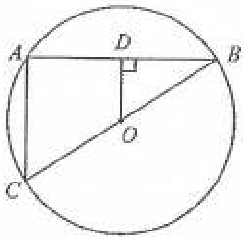
$$CL = \frac{1}{2}\, CD = \frac{1}{2} \times 30 = 15 \text{ cm}$$

$$OL^2 = OC^2 - CL^2 = 17^2 - 15^2$$
$$= 289 - 225 = 64$$

$\Rightarrow$ $OL = \sqrt{64} = 8$ cm

Distance of CD from $AB = 8$ cm

Example 7: In this fig, AB is a chord of circle with centre O. BOC is diameter. If $OD \perp AB$ such that $OD = 6$ cm, what is the length of AC.



Solution: Here $OD \perp AB$

So, D is mid-point of AB.

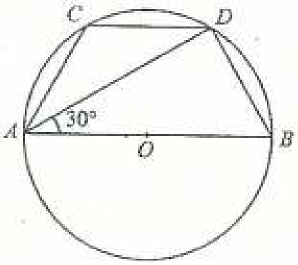
O is mid point of BC.

Hence $OD = \frac{1}{2} AC$

$\Rightarrow$ $2 \times 6 = AC \Rightarrow AC = 12$ cm.

Example 8: In the given fig, AOB is diameter. $CD \parallel AB$.

$\angle BAD = 30°$ then find $\angle CAD$.



Solution: Given $CD \parallel AB$.

$$\angle ADC = \angle BAD = 30°$$

$$\angle ADB = 90°$$

(angle in a semi circle)

$$\angle CDB = 30° + 90° = 120°$$

$ABCD$ is cyclic quadrilateral.

$$\angle BAC + \angle CDB = 180°$$

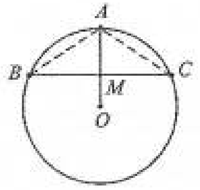
$\Rightarrow$ $\angle BAD + \angle CAD + \angle CDB = 180°$

$\Rightarrow$ $30° + \angle CAD + 120° = 180°$

$\Rightarrow$ $\angle CAD = 180° - 150° = 30°$

Multiple Choice Questions

1. The largest chord of a circle is called its:
   (a) Segment (b) Chord
   (c) Diameter (d) Radius
2. The radius of a circle is 26 cm and length of the perpendicular from the centre to the chord AB is equal to 10 cm. The length of AB is:
   (a) 24 cm (b) 20 cm
   (c) 44 cm (d) 48 cm
3. AB and CD are two parallel chords of a circle such that $AB = 8$ cm, and $CD = 6$cm. If the chords are on the opposite sides of the centre and the distance between them is 7cm, then the diameter of the circle is:
   (a) 5 cm (b) 10 cm (c) 8 cm (d) 12 cm
4. The radius of circumcircle of an equilateral triangle having length of each side equal to 'a' is :
   (a) $\sqrt{3}a$ (b) $\frac{2a}{\sqrt{3}}$
   (c) $\frac{\sqrt{3}a}{2}$ (d) $\frac{a}{\sqrt{3}}$
5. Two circles of radii 13cm and 15cm intersect and the length of common chord is 24 cm, then the distance between their centres is:
   (a) 15 cm (b) 14 cm
   (c) 16 cm (d) 17 cm
6. $AB \cong AC$ and O is the centre of the circle, then,

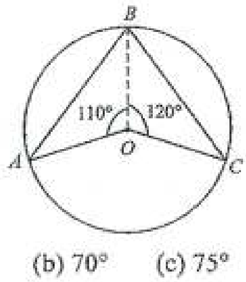


(a) $BM = MC$ (b) $BM \neq MC$
(c) OM is not perpendicular to BC
(d) None of these

7. Bisector AD of $\angle BAC$ of ΔABC passes through the centre of the circumcircle of ΔABC, then,
(a) $AB \neq AC$ (b) $AB = AC$
(c) $BC = AC$ (d) $BC = AB$

8. AB and AC are two equal chords of a circle whose centre is O. If $AB \perp OD$ and $OE \perp AC$, then,
(a) ΔABE is an isosceles triangle
(b) ΔADE is an equilateral triangle
(c) ΔADC is an isosceles triangle
(d) ΔADE is an isosceles triangle

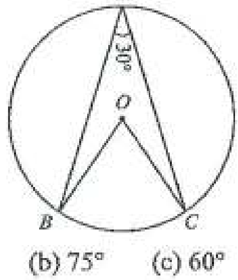
9. Find the measure of $\angle ABC$.



(a) 85° (b) 70° (c) 75° (d) 65°

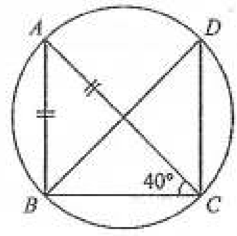
10. Any cyclic parallelogram is a:
(a) rhombus (b) rectangle
(c) square (d) trapezium

11. The measure of $\angle BOC$ is



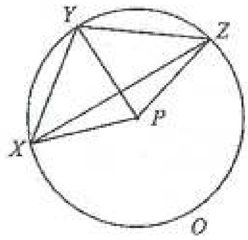
(a) 90° (b) 75° (c) 60° (d) 120°

12. In the adjoining figure, $AB = AC$ and $\angle ACB = 40°$, then $\angle BDC =$?



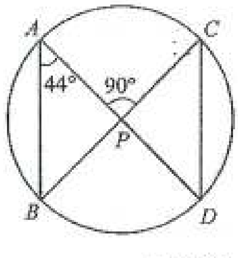
(a) 40° (b) 80°
(c) 90° (d) 100°

13. P is the centre of the circle, and $\angle XPZ = 120°$, $\angle XZY = 35°$, then the measure of $\angle YXZ$ is :



(a) 50° (b) 25°
(c) 35° (d) 60°

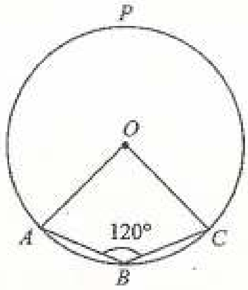
14. Chords AD and BC intersects each other at right angles at point P. If $\angle DAB = 44°$, then $\angle ADC =$?



(a) 44° (b) 88°
(c) 46° (d) 54°

15. $PQRS$ is a cyclic quadrilateral such that PR is a diameter of circle. If $\angle QPR = 64°$ and $\angle SPR = 31°$, then, $\angle R =$?
(a) 95° (b) 64°
(c) 85° (d) 31°

16. If the length of an arc of a circle is proportional to angle subtended by it at the centre. Then, the ratio of ABC : circumference = ?



(a) 1 : 3 (b) 2 : 3 (c) 1 : 2 (d) 3 : 4

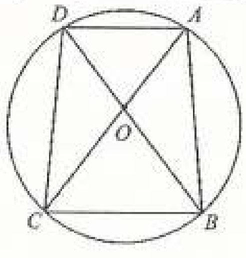
17. If A, B, C are three points on a circle with centre O such that $\angle AOB = 90°$ and $\angle BOC = 120°$, then $\angle ABC =$?

(a) 60° (b) 90° (c) 135° (d) 75°

18. The chord of a circle is equal to its radius. The angle subtended by this chord at the mid arc of the circle is

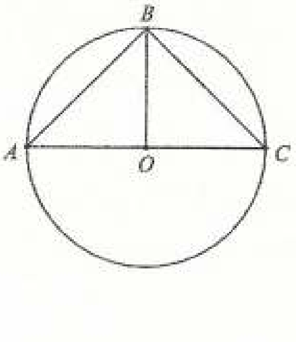
(a) 60° (b) 120° (c) 150° (d) 75°

19. O is the centre of circle, with $AC = 30$ cm and $DA = 10\sqrt{5}$ cm, then the measure of DC is



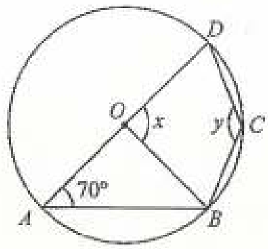
(a) $10\sqrt{5}$ cm (b) 20 cm
(c) $20\sqrt{5}$ cm (d) 25 cm

20. In the adjoining figure, O is the circumcentre of ΔABC, then the value of $\angle OBC + \angle BAC$ is



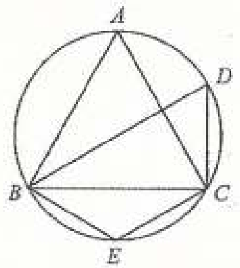
(a) 60° (b) 90°
(c) 120° (d) 150°

21. Find the value of $(x + y)$.



(a) 230° (b) 240°
(c) 235° (d) 250°

22. In the adjoining figure, $AB = AC$, and $\angle ACB = 64°$, then $\angle BEC =$?



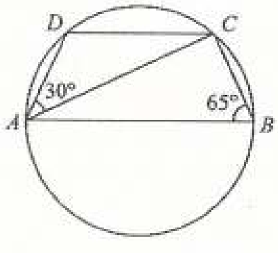
(a) 130° (b) 128°
(c) 122° (d) 120°

23. The sum of the angles in the 4 segments exterior to a cyclic quadrilateral

(a) 360° (b) 450°
(c) 540° (d) 720°

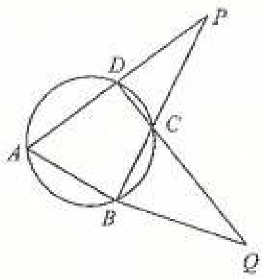
24. $AB \parallel CD$, and $\angle B = 65°$ and $\angle DAC = 30°$

The measure of $\angle CAB$ = ?



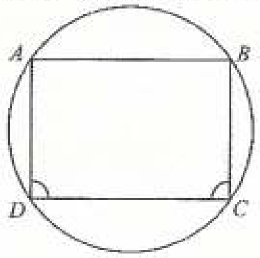
(a) 25° (b) 30°
(c) 40° (d) 35°

25. In figure (a), $\angle A = 60°$, $\angle ABC = 80°$, then the measure of $\angle BQC$ is



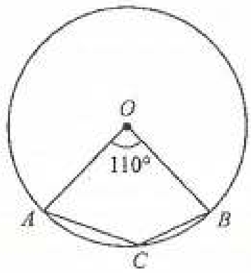
(a) 40° (b) 25° (c) 30° (d) 20°

26. In a cyclic quadrilateral, $AB \parallel CD$, then



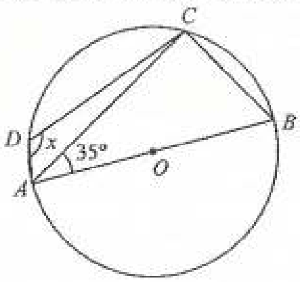
(a) $AD = BC$ (b) $AB = CD$
(c) $AB = AD$ (d) $AD = DC$

27. The measure of $\angle ACB$ is



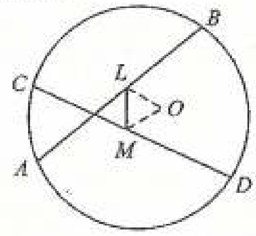
(a) 70° (b) 110° (c) 135° (d) 125°

28. Find x (O is the centre of circle) :



(a) 120° (b) 115°
(c) 125° (d) 145°

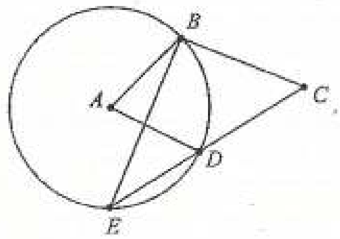
29. The measures of AB and CD are equal, and the measure of $\angle LOM = 160°$. The measure of $\angle OLM$ is :



(a) 12° (b) 10°
(c) 15° (d) 20°

30. If the two diameters of a circle intersect at 90°. The figure formed by joining the end point of the diameters will be a :
(a) rhombus (b) square
(c) rectangle (d) trapezium

31. A is the centre of circle. $ABCD$ is a parallelogram and CDE is a straight line. the ratio $\angle DEB : \angle BCD$ is



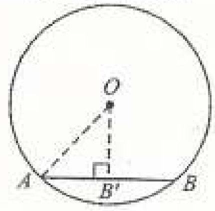
(a) 2 : 1 (b) 1 : 2
(c) $1 : \sqrt{2}$ (d) 1 : 3

Answer Key

1. (c)	2. (d)	3. (b)	4. (d)	5. (b)	6. (a)	7. (b)	8. (d)	9. (d)	10. (b)
11. (c)	12. (d)	13. (c)	14. (b)	15. (b)	16. (a)	17. (d)	18. (c)	19. (c)	20. (b)
21. (d)	22. (b)	23. (c)	24. (d)	25. (d)	26. (a)	27. (d)	28. (c)	29. (b)	30. (b)
31. (b)									

Hints and Solutions

1. **(c)** Diameter is the longest chord of a circle.
2. **(d)** According to question,



$OA = r = 26$ cm.

$OB' = 10$ cm

$\therefore \quad AB' = \sqrt{OA^2 - OB'^2}$

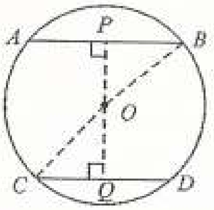
(Using Pythagoras theorem)

$= \sqrt{(26)^2 - (10)^2}$

$= \sqrt{576} = 24$ cm.

$\therefore AB = 2 \times AB' = 2 \times 24$ cm $= 48$ cm.

3. **(b)** Let the radius of the circle be r, and the length OP be x.



$\therefore$ In ΔOPB,

$$OP^2 + PB^2 = OB^2 = r^2$$

$$\Rightarrow \quad x^2 + (4)^2 = r^2 \quad [\because PB = \frac{AB}{2}]$$

$$\Rightarrow \quad x^2 = r^2 - 16 \quad \text{....(i)}$$

In ΔOCQ,

$$OQ^2 + CQ^2 = r^2$$

$$\Rightarrow \quad (7 - x)^2 + (3)^2 = r^2$$

$$[\because OQ = PQ - OP = 7 - x]$$

$$\Rightarrow \quad (7 - x)^2 = r^2 - 9 \quad \text{....(ii)}$$

Subtracting (ii) from (i), we get

$$x^2 - (7 - x)^2 = -7$$

$$\Rightarrow (x - 7 + x)(x + 7 - x) = -7$$

$$\Rightarrow \quad (2x - 7)(7) = -7$$

$$\Rightarrow \quad (2x - 7) = -1$$

$$\Rightarrow \quad x = 3$$

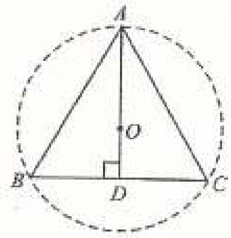
$$\therefore \quad r^2 = x^2 + 16 \quad \text{[using (i)]}$$

$$= (3)^2 + 16 = 25$$

$$\Rightarrow \quad r = 5\text{cm}$$

$$\therefore \quad d = 10 \text{ cm.}$$

4. **(d)** For an equilateral triangle ABC, O lies on the perpendicular from any vertex to the opposite side.



Also,

$AO : OD = 2 : 1$ (for equilateral triangle)

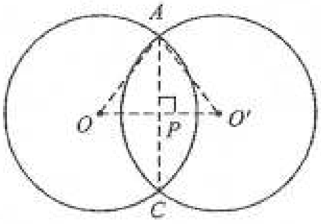
$$\Rightarrow \frac{AO}{OD} = \frac{2}{1}$$

$$\Rightarrow \frac{AO}{AD} = \frac{2}{3} \text{ (using componendo-dividendo)}$$

$\Rightarrow \quad \dfrac{AO}{\frac{\sqrt{3}}{2}a}=\dfrac{2}{3}$

$\Rightarrow \quad AO=\dfrac{2}{3}\times\dfrac{\sqrt{3}}{2}a=\dfrac{a}{\sqrt{3}}$

5. **(b)** In ΔOPA,



$OP=\sqrt{OA^2-AP^2}$

$=\sqrt{(13)^2-\left(\dfrac{AC}{2}\right)^2}$

$=\sqrt{(13)^2-\left(\dfrac{24}{2}\right)^2}=\sqrt{25}=5\text{cm}.$

Similarly,

In $\Delta O'PA$,

$O'P=\sqrt{O'A^2-AP^2}$

$=\sqrt{(15)^2-(12)^2}$

$=9\text{cm}.$

$\therefore OO'=OP+O'P=5+9=14$ cm.

6. **(a)** In $\Delta S\ ABM$ and ACM

$AB\cong AC$ (given)

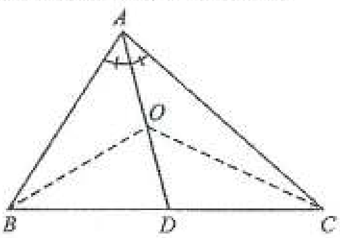
$AM=AM$ (common)

$OM\perp BC$ (given)

$\therefore \quad \Delta ABM\cong\Delta ACM$

$\therefore \quad BM=CM$

7. **(b)** $\because AD$ is the bisector of $\angle BAC$



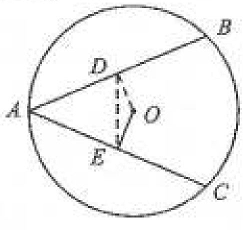
$\therefore AD$ is the $\perp$ bisector of BC.

$\because O$ is the circumcentre of ΔABC.

$\therefore \quad AB=AC\ [\because \angle ABD=\angle ACB]$

(By using $\Delta ABD\cong\Delta ACD$)

8. **(d)** In ΔODE,



$OD=OE$

$\therefore \quad \angle ODE=\angle OED \quad$ ......(i)

Now,

$\angle ODA=\angle ODE=90^\circ \quad$ ....(ii)

Subtracting eq.(i) from (ii), we get

$\angle ODA-\angle ODE=\angle OEA-\angle OED$

$\angle ADE=\angle AED$

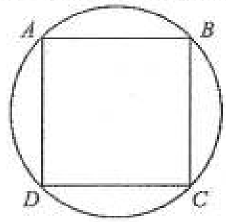
$\therefore AD=AE\Rightarrow\Delta ADE$ is an isosceles triangle.

9. **(d)** Reflex $\angle AOC=360^\circ-(110^\circ+120^\circ)$

$=130^\circ$

$\therefore \angle ABC=\dfrac{\angle AOC}{2}=\dfrac{130^\circ}{2}=65^\circ$

10. **(b)** $\because ABCD$ is a parallelogram,



$\therefore \angle A=\angle C$ and $\angle B=\angle D$

Also,

$ABCD$ is a cyclic quadrilateral,

$\angle A+\angle C=180^\circ$ and $\angle B+\angle D=180^\circ$

$\Rightarrow \angle A=\angle B=\angle C=\angle D=90^\circ$

$\therefore ABCD$ is a rectangle.

11. **(c)** $\angle BOC=2\times\angle BAC$ [Angle subtended at centre is double the angle subtended the circle]

$=2\times 30^\circ=60^\circ$

12. **(d)** In ΔABC,

$$AB = AC,$$

$\Rightarrow \angle ABC = \angle ACB = 40°$

Also,

$\angle ABC + \angle BAC + \angle ACB = 180°$

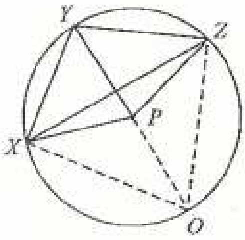
$\Rightarrow \angle BAC = 180° - 40° \times 2$

$= 100°$

$\therefore \angle BAC = \angle BDC = 100°$

(angles in the same segment are equal).

13. **(c)**



$\angle XPY = 2\angle XOP.$

$\because \quad \angle XOP = \angle XZY$

(angles in the same segment)

$\therefore \quad \angle XPY = 2\angle XZY \quad ...(i)$

Similarly,

$\angle YPZ = 2\angle YXZ \quad ...(ii)$

Using (i) and (ii)

$\angle XPZ = 2(\angle XYZ + \angle YXZ)$

$\Rightarrow \angle YXZ = \dfrac{\angle XPZ - 2\angle XYZ}{2} = \dfrac{120° - 2\times 35°}{2}$

$= \dfrac{50°}{2} = 25°$

14. **(b)** $\because \angle APC$ is an exterior angle for ΔABP.

$\therefore \quad \angle ABP + \angle PAB = 90°$

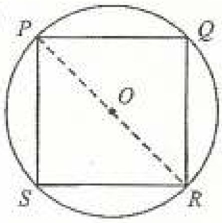
$\Rightarrow \quad \angle ABP = 90° - 44° = 46°$

$\angle ADC = \angle ABP$

(Angles in the same segment)

$\therefore \quad \angle ABP = 46°$

15. **(b)**



$\angle P = \angle QPR + \angle SPR$

$= 64° + 31°$

$= 95°$

$\because \angle Q$ and $\angle S$ are angles of the semicircle.

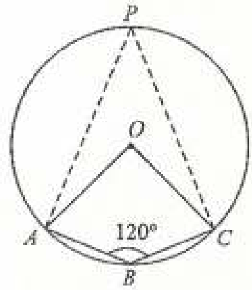
$\therefore \angle Q = \angle S = 90°$

$\because PQRS$ is a cyclic quadrilateral.

$\therefore \quad \angle P + \angle R = 180°$

$\Rightarrow \quad \angle R = 180° - 95° = 85°$

16. **(a)**



$\because ABCP$ is a cyclic quadrilateral.

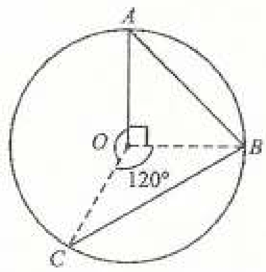
$\therefore \quad \angle B + \angle P = 180°$

$\Rightarrow \angle P = 180° - 120° = 60°$

$\angle AOC = 2\angle P = 2 \times 60° = 120°$

$\therefore \dfrac{\widehat{ABC}}{circumference} = \dfrac{120°}{360°} = 1:3$

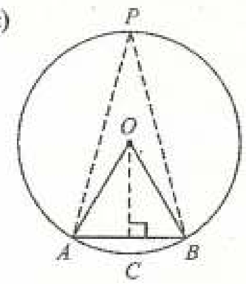
17. **(d)**



Reflex $\angle AOC = 360° - (90° + 120°) = 150°$

$\therefore \angle ABC = \dfrac{reflex \angle AOC}{2} = \dfrac{150°}{2} = 75°$

18. **(c)**



$\because \quad AB = r$

And, $\quad OA = OB = r$

$\therefore$ In ΔOAB,

$$AB = OA = OB = r$$

$\therefore$ ΔOAB is an equilateral Δ.

$\therefore \quad \angle OAB = \angle OBA = \angle AOB = 60°$

$\therefore \quad \angle APB = \dfrac{60°}{2} = 30°$

$\because$ $ACBP$ is a cyclic quadrilateral.

$\therefore \quad \angle C + \angle P = 180°$

$\Rightarrow \quad \angle C = 180° - 30°$

$= 150°$

19. **(c)** In ΔACD,

$\angle ADC = 90°$ [$\because \angle ACD$ is angle in semicircle]

$\therefore \quad AC^2 = DA^2 + DC^2$

$\Rightarrow (30)^2 = \left(10\sqrt{5}\right)^2 + DC^2$

$\Rightarrow DC^2 = 900 - 500$

$\Rightarrow \quad DC = \sqrt{400} = 20$ cm.

20. **(b)** In ΔOAB,

$$OA = OB$$

$\therefore \quad \angle OAB = \angle OBA \quad$ ...(i)

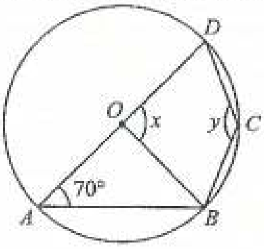
$\because \angle OBC + \angle OBA = \angle ABC$

$\Rightarrow \angle OBC + \angle BAC = \angle ABC$

$\Rightarrow \angle OBC + \angle BAC = 90°$

[$\because \angle ABC$ is the angle in semicircle]

21. **(d)** $\because$ $ABCD$ is a cyclic quadrilateral.



$\therefore \quad \angle A + \angle C = 180°$

$\Rightarrow \quad 70° + y = 180°$

$\Rightarrow \quad y = 110°$

Now, in ΔOAB,

$$OA = OB$$

$\therefore \quad \angle OAB = \angle OBA = 70°$

$\therefore$ $\angle BOD$ is an exterior angle for ΔOAB.

$\therefore$ $\angle OAB + \angle OBA = x$

$\Rightarrow \quad x = 70° + 70°$

$= 140°$

$\therefore \quad x + y = 140° + 110°$

$= 250°$

22. **(b)** $\because AB = BC$

$\therefore \angle ABC = \angle ACB = 64°$

$\therefore \angle BAC = 180° - 64° \times 2 = 52°$

$\therefore$ $ABCD$ is a cyclic quadrilateral.

$\therefore \quad \angle A + \angle E = 180°$

$\Rightarrow \quad \angle E = 180° - 52°$

$= 128°$

23. **(c)** The sum of the angles in the 4 segments of a cyclic quadrilateral $= 6 \times 90° = 540°$

24. **(d)** Let $\angle CAB = x$,

$\therefore \quad \angle ACD = x \quad$ (Alternate $\angle S$)

In ΔACD,

$\angle D = 180° - (30° + x)$

$= 150° - x$

$\because$ $ABCD$ is a cyclic quadrilateral.

$\therefore \quad \angle D + \angle B = 180°$

$\Rightarrow 150° - x + 65° = 180°$

$\Rightarrow \quad x = 35°$

25. **(d)** $\angle A + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 60° = 120°$

$\angle CBA = \angle B - \angle CBA$

$= 180° - 80° = 100°$

$\because \angle DCB$ is an exterior angle for ΔBCQ

$\therefore \angle BQC + \angle CBQ = \angle C$

$\Rightarrow \angle BQC = 120° - \angle CBQ$

$= 120° - 100° = 20°$

26. **(a)** $\because$ $ABCD$ is a cyclic quadrilateral.

$\therefore \quad \angle A + \angle C = \angle B + \angle D = 180° \quad$ ...(i)

Also,

$\angle A + \angle D = \angle B + \angle C = 180° \quad$ ...(ii)

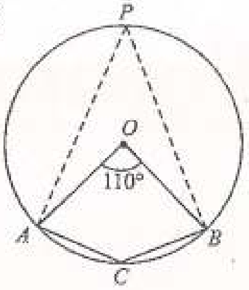
$\therefore \quad \angle B = \angle C \quad$ [using (i) and (ii)]

$\because$ $ABCD$ is a trapezium having $\angle C = \angle D$

$\therefore$ $ABCD$ should be an isosceles trapezium.

$\Rightarrow \quad AD = BC$

27. **(d)**



$$\frac{\angle AOB}{2} = \angle APB$$

$\Rightarrow \angle APB = \frac{110°}{2} = 55°$

$\because$ *ACBP* is a cyclic quadrilateral.

$\therefore \angle ACB + \angle APB = 180°$

$\Rightarrow \angle ACB = 180° - 55°$

$= 125°$

28. **(c)** In ΔCBA,

$\angle A + \angle C + \angle B = 180°$

$\Rightarrow \angle B = 180° - 35° - 90°$

[$\because \angle C = 90°$, i.e., angle in a semicircle]

$\Rightarrow \angle B = 55°$

$\because$ *ABCD* is a cyclic quadrilateral

$\therefore \angle B + \angle D = 180°$

$\Rightarrow x = 180° - 55°$

$= 125°$

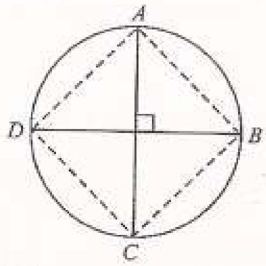
29. **(b)** $\because AB = CD$

$\therefore$ $OL = OM$, as the distance of equal chords from the centre of the circle should be equal.

$\Rightarrow \angle OLM = \angle OML$

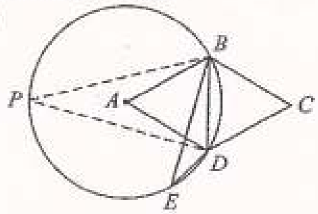
$\Rightarrow \angle OLM = \frac{180° - 160°}{2} = 10°$

30. **(b)** $\because$ Diagonals of the quadrilateral *ABCD* intersect at right angles and are also of equal length i.e., $2r$.



$\therefore$ *ABCD* is a square.

31. **(b)**



$\angle DPB = \frac{\angle BAD}{2}$ (angle subtended at the centre is double the angle at the circumference)

$\Rightarrow \angle BAD = 2\angle DPB$

$\because$ *ABCD* is a parallelogram.

$\therefore \angle BAD = \angle BCD$

$\Rightarrow \angle BCD = 2\angle DPB$

$\angle DEB = \angle BPD$ (angles in the same segment)

$\therefore \frac{\angle DEB}{\angle BCD} = \frac{\angle DPB}{2\angle DPB} = \frac{1}{2} = 1:2$

11. Heron's Formula

Learning Objective:

In this chapter, we will learn about:
*Area
*Heron's Formula

Square

If 'p' is the length of each side of square, then,

Length of diagonal = $\sqrt{2}p$,

Area of square = $p^2 = \frac{1}{2} \times (\text{Diagonal})^2$,

Area of perimeter = $p \times 4 = 4p$

Rectangle

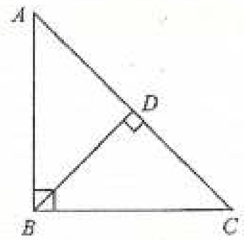
If l is the length and b is the breadth of rectangle, then,

Length of diagonal = $\sqrt{l^2 + b^2}$

Area = lb

Perimeter = $2(l + b)$

Right Angled Triangle



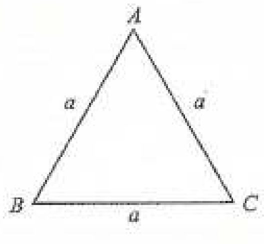
Let ABC be a right angled triangle, right angled at B, then,

(i) Perimeter = $AB + BC + CA$

(ii) $AC = \sqrt{AB^2 + BC^2}$

(iii) area = $\frac{1}{2} \times AB \times BC = \frac{1}{2} \times BD \times AC$

Equilateral Triangle



Let the side length of equilateral triangle be 'a' then,

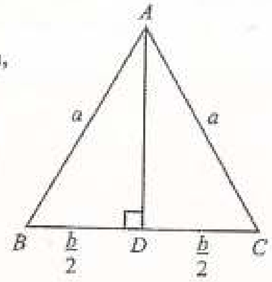
(a) Perimeter = $3a$

(b) Altitude = $\frac{\sqrt{3}a}{2}$

(c) Area = $\frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}a^2}{4}$

Isoceles Triangle

Let the lengths of equal sides be a and length of remaining side be 'b', then,



(a) AD = altitude = $\sqrt{a^2 - \frac{b^2}{4}}$

(b) Perimeter = $a + a + b = 2a + b$

(c) area = $\frac{1}{2} \times \sqrt{a^2 - \frac{b^2}{4}} \times b = \frac{1}{4} \times \sqrt{4a^2 - b^2} \times b$

Parallelogram

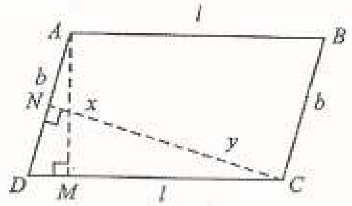
Let $ABCD$ be a parallelogram such that $AB = CD = l$ and $BC = AD = b$, then,

(a) Perimeter = $2\,(l + b)$

(b) Area = Base × Height

$= l \times x = xl$

$= y \times b = yb$



Rhombus

If d_1 and d_2 are the lengths of diagonals of the rhombus and a is the length of side of rhombus, then,

(a) $a = \sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2}$

(b) Perimeter = $4a = \frac{4}{2}\sqrt{d_1^2 + d_2^2} = 2\sqrt{d_1^2 + d_2^2}$

(c) area = $\frac{1}{2}$ × Product of diagonals = $\frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} d_1 d_2$

Heron's Formula

The formula given by Heron about the area of a triangle, is also known as Heron's Formula.
Let a, b, c denote the lengths of the sides of a triangle ABC. Then,

Area = $\sqrt{s(s-a)(s-b)(s-c)}$ where,

$s = \frac{a+b+c}{2}$, is the semi-perimeter of ΔABC.

This formula is valid for any type of triangle.

Example 1: Find the area of triangle whose sides are 13, 14, 15 cm.

Solution: Here $s = \frac{13+14+15}{2} = \frac{42}{2} = 21$ cm

Area = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-13)(21-14)(21-15)}$

$= \sqrt{21 \times 8 \times 7 \times 6} = 84$ cm^2

Example 2: The lengths of sides of a right - angled triangle are 5cm, 12cm and 13cm. Find the length of shortest altitude.

Solution: Here, area of triangle = $\frac{1}{2}\times5\times12=\frac{1}{2}\times side\times(shortest\ altitude)$

$$\Rightarrow \quad \frac{60}{(side)} = \text{shortest altitude.}$$

Altitude is shortest when, side length is largest,

$\therefore$ length of shortest altitude = $\frac{60}{13}$cm.

Example 3: Find the percentage increase in area of triangle if its each side is triple.

Solution: $S_1 = \text{New } S = \frac{3a+3b+3c}{2} = \frac{3(a+b+c)}{2} = 3s$

$$a_1 = 3a,\ b_1 = 3b,\ c_1 = 3c$$

$$A_1 = \text{area} = \sqrt{s_1(s_1-a_1)(s_2-a_2)(s_3-a_3)}$$

$$= \sqrt{3s\times3(s-a)\times3(s-b)\times3(s-b)} = 9A$$

Change in area = $9A - A = 8A$.

$\therefore$ Increase % = $\frac{8A}{A}\times100 = 800\%$

Example 4: Perimeter of an equilateral triangle is 45cm, then area is ---- cm^2.

Solution: Perimeter = $3a = 45$

$$\Rightarrow \quad a = 15$$

$\therefore$ Area = $\frac{\sqrt{3}\times(15)^2}{4} = \frac{225\sqrt{3}}{4}$ cm^2

Example 5: Find the area of rectangle having length 24cm and length of diagonal 26cm.

Solution: Length = 24cm,

Breadth = $\sqrt{(\text{diagonal})^2-(\text{length})^2} = \sqrt{(26)^2-(24)^2}$

$= \sqrt{676-576} = 10\text{cm}.$

$\therefore$ Area of rectangle = length × breadth

$= 24\times10 = 240cm^2$

Example 6: The adjacent sides of parallelogram are 34cm and 20cm and length of diagonal is 42cm. Find the area of parallelogram.

Solution: We know area of parallelogram = 2 (Area of Δ between the parallels)

$$= 2\times\sqrt{s(s-a)(s-b)(s-c)}$$

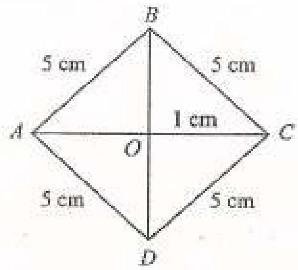
$$s = \frac{34+20+42}{2} = \frac{96}{2} = 48$$

$\therefore$ Area of parallelogram $= 2\times\sqrt{48\times(48-34)(48-20)\times(48-42)}$

$$= 2\times 336\text{ cm}^2 = 672\text{ cm}^2$$

Example 7: Find the area of the blades of the magnetic compass.

[Take $\sqrt{11} = 3.32$]



Solution: In ΔAOB, $OB = \sqrt{AB^2 - OB^2}$

$$= \sqrt{(5)^2 - \left(\frac{1}{2}\right)^2} = \sqrt{25 - \frac{1}{4}} = \frac{\sqrt{99}}{2} = \frac{3}{2}\sqrt{11}\text{ cm}$$

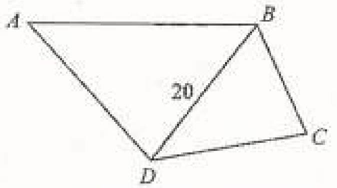
$$BD = 2\times OB = 3\sqrt{11}\text{ cm}$$

$$\therefore \text{area }(ABCD) = \frac{1}{2}\times AC\times BD$$

$$= \frac{1}{2}\times 3\sqrt{11}\times 1 = \frac{3\times 3.32}{2} = 4.98\text{cm}^2$$

Example 8: Find the area of quadrilateral $ABCD$, in which $AB = 42$ cm, $BC = 21$ cm, $CD = 29$cm, $DA = 34$ cm and diagonal $BD = 20$ cm.

Solution:



ar (quad. $ABCD$) = ar (ΔABD) + ar (ΔBDC)

For, ΔABD,

$$s = \frac{AB + BD + AD}{2} = \frac{42 + 20 + 34}{2} = 48\text{ cm.}$$

$$\therefore \text{ar }(\Delta ABD) = \sqrt{48\times(48-42)\times(48-20)\times(48-34)} = 336\text{ cm}^2$$

For ΔBDC, $S = \dfrac{21 + 29 + 20}{2} = 35$cm

$$\therefore \text{ar }(\Delta BDC) = \sqrt{35\times(35-21)\times(35-29)\times(35-20)}$$

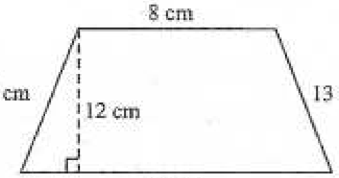
$$= \sqrt{35\times 14\times 6\times 15} = 7\times 2\times 5\times 3$$

$= 210$ cm2

$$\therefore \text{ar (quad. } ABCD) = 336\text{ cm}^2 + 210\text{ cm}^2$$

$$= 546\text{ cm}^2$$

Example 9: Find the area.



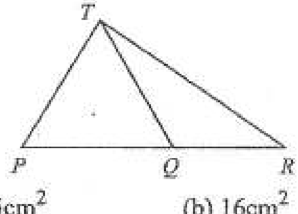
Solution: Area $= \frac{1}{2}\times(18+8)\times12 = 13\times12 = 156\text{ cm}^2$

Example 10: Find the area of square having length of diagonal $5\sqrt{2}$ cm.

Solution: Area of square $= \frac{1}{2}(\text{diagonal})^2 = \frac{1}{2}\times\left(5\sqrt{2}\right)^2 = \frac{1}{2}\times50 = 25\text{cm}^2$

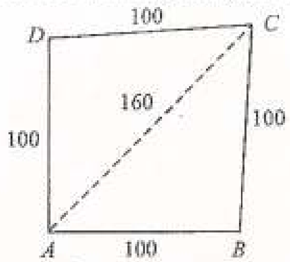
Multiple Choice Questions

1. The area of a triangle, whose two sides are 8 cm and 11cm and the perimeter is 32 cm, will be:
   (a) $6\sqrt{30}$ cm^2 (b) $8\sqrt{30}$ cm^2
   (c) $10\sqrt{30}$ cm (d) $9\sqrt{30}$ cm^2
2. The area of an isosceles triangles whose equal sides are 12cm and the other side is 6cm long, will be :
   (a) $3\sqrt{15}$ cm^2 (b) $6\sqrt{15}$ cm^2
   (c) $9\sqrt{15}$ cm^2 (d) $12\sqrt{15}$ cm^2
3. If the length of each side of a triangle is multiplied by 3, then the % increase in area will be:
   (a) 400% (b) 800%
   (c) 700% (d) 900%
4. The sides of a triangle are 50 cm, 78 cm and 112 cm. the smallest altitude is:
   (a) 50 cm (b) 40 cm
   (c) 30 cm (d) 25 cm
5. The sides of a triangle are 11cm, 15cm and 16 cm. The altitude to the largest side is:
   (a) 30 cm (b) $\frac{15\sqrt{7}}{4}$ cm
   (c) $\frac{15\sqrt{7}}{2}$ cm (d) $20\sqrt{7}$ cm
6. The length of median of an equilateral triangle is $2\sqrt{3}$ cm. The length of its side are :
   (a) 3 cm (b) 6 cm
   (c) 4cm (d) $4\sqrt{3}$ cm
7. The length of median of an equilateral triangle is $\sqrt{3}$ cm. The area of triangle is.
   (a) $2\sqrt{3}$ cm^2 (b) $4\sqrt{3}$ cm^2
   (c) $\sqrt{3}$ cm^2 (d) $3\sqrt{3}$ cm^2
8. The base and hypotenuse of a right triangle are 5cm, 13cm long. The length of altitude from the vertex containing right angle to the hypotenuse will be:
   (a) $\frac{30}{13}$ cm (b) $\frac{90}{13}$ cm
   (c) $\frac{60}{13}$ cm (d) $\frac{120}{13}$ cm
9. In the figure, $PQ : QR = 3 : 2$. If the area of $\Delta PRT = 40$ cm^2, then area of ΔTQR is:



   (a) 15cm^2 (b) 16cm^2
   (c) 35cm^2 (d) 30cm^2

10. Find the area of the given figure :



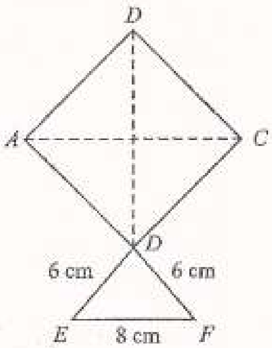
(a) $4800\ cm^2$ (b) $5600\ cm^2$
(c) $9600\ cm^2$ (d) $8800\ cm^2$

11. Find the length *BD*, from the previous question:
(a) 120 cm (b) 60 cm
(c) 80 cm (d) 160 cm

12. If a square and rhombus have same perimeter, and area of square is *S* and area of rhombus is *R*, then
(a) $S > R$ (b) $R > S$
(c) $R = S$ (d) data insufficient

13. If a square and equilateral triangle have same perimeter and, square has area A_1 and equilateral triangle has area A_2, then.
(a) $A_1 = A_2$ (b) $A_1 > A_2$
(c) $A_2 > A_1$ (d) $A_2 = \frac{2}{3} A_1$

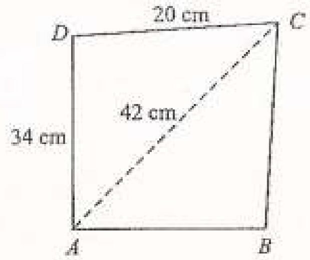
14. The area of kite in the adjoining figure is : $(AC = BD = 32\ cm)$



(a) $512\ cm^2$ (b) $529.84\ cm^2$
(c) $512.84\ cm^2$ (d) $517.84\ cm^2$

15. Two parallel sides of a trapezium are 60cm and 77cm and other sides are 25cm and 26cm. The area of the trapezium is
(a) $622\ cm^2$ (b) $822\ cm^2$
(c) $1244\ cm^2$ (d) $1644\ cm^2$

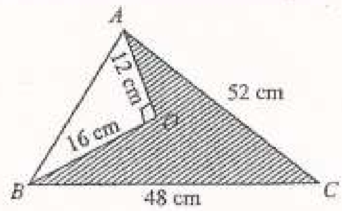
16. Area of parallelogram, in the adjoining figure will be :



(a) $336\ cm^2$ (b) $672\ cm^2$
(c) $1008\ cm^2$ (d) $1080\ cm^2$

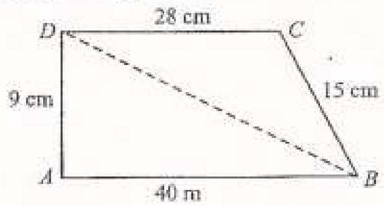
17. The area of rhombus whose perimeter is 80m and one of the diagonal is 24m.
(a) $284m^2$ (b) $384m^2$ (c) $192m^2$ (d) $374m^2$

18. Find the area of the shaded region.



(a) $278\ cm^2$ (b) $384\ cm^2$
(c) $384\ cm^2$ (d) $284\ cm^2$

19. Area of the figure is :



(a) $216\ m^2$ (b) $316\ m^2$
(c) $306\ m^2$ (d) $206\ m^2$

20. A square and an equilateral triangle have equal perimeters. If the diagonal of the square is $12\sqrt{2}$ cm, the area of the triangle is :
(a) $24\sqrt{2}\ cm^2$ (b) $48\sqrt{3}\ cm^2$
(c) $24\sqrt{3}\ cm^2$ (d) $64\sqrt{3}\ cm^2$

21. Find the cost of fencing a triangular park having area = $20\sqrt{2}$ m^2. And two of its sides as 11m and 6 m. (Cost of fencing = ₹ 10/m).
(a) ₹ 280 (b) ₹ 400
(c) ₹ 320 (d) ₹ 270

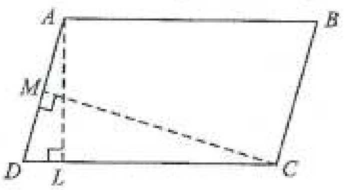
22. The third side of triangle whose two sides are 26 and 28 cm and area is 336 cm^2, is
(a) 29 cm (b) 27 cm
(c) 30 cm (d) 32 cm

23. The area of a trapezium whose parallel sides are 25 cm and 13 cm and other sides are 15 cm, 15 cm, is :
(a) $56\sqrt{20}$ cm^2 (b) $56\sqrt{21}$ cm^2
(c) $57\sqrt{21}$ cm^2 (d) $61\sqrt{21}$ cm^2

24. The length of sides of a triangle are in the ratio 3 : 4 : 5 and its perimeter is 144 cm, then, the height corresponding to the length side is:
(a) 27.8 cm (b) 26.8 cm
(c) 28.8 cm (d) 30.8 cm

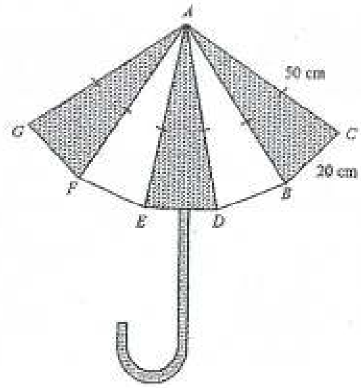
25. The diagonal of a parallelogram divide it in 2 parts, the area of the two parts:
(a) will be equal
(b) will be unequal
(c) cannot be compared
(d) $\frac{2}{3}$ of the area of parallelogram

26. *ABCD* is a parallelogram, where,



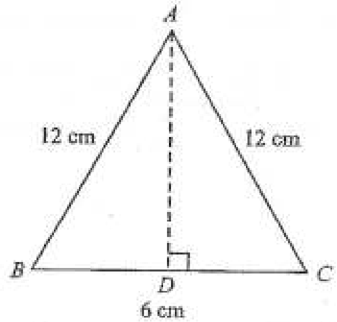
$AL = 8$ cm, $CM = 10$ cm, $AD = 6$ cm. find AB.
(a) 6.5 cm
(b) 6 cm
(c) 7 cm
(d) 7.5 cm

27. Find the area of the umbrella.



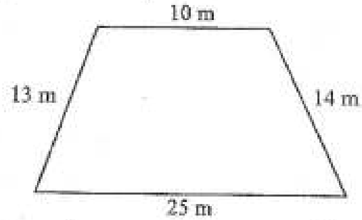
(a) $200\sqrt{6}$ cm^2 (b) $1000\sqrt{6}$ cm^2
(c) $300\sqrt{6}$ cm^2 (d) $400\sqrt{6}$ cm^2

28. The area of the adjoining figure will be :



(a) $9\sqrt{15}$ cm^2 (b) $9\sqrt{11}$ cm^2
(c) $9\sqrt{17}$ cm^2 (d) $11\sqrt{6}$ cm^2

29. The area of the trapezium in the adjoining figure will be equal to :



(a) 98 m^2 (b) 196 m^2
(c) 392 m^2 (d) 49 m^2

30. If each side of Δ is doubled, then the area will become how many times?
(a) 2 times (b) 3 times
(c) 4 times (d) 8 times

Answer Key

1. (b)	2. (c)	3. (b)	4. (c)	5. (b)	6. (c)	7. (c)	8. (c)	9. (b)	10 (c)
11. (a)	12. (a)	13. (a)	14. (b)	15. (d)	16. (b)	17. (b)	18. (d)	19. (c)	20. (d)
21. (c)	22. (b)	23. (c)	24. (c)	25. (a)	26. (d)	27. (b)	28. (a)	29. (b)	30. (c)

Hints and Solutions

1. **(b)** Here perimeter $= 2S = 32$ cm

$\Rightarrow \quad S = \frac{32}{2} = 16$ cm.

Now, sides of Δ are 8 cm, 11 cm and (32 – (11 + 8)) cm, i.e., 13 cm

8cm, 11cm and 13cm.

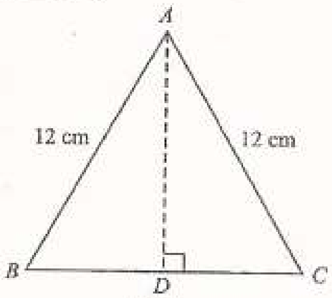
$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16\times(16-8)(16-11)(16-13)}$

$= \sqrt{16\times8\times5\times3}$

$= 8\sqrt{30}\ \text{cm}^2$

2. **(c)** In ΔABD,



$AB^2 + BD^2 = AD^2$

$\Rightarrow \quad AD^2 = AB^2 - BD^2$

$= (12)^2 - \left(\frac{6}{2}\right)^2 = 144 - 9 = 135$

$\Rightarrow AD = \sqrt{135} = 3\sqrt{15}$ cm.

$\therefore$ area $(\Delta ABC) = -\times 6\times 3\sqrt{15}$

$= 9\sqrt{15}\ \text{cm}^2$

3. **(b)** Let the sides be a, b, c.

$\therefore$ New sides $= 3a, 3b, 3c$.

$\therefore \quad s_{new} = 3s$.

New area $= \sqrt{3s(3s-3a)(3s-3b)(3s-3c)}$

$= 9\sqrt{s(s-a)(s-b)(s-c)}$

$= 9\Delta$

$\therefore$ Increase in area $= 9\Delta - \Delta = 8\Delta$

$\therefore$ % Increase in area $= \frac{8\Delta}{\Delta}\times100 = 800\%$

4. **(c)** Here $S = \frac{50+78+112}{2} = 120$cm.

$\therefore$ Area

$= \sqrt{120(120-50)(120-78)(120-112)}$

$= \sqrt{120\times70\times42\times8}$

$= \sqrt{2\times2\times3\times2\times5\times7\times2\times5\times7\times3\times2\times2\times2\times2}$

$= 2\times3\times2\times2\times2\times5\times7$

$= 240\times7\ \text{cm}^2 = 1680\ \text{cm}^2$

Area $= \frac{1}{2}\times\text{base}\times\text{altitude} = 1680\ \text{cm}^2$

$\Rightarrow$ Altitude $= \frac{2\times1680}{\text{base}}$ cm

$= \frac{2\times1680}{112}$ cm

$= 30$ cm

5. **(b)** $S = \frac{11+15+16}{2} = \frac{42}{2} = 21$cm

$\therefore$ Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-11)(21-15)(21-16)}$

$= \sqrt{21\times10\times6\times5}$

$= 5\times2\times3\sqrt{7}$

$= 30\sqrt{7}\text{ cm}^2$

$\frac{1}{2}\times$ Altitude to the largest side $\times$ largest side

$= 30\sqrt{7}\text{ cm}^2$

$\Rightarrow$ Altitude to the largest side

$= \frac{60\sqrt{7}}{16} = \frac{15}{4}\sqrt{7}$ cm.

6. **(c)** Length of median of equilateral triangle $= \frac{\sqrt{3}a}{2} = 2\sqrt{3}$ cm.

$\Rightarrow \quad a = \frac{2\sqrt{3}\times2}{\sqrt{3}}$ cm $= 4$ cm.

7. **(c)** Length of median of equilateral triangle

$= \frac{\sqrt{3}a}{2} = \sqrt{3}$

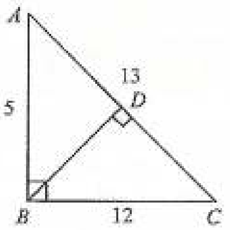
$\Rightarrow \quad a = 2$ cm.

$\therefore$ Area of triangle

$= \frac{\sqrt{3}a}{4} = \frac{\sqrt{3}\times(2)^2}{4} = \sqrt{3}\text{ cm}^2$

8. **(c)** $BC = \sqrt{AC^2 - AB^2}$

$= \sqrt{(13)^2-(5)^2} = 12$cm.



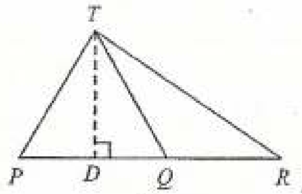
Area of $\Delta = \frac{1}{2}\times AC\times BD = \frac{1}{2}\times AB\times AC$

$\Rightarrow AC\times BD = AB\times AC$

$\Rightarrow 13\times BD = 5\times 12$

$\Rightarrow \quad BD = \frac{60}{13}$ cm

9. **(b)**



$\frac{Ar(\Delta PQT)}{Ar(\Delta PRT)} = \frac{\frac{1}{2}\times TD\times PQ}{\frac{1}{2}\times TD\times PR}$

$\Rightarrow \frac{Ar(\Delta PQT)}{40} = \frac{3}{3+2} = \frac{3}{5}$

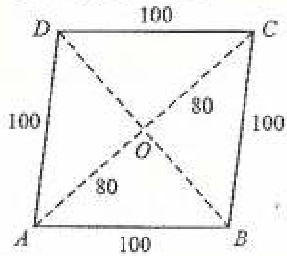
$\Rightarrow \quad$ ar $(\Delta PQT) = 24\text{ cm}^2$

$\therefore$ area (ΔTQR) = ar (ΔPRT) − ar (ΔPQT)

$= 40 - 24 = 16\text{cm}^2$

10. **(c)** In ΔDOC,

$OD^2 + OC^2 = DC^2$



$\Rightarrow OD = \sqrt{DC^2 - OC^2}$

$= \sqrt{(100)^2-(80)^2} = 60$ cm.

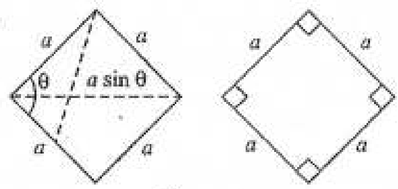
$\therefore \ DB = 2\times OD = 2\times 60 = 120$ cm.

Area of rhombus $= \frac{1}{2}\times d_1\times d_2 = \frac{1}{2}\times160\times120$

$= 9600\text{ cm}^2$

11. **(a)** $BD = 2\times OD = 2\times 60 = 120$ cm

12. **(a)**



Area of square $= a^2$

Area of rhombus $= a^2\sin\theta$

$\because \quad \sin\theta < 1$

$\therefore \quad a^2 > a^2 \sin\theta$

ar (square) > ar (rhombus)

$\Rightarrow \quad S > R$

13. **(b)** Perimeter of square

= Perimeter of equilateral $\Delta = x$.

$\therefore$ length of side of square $= \dfrac{x}{4}$

Length of side of equilateral $\Delta = \dfrac{x}{3}$

$$A_1 = \left(\frac{x}{4}\right)^2,\quad A_2 = \frac{\sqrt{3}\left(\frac{x}{3}\right)^2}{4} = \frac{\sqrt{3}x^2}{36}$$

Clearly, $A_1 > A_2$

14. **(b)** Area of square $ABCD = \dfrac{1}{2} \times AC \times BD$

$= \dfrac{1}{2} \times 32 \times 32$

$= 16 \times 32 \text{ cm}^2$

$= 512 \text{ cm}^2$

Here $\quad s = \dfrac{6+6+8}{2} = \dfrac{20}{2} = 10$

$\therefore$ Area (ΔDEF)

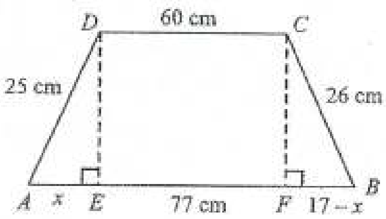
$= \sqrt{\left(\dfrac{6+6+8}{2}\right)(10-6)(10-6)(10-8)}$

$= \sqrt{10\times4\times4\times2}$

$= 8\sqrt{5} \text{ cm}^2 = 17.84 \text{ cm}^2$

$\therefore$ Total area $= 512 + 17.84 = 529.84 \text{ cm}^2$

15. **(d)**



$AE = x$, FB $= (17 - x)$ cm.

From ΔAED, and ΔCFB

$CF^2 = DE^2$

$\Rightarrow \quad (26)^2 - (17-x)^2 = (25)^2 - (x)^2$

$\Rightarrow \quad (26)^2 - (25)^2 = (17-x)^2 - (x)^2$

$\Rightarrow (26-25)(26+25) = (17-x+x)$

$(17-x-x)$

$\Rightarrow \quad 51 = 17(17-2x)$

$\Rightarrow \quad 17 - 2x = 3$

$\Rightarrow \quad 2x = 14 \Rightarrow x = 7\text{cm}.$

$\Rightarrow CF = DE = \sqrt{(25)^2 - (7)^2} = 24\text{cm}.$

$\therefore$ area $= \dfrac{1}{2} \times (60 + 77) \times 24 \text{ cm}^2$

$= 137 \times 12 \text{ cm}^2 = 1644 \text{ cm}^2$

16. **(b)** Area of ΔACD = area of ΔACB

$= \dfrac{1}{2}$ (Area of ∥gm $ABCD$)

$\Rightarrow$ area of $\Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$

$\left\{s = \dfrac{34+20+42}{2} = 48 \text{ cm}\right\}$

$= \sqrt{48\times(48-34)\times(48-20)\times(48-42)}$

$= \sqrt{48\times14\times28\times6}$

$= 4 \times 6 \times 7 \times 2$

$= 4 \times 84 = 336 \text{ cm}^2$

$\therefore$ area of ∥gm $= 2 \times \text{ar}(\Delta ACD)$

$= 2 \times 336 \text{ cm}^2$

$= 672 \text{ cm}^2$

17. **(b)** Side of rhombus $= \dfrac{80}{4} = 20\text{cm}.$

$\therefore$ Length of other diagonal

$= 2\times\sqrt{(20)^2 - \left(\dfrac{24}{2}\right)^2}$

$= 2\times\sqrt{400-144}$

$= 2 \times 16 = 32$ cm.

$\therefore$ Area $= \dfrac{1}{2} \times 32 \times 24$

$= 16 \times 24 \text{ cm}^2 = 384 \text{ cm}^2$

18. **(b)** In ΔAOB,

$AB = \sqrt{OB^2 + OA^2}$

$= \sqrt{(16)^2 + (12)^2}$

$= 20$ cm.

$\therefore$ ar(ΔAOB) = — × 12 × 16 = 96 cm^2

For ΔABC,

$$S = \frac{52+48+20}{2} = 60 \text{ cm.}$$

$\therefore$ ar (ΔABC) = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60\times(60-52)(60-48)(60-20)}$

$= \sqrt{60\times 8\times 12\times 40}$

$= 480$ cm^2

$\therefore$ Area of shaded region

$= 480 - 96$

$= 384$ cm^2

19. **(c)** $BD = \sqrt{AD^2 + AB^2}$

$= \sqrt{(9)^2 + (40)^2} = 41$ cm

$\therefore$ For ΔDBC,

$$S = \frac{28+15+41}{2} = 42 \text{ cm}$$

$\therefore$ ar(ΔDBC) = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42\times(42-28)\times(42-41)\times(42-15)}$

$= \sqrt{42\times 14\times 1\times 27}$

$= 14 \times 3 \times 3$

$= 126$ cm^2

Ar(ΔABD) = $\frac{1}{2}\times 9\times 40$

$= 180$ cm^2

$\therefore$ Total area = (126 + 180) cm^2

$= 306$ cm^2

20. **(d)** Length of equilateral triangle = $\frac{x}{3}$ cm

Length of side of square = $\frac{x}{4}$ cm.

Length of side of square

$$= \frac{\text{length of diagonal}}{\sqrt{2}}$$

$= 12$ cm.

$\therefore$ area of $\Delta = \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2 = \frac{\sqrt{3}}{4}\times\left(\frac{48}{3}\right)^2$

$= \frac{\sqrt{3}}{4}\times 16\times 16 = 64\sqrt{3}$ cm^2

21. **(c)** Let the length of the third side be x.

$\therefore S = \frac{11+6+x}{2} = \frac{17+x}{2}$, and,

Area

$$= \sqrt{\frac{17+x}{2}\left(\frac{17+x}{2}-11\right)\left(\frac{17+x}{2}-6\right)\left(\frac{17+x}{2}-x\right)}$$

$$20\sqrt{2} = \sqrt{\left(\frac{17+x}{2}\right)\left(\frac{x-5}{2}\right)\left(\frac{x+5}{2}\right)\left(\frac{17-x}{2}\right)}$$

$\Rightarrow \quad 800 = \frac{1}{16}(x^2 - 25)(289 - x^2)$

$\Rightarrow 800 \times 16 = (x^2 - 25)(289 - x^2)$

$\Rightarrow \quad x = 15$m.

$\therefore$ cost of fencing = (11 + 6 + 15) × 10

= ₹ 320

22. **(c)** Let the length of third side be x.

$\therefore \quad S = \frac{54+x}{2} = 27 + \frac{x}{2}$

Area

$$= \sqrt{\left(27+\frac{x}{2}\right)\left(27+\frac{x}{2}-26\right)\left(27+\frac{x}{2}-28\right)\left(27-\frac{x}{2}\right)}$$

$$= \sqrt{\left(729-\frac{x^2}{4}\right)\left(\frac{x^2}{4}-1\right)}$$

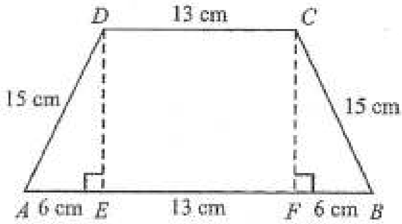
$= 336$ cm^2

$\Rightarrow x = 30$ cm.

$\therefore$ Third side = 30 cm.

23. **(c)** In ΔAED,

$$DE = \sqrt{AD^2 - AE^2} = \sqrt{(15)^2 - (6)^2}$$
$$= \sqrt{189} \text{ cm}$$
$$= 3\sqrt{21} \text{ cm.}$$



$$\therefore \text{ area} = \frac{1}{2} \times (\text{sum of} \parallel \text{sides}) \times DE$$
$$= \frac{1}{2} \times (13 + 25) \times 3\sqrt{21}$$
$$= 19 \times 3\sqrt{21} = 57\sqrt{21} \text{ cm}^2$$

24. **(c)** Lengths of sides of triangle

$$= 3\left(\frac{144}{3+4+5}\right), \left(\frac{144}{3+4+5}\right)4, \left(\frac{144}{3+4+5}\right)5$$
$$= 36, 48, 60$$

$\therefore$ Area

$$= \sqrt{72\times(72-36)\times(72-48)\times(72-60)}$$
$$= \sqrt{72\times36\times24\times12}$$
$$= 12 \times 12 \times 3 \times 2$$
$$= 144 \times 6 \text{ cm}^2$$

$\therefore$ Height corresponding to the longest side

$$= \frac{2\times144\times6}{60} = 28.8 \text{ cm}$$

25. **(a)** The two parts of ||gm are congruent.
$\therefore$ they have equal area.

26. **(d)** Area of ||gm = $AL \times DC = CM \times AD$

$\Rightarrow \quad 8 \times DC = 10 \times 6$

$\Rightarrow \quad DC = \frac{60}{8} = 7.5 \text{ cm} = AB.$

27. **(b)** In ΔABC,
$AB = AC = 50$ cm,

$$S = \frac{50+50+20}{2} = 60 \text{ cm.}$$

ar (ABC)

$$= \sqrt{60\times(60-50)\times(60-50)\times(60-20)}$$
$$= \sqrt{60\times10\times10\times40}$$
$$= 200\sqrt{6} \text{ cm}^2$$

Area of umbrella = $5 \times$ ar (ΔABC)

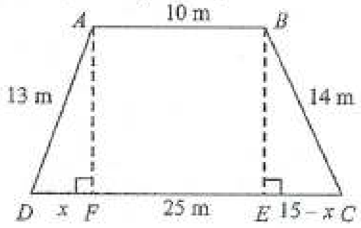
$$= 5 \times 200\sqrt{6}$$
$$= 1000\sqrt{6} \text{ cm}^2$$

28. **(a)** Here $S = \frac{12+12+6}{2} = 15$ cm

$\therefore$ Required area

$$= \sqrt{15\times(15-12)\times(15-12)\times(15-6)}$$
$$= \sqrt{15\times3\times3\times9}$$
$$= 9\sqrt{15} \text{ cm}^2$$

29. **(b)** $BE = AF$
$DF = x,\ CE = (15 - x)$



$\therefore$ In ΔAFD and ΔBEC

$$BE^2 = AF^2$$
$$\Rightarrow (14)^2 - (15-x)^2 = (13)^2 - (x)^2$$
$$\Rightarrow (14)^2 - (13)^2 = (15-x)^2 - (x)^2$$
$$\Rightarrow (14-13)(14+13) = (15-x+x)(15-x-x)$$
$$\Rightarrow 27 = (15)(15-2x)$$
$$\Rightarrow 2x - 15 = \frac{-27}{15}$$
$$\Rightarrow 2x = \frac{-27}{15} + 15$$
$$\Rightarrow 2x = 15 - \frac{9}{5} = \frac{75-9}{5} = \frac{66}{5}$$

$\Rightarrow \quad x = \frac{33}{5}$ m.

$\therefore \sqrt{(13)^2 - x^2} = \sqrt{169 - \left(\frac{33}{5}\right)^2} = \frac{56}{5}$

$\therefore$ Area $= \frac{1}{2} \times (10 + 25) \times \frac{56}{5}$

$= 35 \times \frac{28}{5} = 28 \times 7 = 196 \text{ m}^2$

30. **(c)** If the sides of Δ are a, b and c.

$\therefore$ New sides are $2a$, $2b$, $2c$.

$\therefore$ New $S = 2\left(\frac{a+b+c}{2}\right) = 2s$.

$\therefore$ New area $= \sqrt{S_1(S_1 - a_1)(S_1 - b_1)(S_1 - c_1)}$

$= \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$

$= 4\sqrt{s(s-a)(s-b)(s-c)} = 4\Delta$

12. Surface Areas and Volumes

Learing Objective:

In this chapter,we shall learn about:

*Volumes

*Surface Area

*Cuboid and Cube

*Right Circular Cylinder

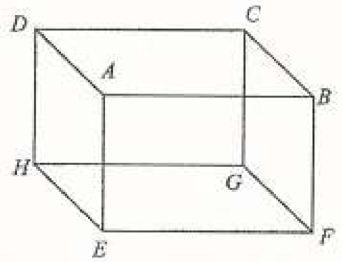
*Sphere

Cuboid and Cube

A cuboid is a solid bounded by six rectangular faces.

Faces

The adjoining figure is made of six rectangular faces, namely, *ABCD, EFGH, AEHD, CGFB, AEFB* and *CDHG*.



Edges

Any two adjacent faces of a cuboid meet in a line segment, which is called an edge of the cuboid. In the above figure, the cuboid has 12 edges, namely, *AB, AD, AE, HD, HE, HG, GE, GC, FE, FB, EF* and *CD*.

Vertex

For any two edges that meet at an end point, there is a third edge, that also meets them at end points. The point of intersection of three edges of a cuboid is called a vertex of the cuboid. A cuboid has 8 vertices.

Base and Lateral Faces

Any face of the cuboid can be considered as base of the cuboid. The four faces meeting the base will be considered as the lateral faces of the cuboid.

Surface area of a cuboid

Surface area of cuboid having length, breadth and height as, l, b, and h respectively $= 2\,(lb + bh + lh)$.

Lateral surface area of a cuboid

Lateral surface area of a cuboid = perimeter of base × height $= 2\,(l + b) \times h$

Formulae for Cuboid and Cube

(i) Total surface area of cuboid = $2(lb + bh + lh)$,

(ii) Lateral surface area of cuboid = $2(lh + bh)$

(iii) Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$

(iv) Perimeter of cuboid = $4(l + b + h)$

(v) Volume of cuboid = lbh

For a cube, $l = b = h = a$, and rest of the properties of cube and cuboid are same. Therefore,

(v) Total surface area of cube = $6a^2$

(vi) Lateral surface area of cube = $4a^2$

(vii) Diagonal of cube = $\sqrt{a^2 + a^2 + a^2} = \sqrt{3}a$

(viii) Perimeter of cube = $4(a + a + a) = 12a$

(ix) Volume of cube = a^3

Example 1: The dimensions of a cuboid are in the ratio of 1 : 2 : 3 and its total surface area is 88 m^2. Find the lateral surface area and volume of the cuboid.

Solution: Let the dimensions of cuboid be x, $2x$, $3x$.

Total surface area = $2(lb + bh + lh)$

$= 2(2x^2 + 6x^2 + 3x^2) = 22x^2$

$\Rightarrow \quad 22x^2 = 88$ (According to question)

$\Rightarrow \quad x^2 = 4$

$\Rightarrow \quad x = 2$

$\therefore$ Dimensions of cuboid are 2 m, 4 m and 6 m.

$\therefore$ Lateral surface area = $2(2 + 4) \times 6 = 2 \times 6 \times 6 = 72\ m^2$

Volume = $lbh = 2 \times 4 \times 6 = 48\ m^3$

Example 2: Find the number of cubes of side 3cm that can be cut from a cuboid of dimensions 10 cm × 9 cm × 6 cm.

Solution: Volume of cuboid = $10 \times 9 \times 6\ cm^3$

$= 2 \times 5 \times 3 \times 3 \times 3 \times 2\ cm^3$

Volume of cube = $3 \times 3 \times 3\ cm^3$

Let the number of cubes be n.

$\therefore$ Volume of cuboid = Total volume of cubes.

$\Rightarrow 2 \times 5 \times 3 \times 3 \times 3 \times 2 = n \times 3 \times 3 \times 3$

$\Rightarrow \quad n = 20$

Example 3: If the sum of all the edges of a cube is 36cm, then the volume of cube and the length of diagonal of cube will be equal to ----.

Solution: $12l = 36$

$\Rightarrow \quad l = 3$ cm

$\therefore$ Volume of cube = $(3)^3 = 27\ cm^3$

Length of diagonal of cube = $\sqrt{3} \times 3 = 3\sqrt{3}$ cm.

Example 4: If the sum of length, breadth and depth of a cuboid is 9cm and length of its diagonal is $\sqrt{29}$ cm, then, its surface area will be ......

Solution: Given

$$l + b + h = 9 \text{ cm} \quad \text{...(i)}$$

and, $$l^2 + b^2 + h^2 = 29 \text{ cm}^2 \quad \text{...(ii)}$$

Squaring equation (i) and using equation, (ii) we have,

$$(l + b + h)^2 = (9)^2 = l^2 + b^2 + h^2 + 2(lb + bh + lh)$$

$$\Rightarrow \quad 81 = 29 + 2(lb + bh + lh)$$

$\Rightarrow$ Surface area of cuboid = $2(lb + bh + lh) = 81 - 29 = 52$ cm^2

Example 5: The cost of preparing the point for four walls of a room at ₹ 2 per square metre is ₹ 252. The height of the room is 4.5m. Find the length and breadth of the room if they are in the ratio 4 : 3.

Solution: Let the length and breadth of room be 4x, 3x m.

Area of 4 walls = $2(l + b) \times h$

$$= 2(4x + 3x) \times h = \frac{252}{2}$$

$$\Rightarrow \quad 7x = \frac{252}{2 \times 2 \times 4.5} = 14$$

$$\Rightarrow \quad x = 2$$

$\therefore$ Length of the room = $4 \times 2 = 8$m

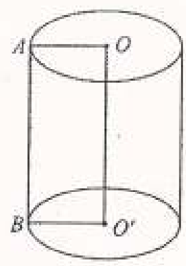
Breadth of the room = $3 \times 2 = 6$m

Right Circular Cylinder

A solid bounded by a curved lateral surface and two parallel plane circular ends, is called a right circular cylinder. It is basically generated by the revolution of a rectangle about one of its sides, or, by arranging number of circles one over another, such that each circle overlaps the other.

Axis

The line segment joining the centres of two bases is called the axis of the cylinder. Here, OO' is the axis of the cylinder.



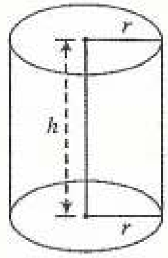
Height

The length of the axis of the cylinder is called the height of the cylinder.

Lateral Surface

The curved surface joining the two bases of a right circular cylinder is called its lateral surface.

Surface area of a right circular cylinder



(i) Surface area (lateral) of a right circular cylinder = $2\pi rh$ sq. units

(ii) Total surface area of a right circular

cylinder = $(2\pi rh + 2\pi r^2)$ sq. units.

$= 2\pi r\,(r + h)$ sq. units.

Example 6: The curved (lateral) surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of cylinder. Also find the total surface area of the cylinder.

Solution: $2\pi rh = 88$

$\Rightarrow \quad 2\times\frac{22}{7}\times r\times 14=88$

$\Rightarrow \quad 2r=\frac{88\times 7}{22\times 14}=2$ cm

$\Rightarrow \quad d = 2$ cm.

and $\quad r = 1$ cm

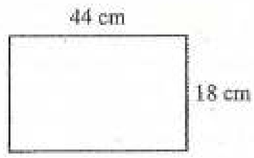
$\therefore$ Total surface area = $2\pi r\,(r + h)$ sq. cm

$$=2\times\frac{22}{7}\times 1\times(1+14)$$

$$=\frac{44\times 15}{7}=\frac{660}{7}\text{cm}^2$$

Example 7: A rectangular sheet of paper 44cm × 18cm is rolled along its length and a cylinder is generated. Find the radius of the resulting cylinder.

Solution: $\because$ Rectangle is rolled along its length.



$\therefore$ Length of rectangle will become the circumference of the base of the resulting cylinder.

$\therefore \quad 2\pi r = 44$

$\Rightarrow \quad 2\times\frac{22}{7}\times r=44$

$\Rightarrow \quad r = 7$cm

Example 8: Find the area covered by a roller in 5 revolutions of it covers a distance 4.4m on the ground and it is 2m long. Also find the volume of the roller.

Solution: Distance covered in one revolution = $2\pi r = 4.4$m

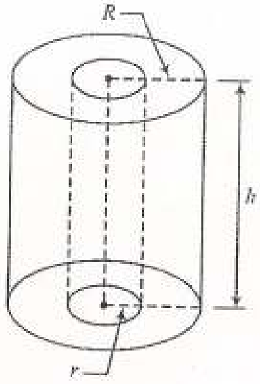
Area covered in one revolution = $2\pi rh = 4.4 \times 2 = 8.8\ m^2$

$\therefore$ Area covered in 5 revolutions = $8.8m^2 \times 5 = 44\ m^2$

Volume of cylinder (roller) = $\pi r^2 h$

$$=\frac{22}{7}\times 0.7\times 0.7\times 2=3.08\text{m}^3$$

Hollow Cylinder



Let r and R be inner and outer radii of the bases of the hollow cylinder, and h be the height of the cylinder.

(i) Area of base = $\pi (R^2 - r^2)$ sq. units.

(ii) Curved (lateral) surface area

= External surface area + internal surface area

$= 2\pi Rh + 2\pi rh$

$= 2\pi h (R + r)$ sq. units.

(iii) Total surface area $= 2\pi Rh + 2\pi rh + 2\pi (R^2 - r^2)$

$= 2\pi (R^2 - r^2 + rh + Rh)$

$= 2\pi \{(r + R)(R - r) + h(r + R)\}$

$= 2\pi (-r + R + h)(R + r)$ sq. units.

(iv) Volume $= \pi R^2 h - \pi r^2 h$

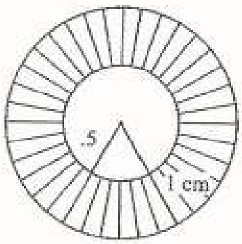
$= \pi h (R^2 - r^2)$ cubic units.

Example 9: A well with 10m inside diameter is dug 14m deep. Earth (soil) taken out of it is spread all around to a width of 5m to form an embankment. Find the height of the embankment. Also find total surface area of the embankment.

Solution: Volume of the earth dug out $= \pi r^2 h$ m^3

$$= \frac{22}{7} \times 5 \times 5 \times 14 \text{ m}^3$$

$$= 1100 \text{ m}^3$$



Area of the embankment $= \pi (R^2 - r^2)$

$= \pi \{(5 + 5)^2 - 5^2\}$

$= \pi (15)(5)$

$= 75\pi$ m^2

$\therefore$ Height of the embankment $= \frac{\text{volume}}{\text{Area}} = \frac{1100}{75\pi} = 4.66\text{m}$

Now,

Total surface area of the embankment $= 2\pi (R + r)(R - r + h)$

$$= 2\times\frac{22}{7}\times(5+10)(10-5+4.66)$$

$$= 2\times\frac{22}{7}\times 15\times 9.66$$

$$= 909.972 \text{ m}^2$$

Example 10: Water flows out through a circular pipe of internal radius 1cm, at the rate of 6 m/s into a cylindrical tank, the radius of whose base is 60cm. Find the rise in the level of water in 1 hour.

Solution: Volume flow rate of water = πr^2 (speed)

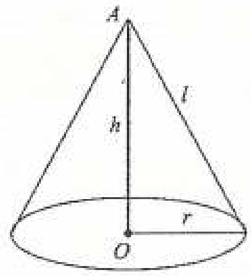
$$= \frac{22}{7}\times 1\times 1\times 600 \text{ cm}^3/\text{s}$$

$\therefore$ Volume (inlet) in 1 hour = $\frac{22}{7}\times 1\times 1\times 600\times 3600 \text{ cm}^3$

$$\therefore \text{Rise of height} = \frac{\frac{22}{7}\times 600\times 3600 \text{ cm}^3}{\frac{22}{7}\times 60\times 60 \text{ cm}^2} = 600 \text{ cm} = 6 \text{ m}$$

Right Circular Cone

A right circular cone is a solid generated by revolving a line segment which passes through a fixed point and which makes a constant angle with a fixed line.



Vertex

The fixed point, here A, is called the vertex of the cone.

Axis

The fixed line AO is the axis of the cone.

Base

The right circular cone has a plane end, which is circular in shape.

Height

The length of axis is called the height of the cone.

Slant height

The length of line segment joining the vertex, to any point on the circular base of the cone, is called the slant height of the cone. It is denoted by l.

$$l = \sqrt{h^2 + r^2}$$

Formulae Related to Right Circular Cone

(i) Surface area of a right circular cone = $\pi rl + \pi r^2$

$= \pi r\,(l + r)$ sq. units.

(ii) Curved surface area of right circular cone = πrl sq. units

(iii) Volume of cone = $\frac{1}{3}\pi r^2 h$ cu.units.

Example 11: The radius and height of a cone are in the ratio 3 : 4. If its volume is 301.44 cm^3, what is the radius and total surface area of the cone?

Solution: Let the radius and height of the cone be $3x$ and $4x$ cm respectively.

Volume = 301.44 cm^3

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times 3x\times 3x\times 4x = 301.44$$

$$\Rightarrow \quad x^3 = \frac{301.44\times 7}{3\times 4\times 22} = 8$$

$$\Rightarrow \quad x = 2$$

Radius = $r = 3x = 6$ cm, height = $4x = 8$ cm

$\therefore$ Total surface area = $\pi r\,(l + r)$

$$= \pi r\left(\sqrt{r^2+h^2} + r\right)$$

$$= \frac{22}{7}\times 6\times\left(\sqrt{6^2+8^2}+6\right)$$

$$= \frac{22}{7}\times 6\times(10+6)$$

$$= \frac{22}{7}\times 6\times 16 \text{ cm}^2$$

$$= 301.44\text{cm}^2$$

Example 12: A cone of radius 5cm is filled with water. If water is poured in a cylinder of radius 10cm, the height of the water rises 2cm, find the height of the cone.

Solution: Volume of cylinder = Volume of cone

$$\Rightarrow \quad \pi R^2 H = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad 3R^2 H = r^2 h$$

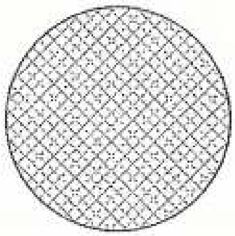
$$\Rightarrow \quad 3\times(10)^2\times 2 = (5)^2\times h$$

$$\Rightarrow \quad \frac{3\times 100\times 2}{25} = h$$

$$\Rightarrow \quad h = 6\times 4$$

$$= 24 \text{ cm.}$$

Sphere



The set of all points in space which are equidistant from a fixed point, is called a sphere.

Diameter

A line segment through the centre of a sphere, and with end-points on the sphere is called a diameter of the sphere.

Hemisphere

A plane through the centre of a sphere divides the sphere into two equal parts, which is called a hemisphere.

Spherical shell

The difference of two solid concentric spheres is called a spherical shell.

Formulae Related to Surface Areas and Volumes of Hemisphere, Sphere and Spherical Shell

(i) Surface area of sphere of radius 'r' is given by:

$S = 4\pi 7^2$ sq. units = curved surface area of sphere.

(ii) Curved surface area of a hemisphere of radius 'r' is:

$S = 2\pi r^2$ sq. units.

(iii) Total surface area of a hemisphere of radius 'r' is:

$S = 2\pi r^2 + \pi r^2 = 3\pi r^2$ sq. units.

(iv) If R and r are outer and inner radii of a spherical shell, then,

Outer surface area = $4\pi R^2$ sq. units.

$$\text{Volume} = \frac{4}{3}\pi\left(R^3 - r^3\right) \text{ cubic units.}$$

(v) Volume of a sphere of radius R is :

$$V = \frac{4}{3}\pi R^3 \text{ cubic units.}$$

(vi) Volume of a hemisphere of radius R is:

$$V = \frac{2}{3}\pi R^3 \text{ cubic units.}$$

Example 13: A toy is in the form of a cone surmounted on a hemisphere. The diameter of the base of cone is 6 cm and its height is 4 cm. Find the total surface area and the volume of the toy.

Solution: Total surface area of the toy

= Surface area (lateral) of the cone + Curved surface area of the hemisphere.

$= \pi rl + 2\pi r^2 = \pi r\,(l + 2r)$

$$= \frac{22}{7} \times \frac{6}{2}\left(\sqrt{4^2 + 3^2} + 6\right) \text{ cm}^2$$

$$= \frac{22}{7} \times 3 \times 11 \text{ cm}^2$$

$$= \frac{726}{7} \text{ cm}^2 = 103.71 \text{cm}^2$$

$$\text{Volume of the toy} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 (h+2r)$$
$$= \frac{1}{3}\times\frac{22}{7}\times 3\times 3\,(4+2\times 3)\text{ cm}^3$$
$$= \frac{1}{3}\times\frac{22}{7}\times 9\times 10\text{ cm}^3$$
$$= \frac{220\times 9}{21}$$
$$= 10.47\times 9\text{cm}^3$$
$$= 94.28\text{ cm}^3$$

Multiple Choice Questions

1. The length of a cold storage is double its breadth. Its height is 3 meters. If the area of four walls including doors is 108 m^2, what is its volume?
   (a) 216 m^3 (b) 264 m^3
   (c) 232 m^3 (d) 218 m^3
2. A small indoor greenhouse is made entirely of glass panes including base held together with tape. How much of tape is needed for all the 12 edges?
   (a) 440 cm (b) 320 cm
   (c) 324 cm (d) 360 cm
3. The paint in a certain container is sufficient to paint an area of 9.375 m^2. How many bricks of dimension 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?
   (a) 80 (b) 100 (c) 120 (d) 150
4. How many 3 metre cubes can be cut from a cuboid measuring 18 m × 12 m × 9 m ?
   (a) 72 (b) 70 (c) 76 (d) 92
5. A solid cube is cut into two cuboids of equal volumes. Find the ratio of total surface area of given cube and that of one cuboid.
   (a) 3 : 2 (b) 2 : 3 (c) 3 : 1 (d) 1 : 3
6. A rectangular reservoir is 120m long and 75m wide. At what speed per hour must water flow into it through a square pipe of 20 cm wide so that the water rises by 2.4 m in 18 hours?
   (a) 40 km/hour (b) 30 km/hour
   (c) 45 km/hour (d) 60 km/hour
7. The diameter of roller 1.5 m long is 84 cm. if it takes 100 revolutions to level a playground, what is the cost of leveling the playground at the rate of 50 paise per square meter?
   (a) ₹ 198 (b) ₹ 168
   (c) ₹ 192 (d) ₹ 208
8. The thickness of a hollow wooden cylinder is 2cm. It is 35cm long and its inner radius is 12cm. What is the volume of the wood required to make the cylinder if it is open at either end?
   (a) 5120 cm^3 (b) 5720 cm^3
   (c) 5820 cm^3 (d) 5620 cm^3
9. The volume of a cylinder is 448 π cm^3 and height 7 cm. What is the lateral surface area of the cylinder?
   (a) 352 cm^2 (b) 356 cm^2
   (c) 342 cm^2 (d) 362 cm^2
10. A solid cylinder has total surface area of 462 m^2. Its curved surface area is one - third of total surface area. What is the volume of the cylinder?
    (a) 569 cm^3 (b) 539 cm^3
    (c) 529 cm^3 (d) 549 cm^3
11. At a mela, a stall keeper in one of the food stalls has large cylindrical vessel of base radius 15cm filled up to a height of 32 cm

with fruit juice. The juice is filled in small cylindrical glasses of radius 3cm upto height of 8cm. how many glasses will be filled by selling the juice completely?

(a) 100 (b) 125
(c) 150 (d) 200

12. The height of a right circular cylinder is 10.5 m Three times the sum of the areas of its two circular faces is twice the area of the curved surface. What is the volume of the cylinder?

(a) 1617 m^3 (b) 1651 m^3
(c) 1631 m^3 (d) 1637 m^3

13. How many metres of cloth of 5 m width will be required to make a conical tent. The radius of whose base is 7 m and heights is 24 m ?.

(a) 120 m (b) 110 m (c) 125 m (d) 130 m

14. The diameter of a sphere is 6cm. it is melted and drawn into a wire of diameter 0.2 cm. What is the length of the wire?

(a) 18 m (b) 26 m
(c) 36 m (d) 30 m

15. The diameter of the moon is approximately $\frac{1}{4}$th of the diameter of the earth. What fraction of the volume of earth is the volume of moon?

(a) $\frac{1}{16}$ (b) $\frac{1}{32}$
(c) $\frac{1}{64}$ (d) None of these

16. How many planks each of which is 2m long, 2.5cm broad and 4cm thick can be cut off from a wooden block 6m long, 15cm, broad and 40cm thick?

(a) 100 (b) 180 (c) 140 (d) 200

17. Water flows in a tank 150 m × 100 m at the base through a pipe whose cross-section is 2 dm by 1.5 dm at the speed of 15 km per hour. In what time will the water be 3 meters deep?

(a) 100 hours (b) 120 hours
(c) 80 hours (d) 150 hours

18. What is the length of diagonal of a cube each of whose edge measures 20cm?

(a) 32.64 cm (b) 17.32 cm
(c) 28.28 cm (d) None of these

19. In a shower, 5cm of rain falls. What is the volume of water that falls on 2 hectares of ground?

(a) 2000 m^3 (b) 1200 m^3
(c) 1000 m^3 (d) None of these

20. Total surface area of a cube is 486cm^2. What is its lateral surface area?

(a) 324cm^2 (b) 364 cm^2
(c) 332cm^2 (d) 348 cm^2

21. The curved surface area and the volume of a pillar are 264m^2 and 396m^3. What is the height of the pillar?

(a) 12 m (b) 14 m (c) 6 m (d) 8 m

22. Find the number of coins 1.5cm in diameter and 0.2 cm thick to be melted to form a right circular cylinder of height 5cm and diameter 4.5 cm.

(a) 225 (b) 175 (c) 215 (d) 275

23. The volume of a cone is 1232 cm^3 and diameter of its base is 14cm. What is its slant height?

(a) 24 cm (b) 25 cm (c) 26 cm (d) 27 cm

24. What is the length of longest rod that can placed in a room of dimension 10 m × 10 m × 5 m?

(a) 16 m (b) 15 m
(c) 12 m (d) $10\sqrt{5}$ m

25. The radius of a wire is decreased to one third. If volume remains the same, the length will become how many times?

(a) 2 times (b) 3 times
(c) 6 times (d) 9 times

26. How many spherical bullets can be made out of a solid cube of lead whose edge measures 44 cm, each bullet being 4cm in diameter?

(a) 1541 (b) 2541 (c) 2041 (d) 2341

27. The volume of a cube is 512 cm^3. What is its surface area?

(a) 256 cm^2 (b) 384 cm^2
(c) 512 cm^2 (d) 264 cm^2

28. If the length of diagonal of a cube is $8\sqrt{3}$ cm. What is its surface area?

(a) 192 cm^2 (b) 512 cm^2
(d) 384 cm^2 (d) 768 cm^2

29. A solid metallic cylinder of base radius 3cm and height 5cm is melted to make a solid cone of height 1cm and base radius 1mm. What is the number of cones?

(a) 1350 (b) 4500 (c) 13500 (d) 450

30. A metallic sphere of radius 10.5cm is melted and then recast into small cones each of radius 3.5cm and length 3cm. what is the number of such cones?

(a) 126 (b) 63 (c) 130 (d) 123

31. A cone and a hemisphere have equal bases and equal volumes. What is the ratio of their heights?

(a) 1 : 2 (b) 2 : 1
(c) $\sqrt{2}$: 1 (d) 4 : 1

32. How many lead shots each 0.3 cm in diameter can be made from a cuboid of dimension 18cm × 22cm × 6cm?

(a) 84000 (b) 168000
(c) 160000 (d) None of these

33. The diameter of a roller 1m long is 84 cm. If it takes 200 complete revolutions to level a ground,what is the area of the ground?

(a) 1320 m^2 (b) 628 m^2
(c) 528 m^2 (d) 264 m^2

34. What is the length of longest rod that can fit in a cubical vessel of side 20cm?

(a) $10\sqrt{3}$ (b) $20\sqrt{2}$
(c) $20\sqrt{3}$ (d) None of these

35. The curved surface area of a cylindrical pillar is 264 m^2 and its volume is 924 m^3. What is the height of the pillar?

(a) 6 m (b) 8 m (c) 4 m (d) 9 m

Answer Key

1. (a)	2. (b)	3. (b)	4. (a)	5. (a)	6. (b)	7. (a)	8. (a)	9. (a)	10 (b)
11. (a)	12. (a)	13. (a)	14. (c)	15. (c)	16. (b)	17. (a)	18. (d)	19. (a)	20. (a)
21. (b)	22. (a)	23. (b)	24. (b)	25. (d)	26. (b)	27. (b)	28. (c)	29. (c)	30. (a)
31. (b)	32. (b)	33. (c)	34. (c)	35. (a)					

Hints and Solutions

1. **(a)** Let $l = 2b$, $h = 3$m

   Area of four walls = 108

   $2(l + b)h = 108$

   $\Rightarrow 2(2b + b)3 = 108$

   $\Rightarrow \quad 3b = \frac{108}{6} = 18$

   $\Rightarrow \quad b = \frac{18}{3} = 6$

   $\therefore \; l = 2b = 2 \times 6 = 12$

   $\therefore$ Volume of cold storage

   $= lbh = 12 \times 6 \times 3 = 216 \text{ m}^3$

2. **(b)** Length of the tape $= 4(l + b + h)$

   $= 4(30 + 25 + 25)$

   $= 4 \times 80 = 320$ cm.

3. **(b)** No. of bricks $= \frac{9.375 \times 100 \times 100}{22.5 \times 10 \times 7.5} = 100$

4. **(a)** Number of cubes $= \frac{\text{volume of the cuboid}}{\text{volume of each cube}}$

   $= \frac{18 \times 12 \times 9}{3 \times 3 \times 3} = 72$

5. **(a)** Volume of cuboid $= \frac{a^3}{2} = lbh$

   $\therefore$ Surface area of each cuboid

   $= 2(lb + bh + lh)$

   $= \left(\frac{a}{2} \times a + a \times a + \frac{a}{2} \times a\right)2$

   $= 2(2a^2) = 4a^2$

   Total surface area of cube $= 6a^2$

   $\therefore$ Required ratio $= 6a^2 : 4a^2 = 3 : 2$

6. **(b)** Volume of the water accumulated the reservoir 18 hours $= (120 \times 75 \times 2.4) \text{ m}^3$

   Let speed of water = v km/hour.

   The width of cuboid $= b = \frac{20}{100} = \frac{1}{5}$ m

   Height $= h = \frac{20}{100} = \frac{1}{5}$ m

   Length of water cuboid formed in 18 hours

   $= 18\,v \text{ km} = 18 \times 1000\,v \text{ m}$

   $= 18000\,v \text{ m}$

   Volume of the water accumulated in reservoir in 18 hours

   $= 18000v \times \frac{1}{5} \times \frac{1}{5} = 720\,v \text{ m}^3$

   $\Rightarrow 720v = 120 \times 75 \times 2.4$

   $\Rightarrow \quad v = \frac{120 \times 75 \times 24}{720 \times 10} = 30$ km/hour

7. **(a)** Curved surface area of the roller $= 2\pi rh$

   $= 2 \times \frac{22}{7} \times \frac{150}{2} \times 84$

   $= 44 \times 150 \times 6$

   Area covered by roller in 100 revolutions

   $= \frac{44 \times 150 \times 6 \times 100}{100 \times 100} \text{ m}^2$

   Cost of leveling the playground

   $= \frac{44 \times 150 \times 6}{100} \times \frac{50}{100} = 11 \times 6 \times 3$

   $=$ ₹ 198

8. **(b)** Let r be the inner radius of the cylinder.

   $r = 12$cm

   Outer radius $= R = 12 + 2 = 14$cm

   $h = 35$cm

   Volume of wood $= \pi(R^2 - r^2)h$

   $= \frac{22}{7}(14^2 - 12^2)35$

   $= \frac{22}{7} \times 2 \times 26 \times 35$

   $= 44 \times 130 = 5720 \text{ cm}^3$

9. **(a)** Volume of the cylinder $= 448\pi$

   $\Rightarrow \quad \pi r^2 h = 448\pi$

   $\Rightarrow \quad r^2 h = 448$

$\Rightarrow \quad r^2 = \frac{448}{h} = \frac{448}{7} = 64$

$\Rightarrow \quad r = 8$ cm

Lateral surface area $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 8 \times 7$

$= 22 \times 16 = 352 \text{ cm}^2$

10. **(b)** Curved surface area

$= \frac{1}{3} \times$ total surface area

$= \frac{1}{3} \times 462 = 154$

$\Rightarrow \quad 2\pi rh = 154 \quad \ldots(1)$

Total surface area = 462

$\Rightarrow \quad 2\pi rh + 2\pi r^2 = 462$

$\Rightarrow \quad 154 + 2\pi r^2 = 462$

$\Rightarrow \quad 2\pi r^2 = 308$

$\Rightarrow \quad r^2 = \frac{308 \times 7}{2 \times 22} = 49$

Putting this value in (i), we get

$2\pi rh = 154$

$h = \frac{154 \times 7}{2 \times 22 \times 7} = \frac{7}{2}$

$\therefore$ Volume of cylinder $= \pi r^2 h = \frac{22}{7} \times 7^2 \times \frac{7}{2}$

$= 11 \times 49 = 539 \text{ cm}^3$

11. **(a)** Let the number of glasses be n.

$\therefore$ Total volume in the vessel

= volume of juice in glasses

$\Rightarrow \quad \pi R^2 H = n \times \pi r^2 h$

$\Rightarrow \quad R^2 H = nr^2 h$

$\Rightarrow \quad (15)^2 \times 32 = n \times (3)^2 \times 8$

$\Rightarrow \quad n = 5 \times 5 \times 4 = 100$

12. **(a)** $h = 10.5$ m.

Let the area of each circular face be A m^2

$\therefore$ According to question,

$3(A + A) = 2 \times 2\pi rh$

$\Rightarrow \quad 3 \times 2\pi r^2 = 4\pi rh$

$\Rightarrow \quad 6\pi r^2 = 4\pi rh$

$\Rightarrow \quad 3r = 2h$

$\Rightarrow r = \frac{2}{3} h = \frac{2}{3} \times 10.5\text{m} = 7$ m

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 10.5 \text{ m}^3$

$= 154 \times 10.5\text{m}^3 = 1617 \text{ m}^3$

13. **(b)** $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576}$

$= \sqrt{625} = 25$ m

Curved surface $= \pi rl = \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$

Length of canvas used

$= \frac{\text{Area}}{\text{Width}} = \frac{550}{5} = 110$ m

14. **(c)** Radius of sphere = 3 cm.

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3$

$= \frac{88 \times 9}{7}$

Radius of cylindrical wire $= \frac{0.2}{2} = 0.1$cm

Volume of wire $= \pi r^2 h$

$= \frac{22}{7} \times (0.1)^2 \times h$

$= \frac{22}{7} \times \frac{1}{10} \times \frac{1}{10} \times h = \frac{88 \times 9}{7}$

$\Rightarrow \quad h = \frac{88 \times 9 \times 10 \times 10}{22} = 3600$cm

$\Rightarrow \quad h = \frac{3600}{100} = 36$ m

15. **(c)** Let the diameter of the earth be x m.

$\therefore$ Diameter of moon $= \frac{x}{4}$ m

$$\frac{\text{volume of moon}}{\text{volume of earth}} = \frac{\frac{4}{3}\pi\left(\frac{x}{4}\right)^3}{\frac{4}{3}\pi x^3} = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

16. **(b)** No. of planks $= \frac{\text{volume of wooden block}}{\text{volume of each plank}}$

$$= \frac{600 \times 15 \times 40}{200 \times 2.5 \times 4} = 180$$

17. **(a)** Volume of water in the tank

$= 150 \times 100 \times 3 = 45000\ m^3$

Area of cross – section of the pipe

$= 2dm \times 1.5dm$

$= \frac{2}{10} \times \frac{1.5}{10} = \frac{3}{100}\ m^2$

Let the time taken be t hours.

Volume of water that flows in tank in t hours.

$= \frac{3}{100} \times 15\ km/h \times t\ m^3$

$= \frac{3}{100} \times 15 \times 1000 \times t\ m^3$

$= 450\ t\ m^3$

$\Rightarrow 450t = 45000$

$\Rightarrow \quad t = \frac{45000}{450} = 100$ hours

18. **(d)** Length of diagonal of a cube

$= \sqrt{3}\,(Edge)$

$= \sqrt{3} \times 20cm$

$= 1.732 \times 20cm$

$= 34.64$ cm

19. **(c)** Volume of water

$$= 2 \times 10000 \times \frac{5}{100} = 1000\ m^3$$

20. **(a)** Total surface area of a cube = 486

$\Rightarrow \quad 6a^2 = 486$

$\Rightarrow \quad a^2 = 81$

$\Rightarrow \quad a = 9cm$

Lateral surface area of cube $= 4a^2$

$= 4 \times 81$

$= 324\ cm^2$

21. **(b)** Curved surface area of pillar = 264

$\Rightarrow \quad 2\pi rh = 264$

Volume of pillar = 396

$\Rightarrow \quad \pi r^2 h = 396$

$\therefore \quad \frac{\pi r^2 h}{2\pi rh} = \frac{396}{264}$

$r = \frac{2 \times 396}{264} = 3$ m

$\therefore \quad 2\pi rh = 264$

Now $\quad h = \frac{264}{2\pi r}$

$= \frac{264 \times 7}{2 \times 22 \times 3} = 14$ m

22. **(a)** Radius of coin $= \frac{1.5}{2} = 0.75$ cm

Thickness of coin = 0.2 cm

Volume of each coin $= \pi r^2 h$

$= \pi \times 0.75 \times 0.75 \times 0.2$

Radius of new cylinder $= \frac{4.5}{2} = 2.25$ cm

Height of new cylinder = 5 cm

Volume of new cylinder $= \pi\,(2.25)^2 \times 5$

No. of coins $= \frac{\text{volume of new cylinder}}{\text{volume of each coin}}$

$= \frac{\pi \times 2.25 \times 2.25 \times 5}{\pi \times 0.75 \times 0.75 \times 0.2}$

$= \frac{225 \times 225 \times 5 \times 10}{75 \times 75 \times 2}$

$= 3 \times 3 \times 5 \times 5$

$= 225$

23. **(b)** Volume of the cone = 1232

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 1232$

$\Rightarrow \frac{1}{3}\times\frac{22}{7}\times\left(\frac{14}{2}\right)^2\times h=1232$

$\Rightarrow \quad h=\frac{1232\times3\times7}{22\times7\times7}$

$=24$ cm

Slant height $= l$

$=\sqrt{h^2+r^2}=\sqrt{576+49}=\sqrt{625}$

$l=25$cm

24. **(b)** Length of longest rod = length of diagonal

$=\sqrt{l^2+b^2+h^2}$

$=\sqrt{(10)^2+(10)^2+(5)^2}$

$=\sqrt{225}=15$m

25. **(d)** Let radius be r and height be h.

New radius be $\frac{r}{3}$ and height H.

$\therefore \quad \pi r^2h=\pi\left(\frac{r}{3}\right)^2H$

$\Rightarrow \quad r^2h=\frac{r^2H}{9}$

$\Rightarrow \quad H=9h$

Length will become 9 times.

26. **(b)** Volume of each bullet $=\frac{4}{3}\pi r^3$

$=\frac{4}{3}\pi\times2^3$

$=\frac{4\times8\times\pi}{3}$

Volume of cube $= 44\times44\times44$

No. of bullets $=\frac{44\times44\times44}{\frac{4\times8\times\pi}{3}}$

$=\frac{44\times44\times44\times7\times3}{4\times8\times22}=2541$

27. **(b)** Volume of the cube = 512

$\Rightarrow \quad a^3=512=8^3$

$\Rightarrow \quad a=8$

$\therefore$ Total surface area $=6a^2=6\times8^2$

$=6\times64=384\text{cm}^2$

28. **(c)** Length of diagonal of a cube $=8\sqrt{3}$

$\Rightarrow \quad \sqrt{3}\,a=8\sqrt{3}$

$\Rightarrow \quad a=8$ cm

Surface area $=6a^2=6\times8^2$

$=6\times64=384\ \text{cm}_2$

29. **(c)** Number of cones $=\frac{\text{volume of cylinder}}{\text{volume of one cone}}$

$=\frac{\pi r^2h}{\frac{1}{3}\pi r^2h}$

$=\frac{\pi\times3\times3\times5}{\frac{1}{3}\times\pi\times\frac{1}{10}\times\frac{1}{10}\times1}$

$=3\times3\times3\times5\times10\times10$

$=135\times100$

$=13500$

30. **(a)** No. of cones $=\frac{\text{volume of sphere}}{\text{volume of one cone}}$

$=\frac{\frac{4}{3}\times\pi\times\frac{21}{2}\times\frac{21}{2}\times\frac{21}{2}}{\frac{1}{3}\times\pi\times\frac{7}{2}\times\frac{7}{2}\times3}$

$=\frac{4\times21\times21\times21\times2\times2}{2\times2\times2\times7\times7\times3}$

$=3\times21\times2=126$

31. **(b)** Cone and hemisphere have equal base means equal radii, of R cm,

Height of the cone be H cm.

Height of hemisphere $= R$ cm.

Volume of cone = volume of hemisphere

$\Rightarrow \quad \frac{1}{3}\pi R^2H=\frac{2}{3}\pi R^3$

$\Rightarrow \quad \frac{R^2H}{R^3}=2$

$\Rightarrow \qquad \frac{H}{R} = \frac{2}{1} = 2:1$

32. **(b)** No. of lead shots

$$= \frac{18 \times 22 \times 6 \times 7 \times 8}{\frac{4}{3} \times 22 \times 0.3 \times 0.3 \times 0.3}$$

$$= \frac{18 \times 42 \times 8 \times 3 \times 1000}{4 \times 27}$$

$= 168000$

33. **(c)** Radius of roller $= \frac{84}{2} = 42\text{cm}$

$h = 100\text{cm}$

Area covered by the roller in 200 Revolutions $= 200 \times 2\pi rh$

$$= \frac{200 \times 2 \times 22 \times 42 \times 100}{100 \times 100 \times 7}$$

$= 4 \times 22 \times 6$

$= 88 \times 6 = 528 \text{ m}^2$

34. **(c)** Length of longest rod

= length of the diagonal

$= \sqrt{3}\, a = \sqrt{3} \times 20$

$= 20\sqrt{3}$ cm.

35. **(a)** Here $2\pi rh = 264 \text{ m}^2$

and $\qquad \pi r^2 h = 924 \text{ m}^3$

$\therefore \qquad \frac{\pi r^2 h}{2\pi rh} = \frac{924}{264}$

$\Rightarrow \qquad r = \frac{2 \times 924}{264} = 7 \text{ m}$

$\therefore$ Patting $\quad r = 7$

$2\pi rh = 264$

them $\qquad h = \frac{264 \times 7}{2 \times 22 \times 7} = 6 \text{ m}$

13. Statistics

Learning Objective:

In this chapter, we will learn about:

*Data

*Frequency Distribution

*Exclusive Method

*Cumulative Frequency Distribution

*Graphical Representation of Data

*Measures of Central Tendency

*Important Formulae

The word 'statistics' is derived from the Latin word status which means political state. Political state had to collect information about its citizens to facilitate Governance and plan for their development.

Data

The word data means information in the form of numerical figures or a set of given facts.

Primary data

When an investigator collects data himself with a definite plan in his (her) mind, it is called primary data.

Secondary data

Data which are not originally collected rather obtained from published or unpublished sources are known as secondary data.

Array

The raw data when put in ascending or descending order of magnitude is called an array or arrayed data.

Frequency Distribution

It is a method to represent raw data in the form from which one can easily understand the information contained in the raw data.

Frequency distributions are of two types:

Discrete frequency distribution, and Continuous or grouped frequency distribution.

(a) The process of preparing this type of distribution is very simple. The construction of a discrete frequency distribution is done by the use of the method of tally marks.

(b) The method of condensing the raw data is convenient only where the values in the raw data are largely repeating and the difference between the greatest and the smallest observations is not very large.

Example 1: The number of children in 20 families are,

1, 1, 2, 3, 4, 3, 2, 1, 1, 4, 5, 2, 4, 2, 2, 1, 3, 4, 2, 3.

Represent this raw data in discrete frequency distribution.

Solution:

No. of children	Tally bars	Frequency
1	𝍸	5
2	𝍸 |	6
3	||||	4
4	||||	4
5	|	1

Example 2: The marks obtained by 30 students in a class is :

25, 39, 5, 33, 19, 21, 12, 48, 13, 21, 9, 1, 8, 10, 17, 19, 12, 17, 40, 41, 12, 46, 37, 17, 30, 27, 2, 6, 23, 19. Represent this raw data in continuous frequency distribution.

Solution:

No. of children	Tally bars	Frequency
0 - 10	𝍸	5
10 - 20	𝍸 𝍸 |	11
20 - 30	𝍸	5
30 - 40	||||	4
40 - 50	||||	4

Exclusive Method of Dividing Class Intervals

When the class intervals are fixed so that the upper limit of one class is the lower limit of the next class it is known as the exclusive method of dividing class intervals.

Inclusive method: In this method the classes are so formed that the upper limit of a class is included in that class. The following example represent this method.

Marks	No. of students
51 – 60	20
61 – 70	27
71 – 80	26
81 – 90	23
91 – 100	24

In class 51 – 60 we include the students having marks between 51 and 60. If the marks obtained by a student is exactly 61, he (she) is included in the next class.

Note:

If $a - b$ is a class in inclusive method, then in exclusive, method it becomes $\left(a-\frac{h}{2}\right)-\left(b+\frac{h}{2}\right)$ where,

$$h = \frac{\text{lower limit of a class} - \text{upper limit of previous class}}{2}$$

$$\text{class mark} = \frac{\text{lower limit} + \text{upper limit}}{2}$$

Example 3: Convert this inclusive form into exclusive form of classification.

Wages (₹)	No. of Workers
1000 – 1099	125
1100 – 1199	150
1200 – 1299	200
1300 – 1399	250
1400 – 1499	175
1500 – 1599	100

Solution: $h = \frac{1100 - 1099}{2} = \frac{1}{2}$

$\therefore$ In exclusive method, the representation, would be,

Wages (₹)	No. of Workers
999.5 – 1099.5	125
1099.5 – 1199.5	150
1199.5 – 1299.5	200
1299.5 – 1399.5	250
1399.5 – 1499.5	175
1499.5 – 1599.5	100

Cumulative Frequency Distribution

If the frequency of a class is added to the sum of preceding class frequencies, and then it is represented in tabular form, then, this distribution is known as cumulative frequency distribution.

Wages (₹)	No. of workers
Less than 500	20
Less than 600	25
Less than 700	55

The above cumulative frequency distribution is known as less than cumulative frequency distribution. For greater than cumulative frequency distribution 'Greater than' is used instead of 'Less than' and in that case, the cumulative frequency column will be changed.

Example 4: The age of 20 students (in years) are as follows :

14, 15, 16, 17, 14, 18, 16, 15, 16, 17, 14, 18, 16, 17, 18, 17, 16, 15, 18, 16.

Prepare frequency and cumulative frequency tables.

Solution:

Age (in year)	Tally marks	Frequency
14	|||	3
15	|||	3
16	~~||||~~ |	6
17	||||	4
18	||||	4
		Total = 20

Cumulative frequency table : (Less than type)

Age (in years)	Cumulative frequency
Less than 15	3
Less than 16	6
Less than 17	12
Less than 18	16
Less than 19	20

Greater than type :

Age (in years)	Cumulative frequency
Greater than 13	20
Greater than 14	17
Greater than 15	14
Greater than 16	8
Greater than 17	4

Example 5: Given below the marks obtained by 10 students during a class test :

20, 22, 20, 21, 20, 22, 27, 24, 23, 21.

Find range and number of class intervals if the magnitude of class interval is 2.

Solution: Range = upper limit – lower limit

$= 27 - 20 = 7$

$$\text{Class intervals number} = \frac{\text{range}}{\text{magnitude of class interval}} = \frac{7}{2} = 3.5$$

Example 6: Given below the marks obtained by 6 students in a weekly test. Find the upper and lower limits of the first class.

50, 55, 60, 65, 70, 75.

Solution: h = difference between two consecutive marks.

$= 65 - 60 = 5$

$$\therefore \text{Upper limit} = a + \frac{h}{2} = 50 + \frac{5}{2} = 52.5$$

$$\text{Lower limit} = a - \frac{h}{2} = 50 - \frac{5}{2} = 47.5$$

Example 7: The mid value of a class interval is 42. If the class size is 20, find the upper limit of the class.

Solution: Upper limit $= a + \frac{h}{2} = 42 + \frac{20}{2} = 42 + 10 = 52$

Graphical Representation of Statistical Data

Bar Graph

A bar graph is a pictorial representation of the numerical data by a number of bars (rectangles) of uniform width erected horizontally or vertically with equal spacing between them.

Bar graphs may be either horizontal or vertical.

Example 8: The following bar graph represents the heights of 50 students of class IX of a particular school find :

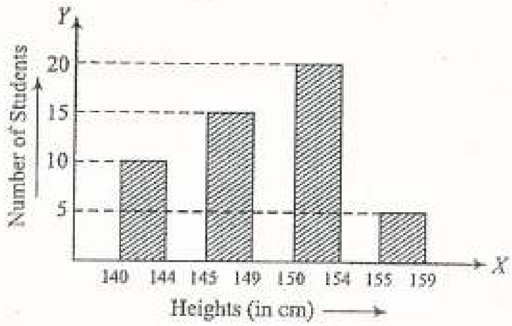
(a) What percentage of the total number of students have their heights more than 149 cm?

(b) In the range of maximum height, there are how many students?

(c) Find the number of students in the range 160 – 164 cm.

Solution: (a) Number of students having their height more than 149cm = 20 + 5 = 25

$\therefore$ Required percentage $= \frac{25}{50} \times 100 = 50\%$



(b) 155 – 159 indicate the range of maximum height,

$\therefore$ Number of students = 5

(c) In the range 160 – 164 cm, there are no students is zero.

Histogram

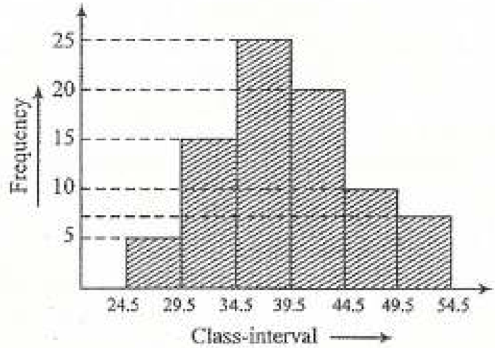
A histogram is graphical representation of a frequency distribution in the form of rectangles with class intervals as bases and height proportional to corresponding frequencies such that there is no gap between any two successive rectangles.

The class intervals may be equal or unequal.

Example 9: Draw a histogram for the following data :

Class interval	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49	50 – 54
Frequency	5	15	25	20	10	7

Solution: $h = \dfrac{29-25}{2} = 2$



Frequency Polygon

Frequency polygon is another method of representing frequency distributions graphically. The frequency polygon can be easily obtained by joining the mid – points of the upper horizontal side of each rectangle in histogram, or by making the class intervals exclusive, and then obtaining class mark and plotting class mark versus frequency on x – and y – axis respectively.

Ogives

It is another method of representing frequency distributions graphically. Ogive are of two types, namely, 'more than' type and 'less than' type. In 'more than' type, cumulative frequency is plotted against the lower limit of the class interval and vice – versa.

Measures of Central Tendency

Methods providing medium values, are called measures of location or central tendency.

The commonly used measures of central tendency (or averages) are: (i) Arithmetic mean (ii) Geometric mean (iii) Harmonic mean (iv) Median (v) Mode

Arithmetic mean

If $x_1, x_2, x_3 \ldots\ldots, x_n$ are n values of a variable X, then the arithmetic mean or simply the mean of these values is denoted by $\overline{X}$ and is defined as,

$$\overline{X} = \frac{x_1 + x_2 + x_3 \ldots\ldots + x_n}{n} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)$$

For a variate X, which takes values $x_1, x_2, x_3, \ldots, x_n$ with corresponding frequencies $f_1, f_2, f_3, \ldots, f_n$ respectively, then,

$$\overline{X} = \frac{\sum_{i=1}^{n} f_i x_i}{N} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

Example 10: If $a \neq 0$, and the mean of n observations is $\overline{X}$, and each number is divided by a, then the new mean will be how much?

Solution: We have,

$$n\overline{X} = x_1 + x_2 + x_3 \ldots\ldots + x_n.$$

$\Rightarrow$ dividing both sides by a,

$$\frac{n\overline{X}}{a} = \frac{x_1}{a} + \frac{x_2}{a} + \frac{x_3}{a} + \ldots\ldots + \frac{x_n}{a}$$

$$\text{New mean} = \frac{\overline{X}}{a} = \frac{\frac{x_1}{a} + \frac{x_2}{a} + \frac{x_3}{a} + \ldots\ldots + \frac{x_n}{a}}{n}$$

Example 11: If the mean of 5 observations x, $x + 2$, $x + 4$, $x + 6$ is 11. Find the mean of 2nd and 3rd observations.

Solution: Here $\frac{x+(x+2)+(x+4)+(x+6)+(x+8)}{5} = 11$

$\Rightarrow \quad 5x + 20 = 55$

$\Rightarrow \quad 5x = 35$

$\Rightarrow \quad x = 7$

$\therefore$ Required mean $= \frac{(x+2)+(x+4)}{2} = \frac{2x+6}{2} = x + 3 = 7 + 3 = 10$

Example 12: If the mean of the following distribution is 6, find the value of p.

x	2	4	6	10	$p+5$
f	3	2	3	1	2

Solution: Mean = 6

$$\therefore \quad \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = 6$$

$$\Rightarrow \quad \frac{2\times3+4\times2+6\times3+10\times1+2\times(p+5)}{3+2+3+1+2} = 6$$

$\Rightarrow \quad 6 + 8 + 18 + 10 + 10 + 2p = 66$

$\Rightarrow \quad 2p = 14$

$\Rightarrow \quad p = 7$

Median

Median of a distribution is the value of the variable which divides the distribution into two equal parts, i.e., it is the value of the variable such that number of observations above it is equal to the number of observations below it.

For n observations,

Median = value of $\left(\frac{n+1}{2}\right)$th observation, if n is odd, and,

$$\text{Median} = \frac{\text{value of } \frac{n}{2}\text{th observation} + \text{value of } \left(\frac{n+1}{2}\right)\text{th observation}}{2}$$

Example 13: Find the median of the following data :

12, 17, 13, 12, 14, 16, 18, 20, 21, 23, 20

Solution: Number of observations = 11,

$\therefore$ median = value of $\left(\frac{n+1}{2}\right)$th observation, after arranging the values in ascending order.

Ascending order $\Rightarrow$ 12, 12, 13, 14, 16, 17, 18, 20, 20, 21, 23.

$\therefore$ Median = value of $\left(\frac{11+1}{2}\right)$th term = value of 6^{th} term = 17.

Mode

It is the value which occurs most frequently in a set of observations and around which the other items of the set cluster densely.

$\therefore$ The observation having maximum frequency is selected as model class of the set, and the value of observation is the mode.

Example 14: Find the mode from the following set of observation :

Marks	20	25	26	29	30	32
No. of Students	17	19	14	18	12	16

Solution: Frequency of students is maximum in the class marks 29, i.e., 18.

$\therefore$ Mode = 29

Important Formulae

(i) Mean = $\dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

(ii) If mean, median are given, then, mode can be calculated by, using the relation,

Mode = 3 median – 2mean.

(iii) Median = $\dfrac{\text{Mode} + 2\text{ Mean}}{3}$

Multiple Choice Questions

1. The range of the following ungrouped data will be :

   30, 32, 45, 54, 74, 78, 108, 112, 66, 76, 86, 41, 14, 15, 35, 112.

   (a) 94 (b) 95 (c) 97 (d) 98

2. The no. of class intervals, if the magnitude of class interval is 4 will be :

   Data : 31, 23, 19, 29, 20, 16, 22, 10, 13, 34, 33, 38, 36, 24, 18, 15, 12, 30, 27, 23, 20

   (a) 6 (b) 5 (c) 7 (d) 8

3. The marks of 40 students in final exam obtained by students of class 9 is given below :

   8, 18, 12, 6, 8, 16, 12, 5, 23, 2, 16, 23, 2, 10, 20, 12, 9, 7, 6, 5, 3, 5, 13, 21, 13, 15, 20, 24, 1, 7, 21, 16, 13, 18, 23, 7, 3, 18, 17, 16.

   The number of students in the class interval 5 – 10 are :

   (a) 6 (b) 8 (c) 10 (d) 12

4. The class marks of a distribution are :

   52, 47, 57, 67, 62, 72, 82, 87, 97, 92, 102.

   the lower and upper limits of first class interval will be :

   (a) 44, 49 (b) 44.5, 49.5
   (c) 45, 50 (d) 46, 51

5. The class marks distribution are:

   25, 26, 27, 31, 36, 41, 46, 51, 57, 59

   The lower limit of first class interval will be:

   (a) 40 (b) 42 (c) 23.5 (d) 24.5

6. Tallys are usually marked in a bunch of :

   (a) 3 (b) 5 (c) 4 (d) 6

7. Let 'l' be the lower limit of a class interval in a frequency distribution and 'm' be the mid-point of the class. Then the upper limit of the class is:

   (a) $m-2l$ (b) $2m-l$
   (c) $\frac{3l+m}{2}$ (d) $\frac{2l+m}{2}$

8. The mid – value and upper limit of a class interval are 41 and 47 respectively. The class size will be :

   (a) 6 (b) 12 (c) 10 (d) 18

9. The mid value of a class interval is 14 and the class size is 2. The lower limit of the class is :

   (a) 15 (b) 13
   (c) 17 (d) 18

10. The x-and y-axes in the histogram represent :

    (a) Class interval and frequency
    (b) Class interval and cumulative frequency
    (c) Frequency and class interval
    (d) Cumulative frequency and class interval

11. A frequency polygon is constructed by plotting frequency of the class interval and the

    (a) Upper limit of the class
    (b) Lower limit of the class
    (c) Mid value of the class
    (d) Any values of the class

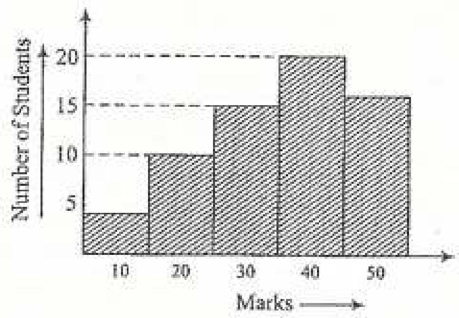
12. In the 'more – than' type of ogive the cumulative frequency is plotted against :

    (a) The lower limit of the concerned class interval
    (b) The mid value of the concerned class interval
    (c) The upper limit of the concerned class interval
    (d) Any value of the concerned class interval

13. Ogives are the graphical representation of

    (a) Cumulative frequency
    (b) Frequency
    (c) Raw data
    (d) Relative frequency

14.



The frequency of students is highest in the class interval :

(a) 0 – 10 (b) 20 – 30
(c) 30 – 40 (d) 40 – 50

15. Total number of students (prob - 14) are :

(a) 66 (b) 76 (c) 56 (d) 54

16. The mean of $x_1, x_2, \ldots\ldots, x_n$ is $\overline{X}$, then the value of:

$$\left(x_1-\overline{x}\right)+\left(x_2-\overline{x}\right)+\left(x_3-\overline{x}\right)+\left(x_4-\overline{x}\right) \ldots\ldots+\left(x_n-\overline{x}\right)=$$

(a) n (b) $n-1$ (c) zero (d) 1

17. If the mean of $x_1, x_2, x_3 \ldots\ldots, x_n$ is $\overline{X}$ and 5 is added to each number, the new mean will be :

(a) $\overline{X}+5n$ (b) $\overline{X}-5n$
(c) $\overline{X}+5$ (d) $\overline{X}-5$

18. If each number in (Prob - 17) is multiplied by k, the new mean will be :

(a) $k\overline{X}$ (b) $k^2\overline{X}$
(c) $k^n\overline{X}$ (d) $\dfrac{\overline{X}}{k}$

19. The mean of 10 numbers is 16. If two consecutive numbers are excluded, the new mean is 18. The sum of the excluded numbers is:

(a) 15 (b) 16 (c) 18 (d) 21

20. If $x_1, x_2, \ldots\ldots x_n$ are n values of variable x, such that,

$\sum_{i=1}^{n}(x_i-2)=110$ and $\sum_{i=1}^{n}(x_i-5)=20$ then value of the mean is :

(a) $\dfrac{16}{3}$ (b) 5 (c) $\dfrac{17}{3}$ (d) $\dfrac{20}{3}$

21. The sum of deviations of a set of n values $x_1, x_2, \ldots\ldots x_n$ measured from 50 is – 10 and the sum of deviations of the values from 46 is 70. The value of n is :

(a) 21 (b) 20
(c) 23 (d) 25

22. The mean of marks scored by 10 students was found to be 43. Later on it was discovered that a score of 30 was misread as 40. The new mean will be (correct).

(a) 41 (b) 44 (c) 43 (d) 42

23. The mean of the following distribution is:

x	10	30	50	70	89
f	7	8	10	15	10

(a) 54 (b) 50 (c) 55 (d) 57

24. The value of p, if the mean of the following distribution is 20.

x	15	17	19	$20+p$	23
f	2	3	4	$5p$	6

(a) 1 (b) 2 (c) 3 (d) 4

25.

x	10	30	70	50	90
f	17	f_1	f_2	32	19

Total = 120

Mean = 50

$f_1, f_2 =$

(a) 24, 28 (b) 28, 24
(c) 26, 28 (d) 26, 24

26. Which of the following is not a measure of central tendency?

(a) Frequency (b) Mean
(c) Mode (d) Median

27. The new median, of the following data, if 37 is replaced by 5.

7, 9, 16, 25, 31, 36, 37, 39, 40, 42, 43

(a) 39 (b) 36
(c) 31 (d) 43

28. If the median of the following data is 63, find x.

29, 32, 48, 50, x, $x+2$, 72, 78, 84, 95

(a) 60 (b) 64
(c) 62 (d) 63

29. The mode of the following data is :

29, 40, 41, 46, 45, 44, 43, 29, 40, 41, 46, 44, 44, 47, 49, 53, 29, 57, 44, 43, 41, 28, 16, 26.

(a) 29 (b) 41
(c) 44 (d) 42

30. If mean of a grouped data is 23 and mode is equal to 14, then the median is equal to :

(a) 19 (b) 21
(c) 22 (d) 20

Answer Key

1. (d)	2. (c)	3. (c)	4. (b)	5. (d)	6. (c)	7. (c)	8. (b)	9. (b)	10. (a)
11. (c)	12. (a)	13. (a)	14. (c)	15. (a)	16. (c)	17. (a)	18. (a)	19. (b)	20. (c)
21. (b)	22. (d)	23. (a)	24. (a)	25. (b)	26. (a)	27. (c)	28. (c)	29. (c)	30. (d)

Hints and Solutions

1. **(d)** Range = uppermost value – lowest value

$$= 112 - 14$$
$$= 98$$

2. **(c)** Range = 38 – 10 = 28

$\therefore$ number of class intervals $= \frac{28}{4} = 7$

3. **(c)** 5 – 10 $\text{卌卌} = 10$

4. **(b)** Upper limit $= a+\frac{h}{2} = 47+\frac{5}{2} = 49.5$

Lower limit $= a-\frac{h}{2} = 47-\frac{5}{2}$

$= 44.5$

[h = difference between any two marks $= 52 - 47 = 5$]

5. **(d)** Lower limit $= a-\frac{h}{2} = 25-\frac{1}{2} = 24.5$

6. **(c)** Tallys are usually marked in a bunch of 4.

7. **(c)** Upper limit $= \frac{l+m}{2}+1 = \frac{3l+m}{2}$

8. **(b)** Upper limit = mid value + $\frac{\text{class size}}{2}$

$$47 = 41 + \frac{\text{class size}}{2}$$

$\therefore$ Class size $= (47 - 41) \times 2$
$= 12$

9. **(b)** Lower limit = mid-value − $\frac{\text{class size}}{2}$

$$= 14 - \frac{2}{2} = 13$$

10. **(a)** X-axis represents class interval,
Y-axis represents frequency.

11. **(c)** Mid-value is always considered for frequency polygon construction.

12. **(a)** The lower limit of the concerned class interval is used for 'more than' type of ogive.

13. **(a)** Ogives represent cumulative frequency.

14. **(c)** $\because$ the class interval, 30 – 40, has highest peak,

$\therefore$ it has highest frequency.

15. **(a)** Total number of students

$= 4 + 10 + 16 + 20 + 16$
$= 66$

16. **(c)** $\frac{x_1 + x_2 + x_3 + x_4 ..... + x_n}{n} = \overline{X}$

$\Rightarrow x_1 + x_2 + x_3 + x_4 .... + x_n = n\overline{X}$
$\Rightarrow x_1 + x_2 + x_3 ...... + x_n$
$= \overline{X} + \overline{X} + \overline{X}$ ...... n times
$\Rightarrow (x_1 - \overline{X}) + (x_2 - \overline{X}) + (x_3 - \overline{X})$
$..... + (x_n - \overline{X}) = 0$

17. **(c)** Given $\frac{x_1 + x_2 + x_3 + x_4 ..... + x_n}{n} = \overline{X}$

$\Rightarrow$ if 5 is added to every number, then,

$$\frac{(x_1 + 5) + (x_2 + 5) + (x_3 + 5) .... + (x_n + 5)}{n}$$
$$= \frac{x_1 + x_2 + x_3 + x_4 ..... + x_n + 5}{n} = \overline{X} + 5$$

18. **(a)** Given $\frac{x_1 + x_2 + x_3 + x_4 ..... + x_n}{n} = \overline{X}$

$$\Rightarrow \frac{kx_1 + kx_2 + kx_3 + ... + kx_n}{n}$$
$$= k\frac{(x_1 + x_2 ..... + x_n)}{n} = k\overline{X}$$

19. **(b)** $\frac{x_1 + x_2 + x_3 .... + x_{10}}{10} = 16$

$\Rightarrow \quad x_1 + x_2 ...... + x_{10} = 16$ .... (i)

Let x_2 and x_3 are removed, then,

$$\frac{x_1 + x_4 .... + x_{10}}{8} = 18$$

$\Rightarrow \quad x_1 + x_4 + ... + x_{10} = 144$ ....(ii)

Now, Eq. (i) – eq (ii)

$x_2 + x_3 = 160 - 144 = 16$

20. **(c)** $\sum_{i=1}^{n} x_i - 2\sum_{i=1}^{n} 1 = \sum_{i=1}^{n} x_i - 2n = 110$ ...(i)

$\sum_{i=1}^{n} x_i - 5\sum_{i=1}^{n} 1 = \sum_{i=1}^{n} x_i - 5n = 20$ ...(ii)

Subtracting eq (ii) from eq (i), we have,

$3n = 90 \Rightarrow n = 30$

$$\therefore \text{mean} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{110 + 2n}{n} = \frac{110 + 60}{30} = \frac{17}{3}$$

21. **(b)** $\sum_{i=1}^{n}(x_i - 50) = -10 \Rightarrow \sum_{i=1}^{n} x_i - 50n = -10$ ...(i)

$\sum_{i=1}^{n}(x_i - 46) = 70 \Rightarrow \sum x - 46n = 70$ ...(ii)

Subtracting eq. (i), from eq. (ii),

$46n - (-50\,n) = 70 - (-10)$
$\Rightarrow \quad 4n = 80$
$\Rightarrow \quad n = 20$

22. **(d)** Sum of marks of 10 students $= 10 \times 43$
$= 430$

Correct summation of marks
$= (430 - 40) + 30$
$= 420$

$\therefore$ correct mean $= \frac{420}{10} = 42$

23. **(c)** Mean $= \frac{\sum_{i=1}^{5} f_i x_i}{\sum_{i=1}^{n} f_i}$

$$= \frac{10 \times 7 + 30 \times 8 + 50 \times 10 + 70 \times 15 + 89 \times 10}{7 + 8 + 10 + 15 + 10}$$
$$= \frac{70 + 240 + 500 + 1050 + 890}{50} = 55$$

24. **(a)** Mean $= \frac{\sum f_i x_i}{\sum f_i}$

$$\Rightarrow 20 = \frac{15 \times 2 + 17 \times 3 + 19 \times 4 + 5p(20 + p) + 23 \times 6}{2 + 3 + 4 + 5p + 6}$$

$\Rightarrow 20\,(15 + 5p) = 30 + 51 + 76 + 100p + 5p^2 + 138$

$\Rightarrow 300 + 100p = 295 + 100p + 5p^2$

$\Rightarrow \quad 5p^2 = 5$

$\Rightarrow \quad p = 1$

25. **(b)** $f_1 + f_2 + 17 + 32 + 19 = 120$

$\Rightarrow \quad f_1 + f_2 = 52 \quad \ldots\text{(i)}$

Mean

$$= \frac{10\times17+30f_1+70f_2+50\times32+90\times19}{120}$$

$\Rightarrow 50 \times 120 = 170 + 1600 + 1710 + 30f_1 + 70f_2$

$\Rightarrow \quad 30f_1 + 70f_2 = 2520$

$\Rightarrow \quad 37_1 + 7f_2 = 252 \quad \ldots\ldots\text{(ii)}$

From eq (i) and eq (ii).

$f_1 = 28, f_2 = 24$

26. **(a)** Frequency is not a measure of central tendency.

27. **(c)** If $37 \longrightarrow 5$, then, the new data will be.

5, 7,9, 16, 25, 31, 36, 39, 40, 42, 43

$\because$ No. of numbers = 11

$\therefore \left(\frac{11+1}{2}\right)th = 6^{th}$ number will be the median.

$\therefore$ Median = 31

28. **(c)** $\because$ No. of number = 10

$\therefore$ 5^{th} and 6^{th} number will be considered from median.

$$\therefore \text{median} = \frac{\text{5th term} + \text{6th term}}{2}$$

$$= \frac{x+x+2}{2} = 63$$

$\Rightarrow \quad x + 1 = 63$

$\Rightarrow \quad x = 62$

29. **(c)** $\because$ 44 is repeated 4 times, i.e., maximum no. of times.

$\therefore$ Mode = 44

30. **(d)** Mode = 3 × Median − 2 × Mean

$$\Rightarrow \text{Median} = \frac{\text{Mode} + 2\times\text{Mean}}{3}$$

$$= \frac{14+23\times2}{3}$$

$$= \frac{14+46}{3}$$

$$= \frac{60}{3}$$

$$= 20$$

14. Probabiliy

Learing Objective:

In this chapter, we will learn about:

*Experiment

*Compound Event

*Empirical Probability

The uncertainty of 'probably' etc. can be measured numerically by means of probability in many cases.

Experiment

An operation which can produce some well-defined outcomes; is called an experiment.

Each outcome is called an event.

Random experiment

An experiment in which all possible outcomes are known and the exact outcome cannot be predicted earlier, is called a random experiment.

Trial

The process of performing a random experiment is called a trial.

Compound Event

A collection of two or more possible outcomes of a trial of a random experiment is called a compound event.

Empirical Probability

Empirical probability, $P(A)$, i.e., probability of occurrence of an event A can be mathematically, expressed as,

$$P(A) = \frac{m}{n} = \frac{\text{number of trials in which the event happens}}{\text{total number of trials}}$$

$$0 \le P(A) \le 1$$

Probability, $P(A) = 0$, when the event is impossible to happen and $P(A) = 1$, for sure event.

Example 1: A die is thrown 200 times and the outcomes are :

Outcome	1	2	3	4	5	6
Frequency	20	30	20	70	25	35

The probability of getting 5 is ....

Solution: $P(\text{getting } 5) = \frac{25}{200} = \frac{1}{8}$

Example 2: Probability of getting a prime number (in problem 1) is ....

Solution: P (getting a prime number) $= \frac{30+20+25}{200} = \frac{75}{200} = \frac{3}{8}$

Example 3: A coin is tossed 3 times. The probability of getting 2 heads is ....

Solution: The outcomes of the trial are:

HHH, HHT, HTH, THH, THT, TTT, TTH, HTT.

For two heads, there are 3 favourable conditions.

$\therefore P(2 \text{ heads}) = \frac{3}{8}$

Example 4: A coin is tossed 100 times and tail comes up 25 times. The probability of getting a head is ....

Solution: $P(\text{getting head}) = \frac{100-25}{100} = \frac{75}{100} = \frac{3}{4}$

Example 5: A card is chosen from a well shuffled deck of 52 cards.

The probability of getting a king of red suit is .....

Solution: P (getting a king of red suit) $= \frac{2}{52} = \frac{1}{26}$

Example 6: A card is drawn at random from a pack of 52 cards. Find the probability of getting a diamond.

Solution: P (getting a diamond) $= \frac{13}{52} = \frac{1}{4}$

Example 7: There are 50 cards numbered from 1 to 50. One card is drawn at random. Find the probability that the number is divisible neither by 5 nor by 3.

Solution: Numbers divisible by 5 are 5, 10, 15, ..., 50, i.e., 10.

Numbers divisible by 3 are 3, 6, 9, 12, 15, ..., 18, i.e., 16.

Numbers divisible by 15 are 15, 30, 45, i.e., 3.

$\therefore$ Total numbers, i.e., neither divisible by 5 nor 3

$= 50 - (10 + 16 - 3)$

$= 50 - (23) = 27$

$\therefore$ Required probability $= \frac{27}{50}$

Example 8: A bag contains 2 dozen eggs out of which 2 are defective. One egg is selected at random. Find the probability of the egg to be non – defective.

Solution: P (getting a non – defective egg) $= \frac{24-2}{24} = \frac{22}{24} = \frac{11}{12}$

Example 9: A bag contains 7 red, 5 white and 3 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is neither white nor black.

Solution: If the ball drawn is neither white nor black, then it should be red.

$\because$ Number of red balls = 7

$\therefore$ Required probability $= \frac{7}{7+5+3} = \frac{7}{15}$

Example 10: Two dice are thrown simultaneously. The probability of getting 7 as a sum is ....

Solution: 7 can be obtained as a sum, either by getting (6, 1), or (1, 6), i.e., 2 ways, and by (4, 3), (3, 4), (5, 2), (2, 5), i.e., total 6 ways.

Total outcomes $= 6 \times 6 = 36$

$\therefore$ Required probability $= \frac{6}{36} = \frac{1}{6}$

Multiple Choice Questions

1. The probability of an impossible event is :
   (a) 1 (b) Zero
   (c) Less than 1 (d) −1
2. The probability of a certain event is:
   (a) 0 (b) 1
   (d) Less than 1 (d) − 1
3. Two coins are tossed simultaneously. The probability of getting at least one head is:
   (a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) 1
4. 4 coins are tossed simultaneously. The probability of getting all tails is;
   (a) 1 (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $\frac{3}{8}$
5. In a class, 20 students failed in a certain examination. If the no. of passed students is 80 and 1 student is selected at random, then the probability that the student has passed in the exam is :
   (a) 0.8 (b) 0.6 (c) 0.2 (d) 0.4
6. A dice is rolled 600 times and the occurrence of the outcomes are given below :

Outcomes	1	2	3	4	5	6
Frequency	200	30	120	100	50	100

   The probability of getting a composite number is :
   (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{11}{60}$ (d) $\frac{71}{125}$
7. A dice is rolled twice, and the outcomes are noted down. The probability of getting even no. as a sum is :
   (a) $\frac{19}{36}$ (b) $\frac{17}{36}$ (c) $\frac{1}{2}$ (d) $\frac{5}{9}$
8. The unit place digit of 200 people's mobile number is observed, and the following table is plotted :

Unit place digit	0	1	2	3	4	5	6	7	8	9
Frequency	23	26	23	20	19	11	14	30	14	20

   A number is chosen at random. The probability that its unit place has prime number is:

(a) 0.42 (b) 0.43
(c) 0.84 (d) 0.48

9. The probability of getting odd number as a unit place digit is:

(a) $\frac{103}{200}$ (b) $\frac{97}{200}$ (c) $\frac{107}{200}$ (d) $\frac{121}{200}$

10. A dice is rolled twice. Find the probability of getting prime number as a sum.

(a) $\frac{5}{12}$ (b) $\frac{1}{2}$ (c) $\frac{7}{36}$ (d) $\frac{17}{36}$

11. A coin is tossed 1000 times, if the probability of getting a tail, $\frac{3}{8}$ how many times head is obtained?

(a) 325 (b) 525 (c) 625 (d) 725

12. A coin is tossed 1000 times and the following frequencies are observed:

Head: 455, tail : 545.

The probability for getting tail is:

(a) $\frac{109}{200}$ (b) $\frac{91}{200}$ (c) $\frac{9}{20}$ (d) $\frac{108}{200}$

13. The distribution of marks of 90 students are as follows :

Marks	0-20	20-30	30-40	40-50	50-60
Numbers of students	27	10	10	23	20

The probability that a student obtained 40 or more marks is :

(a) $\frac{27}{90}$ (b) $\frac{47}{90}$ (c) $\frac{43}{90}$ (d) $\frac{53}{90}$

14. Two coins are tossed simultaneously. The probability of getting at least 2 tail is:

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

15. One card is drawn from a well shuffled deck of 52. What is the probability of drawing a red card?

(a) $\frac{6}{13}$ (b) $\frac{1}{2}$ (c) $\frac{2}{13}$ (d) $\frac{4}{13}$

16. One card is drawn from a well – shuffled deck of 52 cards. What is the probability of getting a king?

(a) $\frac{3}{26}$ (b) $\frac{1}{13}$ (c) $\frac{2}{13}$ (d) $\frac{5}{26}$

17. There are 36 students in a class of whom 20 are boys and remaining are girls. What is the probability that a student chosen is a girl?

(a) $\frac{5}{9}$ (b) $\frac{4}{9}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

18. Three coins are tossed simultaneously. The probability of getting exactly 2 heads is:

(a) $\frac{2}{8}$ (b) $\frac{3}{8}$ (c) $\frac{4}{8}$ (d) $\frac{5}{8}$

19. Two dice are thrown simultaneously. What is the probability of getting a doublet?

(a) $\frac{5}{36}$ (b) $\frac{1}{9}$ (c) $\frac{1}{6}$ (d) $\frac{7}{36}$

20. A bag contains numbers 1, 2, 3, 4 ...., 35. What is the probability of getting a multiple of 8?

(a) $\frac{2}{35}$ (b) $\frac{3}{35}$ (c) $\frac{4}{35}$ (d) $\frac{1}{7}$

21. A bag contains 6 red balls, 8 white balls, 5 green balls and 3 black balls. One ball is drawn at random from the bag. The probability that the ball is neither white nor black is:

(a) $\frac{1}{3}$ (b) $\frac{17}{22}$

(c) $\frac{1}{2}$ (d) $\frac{9}{22}$

22. From a pack of 52 playing cards, jacks, queens, kings and aces of red colour are

removed. A card is drawn at random. The probability of drawing a black king is:

(a) $\frac{1}{24}$ (b) $\frac{1}{22}$ (c) $\frac{1}{26}$ (d) $\frac{1}{44}$

23. A bag contains 5 red and some black balls. If the probability of drawing a black ball is thrice that of a red ball, the number of black balls in the bag is:

(a) 5 (b) 10 (c) 15 (d) 20

24. Two men were born in the same year, i.e., 1987. What is the probability that their birthday will fall on different days?

(a) $\frac{364}{366}$ (b) $\frac{364}{365}$

(c) $\frac{1}{365}$ (d) $\left(1+\frac{364}{365}\right)$

25. A box contains 200 balls out of which 20 are defective. A bulb is drawn at random. What is the probability of drawing a non – defective bulb?

(a) $\frac{1}{10}$ (b) $\frac{9}{10}$ (c) $\frac{7}{10}$ (d) $\frac{4}{5}$

26. The probability of getting 53 Fridays in a leap year is:

(a) $\frac{1}{7}$ (b) $\frac{3}{7}$ (c) $\frac{2}{7}$ (d) $\frac{3}{14}$

27. The sum of probabilities of all the outcomes of an experiment is:

(a) Zero (b) Less than zero

(c) 1 (d) Less than 1

28. A bag contains cards marked with numbers 51, 52, ..., 100. A number is selected at random. What is the probability of getting a number which is not a multiple of 5?

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

29. One card is drawn from a well – shuffled deck of 52 cards. The probability of drawing a red face card is:

(a) $\frac{3}{26}$ (b) $\frac{3}{13}$

(c) $\frac{3}{52}$ (d) $\frac{1}{13}$

30. Two dice are rolled simultaneously. Find the probability of getting their product as a odd number.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{5}{36}$ (d) $\frac{1}{4}$

Answer Key

1. (b)	2. (b)	3. (b)	4. (b)	5. (a)	6. (b)	7. (c)	8. (a)	9. (c)	10. (a)
11. (c)	12. (a)	13. (c)	14. (a)	15. (b)	16. (b)	17. (b)	18. (b)	19. (c)	20. (c)
21. (c)	22. (b)	23. (c)	24. (b)	25. (b)	26. (c)	27. (c)	28. (d)	29. (a)	30. (d)

Hints and Solutions

1. **(b)** The probability of an impossible event is zero.
2. **(b)** The probability of a certain event is 1.
3. **(b)** Outcomes of the event are,
HH, TT, TH, HT
No. of favourable cases = 3
$\therefore$ required probability = $\frac{3}{4}$
4. **(b)** The probability of getting all tails
$= \frac{1}{2^4} = \frac{1}{16}$
5. **(a)** P(student passed in the examination)
$= \frac{80}{20+80}$
$= 0.8$
6. **(b)** P (composite number)
$= \frac{100+100}{600} = \frac{200}{600} = \frac{1}{3}$
{$\because$ composite numbers are 4 and 6}
7. **(c)** P (drawing sum as even number)
$= \frac{18}{36} = \frac{1}{2}$
8. **(a)** Prime numbers are 2, 3, 5, 7.
$\therefore$ Sum of frequencies = 23 + 20 + 11 + 30
= 84
$\therefore$ Required probability = $\frac{84}{200} = \frac{42}{100} = 0.42$
9. **(c)** 1, 3, 5, 7 and 9 are odd numbers.
$\therefore$ Required probability
$= \frac{26+20+11+30+20}{200} = \frac{107}{200}$
10. **(a)** Prime numbers which can be get as a sum of the numbers on dice are 2, 3, 5, 7, 11.
$\therefore$ Required probability = $\frac{1+2+4+6+2}{36}$
$= \frac{15}{36} = \frac{5}{12}$
11. **(c)** Tail is obtained $\left(\frac{3}{8}\times 1000\right)$ times = 375 times.
$\therefore$ Head is obtained $(1000-375) = 625$ times.
12. **(a)** P (getting tail) = $\frac{545}{1000} = \frac{109}{200}$
13. **(c)** P (student getting 40 or more marks)
$= \frac{23+20}{90} = \frac{43}{90}$
14. **(a)** Outcomes are HH, TT, TH, HT
P (getting at least 1 tail) = $\frac{3}{4}$
15. **(b)** P (getting a red card) = $\frac{26}{52} = \frac{1}{2}$
16. **(b)** P (getting a king) = $\frac{4}{52} = \frac{1}{13}$
17. **(b)** P (student chosen is girl) = $\frac{36-20}{36}$
$= \frac{16}{36} = \frac{4}{9}$
18. **(b)** Outcomes are HHH, HHT, HTH, HTT, THT, TTH, TTT, THH.
P (getting exactly 2 heads) = $\frac{3}{8}$
19. **(c)** (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) are the doublets.
$\therefore$ P (getting a doublet) = $\frac{6}{36} = \frac{1}{6}$
20. **(c)** Multiples of 8 are 8, 16, 24 and 32.
$\therefore$ P (getting a multiple of 8) = $\frac{4}{35}$
21. **(c)** P (neither white nor black ball)
= P(red or green ball)

$$= \frac{6+5}{6+8+5+3} = \frac{11}{22} = \frac{1}{2}$$

22. **(b)** Remaining cards $= 52 - 2 \times 4$

$$= 52 - 8 = 44$$

$\therefore$ P (drawing a black king) $= \frac{2}{44} = \frac{1}{22}$

23. **(c)** Let the number of black balls in the bag be x.

$$\therefore \quad 3 \times \frac{5}{5+x} = \left(\frac{x}{5+x}\right)$$

$$\Rightarrow \quad x = 15$$

24. **(b)** 1987 is non-leap year.

$\therefore$ Number of days $= 365$

$\therefore$ P (birthday will fall on different days)

$= 1 - $ P(birthday on same day)

$$= 1 - \frac{1}{365} = \frac{364}{365}$$

25. **(b)** P (getting a non – defective bulb)

$$= \frac{200-20}{200}$$

$$= \frac{180}{200} = \frac{9}{10}$$

26. **(c)** Leap year has 366 days, i.e.,

$\frac{364}{7} = 52$ weeks and 2 days.

The 2 days can either be MT, TW, WTh, Th, F, FS, SS, SM.

$\because$ Thursday, Friday and Friday, Saturday are 2 favourable outcomes.

$\therefore$ Required probability $= \frac{2}{7}$

27. **(c)** P (occurrence of a event) + P (non-occurrence of event) = total probability = 1

28. **(d)** Number of multiples of $5 = \frac{100-50}{5} = 10$

$\therefore$ Required probability $= 1 - \frac{10}{50} = 1 - \frac{1}{5} = \frac{4}{5}$

29. **(a)** King, queen and jack are known as face cards.

Number of red face cards $= 2 \times 3 = 6$

$\therefore$ Required probability $= \frac{6}{52} = \frac{3}{26}$

30. **(d)** The outcomes are:

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (5, 5) (5, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

Even × odd = even and odd × odd = odd.

$\therefore$ For product to be odd, both numbers should be odd.

$\therefore$ Required probability $= \frac{9}{36} = \frac{1}{4}$

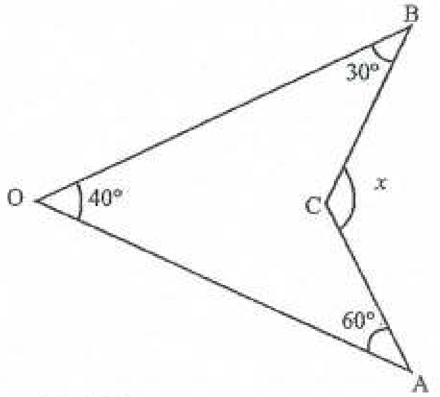
ACHIEVERS SECTION

Multiple Choice Questions

1. If $x+\frac{1}{x+1}=1$ then find the value of $(x+1)^5+\frac{1}{(x+1)^5}$

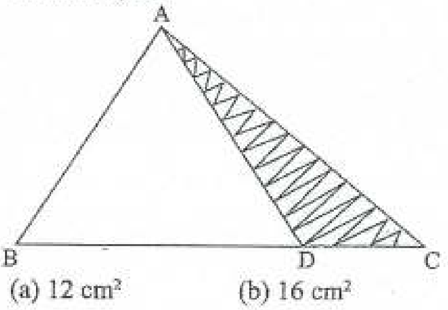
   (a) 1 (b) 2
   (c) 5 (d) None of these

2. In the given figure, what is the value of x?



   (a) 70° (b) 100°
   (c) 90° (d) 130°

3. In the given figure, D is a point between BC in ΔABC such that BD : DC = 3 : 2. If the area of ΔABC is 40 cm². What is the area of shaded region?



   (a) 12 cm² (b) 16 cm²
   (c) 18 cm² (d) 20 cm²

4. If $x=\frac{1}{x-5}$ then what is the value $x^2-\frac{1}{x^2}$ of ?

   (a) 24 (b) 25
   (c) 26 (d) 1

5. If $\sqrt{13-a\sqrt{10}}=\sqrt{8}+\sqrt{5}$. What is the value of a?

   (a) 4 (b) – 2
   (c) – 4 (d) 2

6. Difference between the semi perimeter and the sides of a ΔABC are 8 cm, 7 cm and 5 cm respectively. What is the area of triangle?

   (a) 20 cm² (b) $14\sqrt{20}$ cm²
   (c) $20\sqrt{14}$ cm² (d) $12\sqrt{10}$ cm²

7. If $a=\frac{2^{x-1}}{2^{x-2}}$ and $b=\frac{2^{-x}}{2^{x+1}}$; a – b = 0 then what is the value of x?

   (a) – 2 (b) 1
   (c) – 1 (d) 2

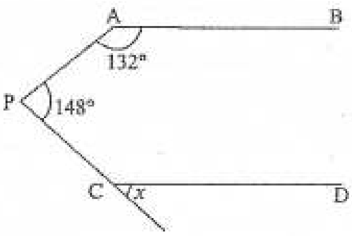
8. If $\frac{x}{y}=\frac{2}{3}$ then what is the value of $\frac{4}{5}+\frac{y-x}{y+x}$?

   (a) 3 (b) 2
   (c) 1 (d) – 1

9. If $x^3+\frac{1}{x^3}=110$ what is the value of $x+\frac{1}{x}$?

   (a) 5 (b) 6
   (c) 7 (d) 9

10. In the figure, AB || CD. What is the value of x?



(a) 115° (b) 100°

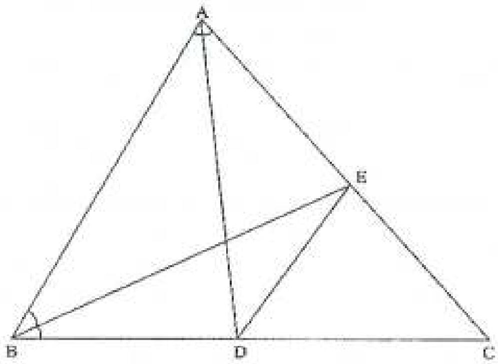
(c) 95° (d) 105°

11. What is the value of K for which $(x-1)$ is a factor of $4x^3+3x^2-4x+K$?

(a) – 3 (b) – 2

(c) 4 (d) 3

12. In ΔABC, $\angle B = 2\angle C$. D is a point on BC such that AD bisects, $\angle$BAC. It is given that AB = CD. BE is the bisector of $\angle$B. What is the measure of $\angle$BAC?



(a) 72° (b) 73°

(c) 95° (d) 75°

13. What is the perpendicular distance of point A(4, 3) from Y – axis?

(a) 6 (b) 5

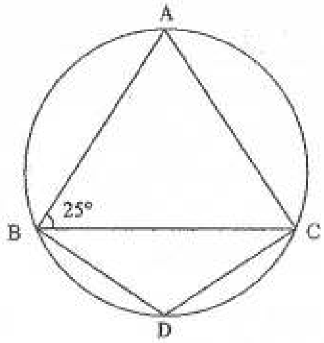
(c) 4 (d) 3

14. If each side of a triangle is doubled, then what is the percentage increase in its area?

(a) 200% (b) 250%

(c) 300% (d) 400%

15. In the given figure BD = DC and $\angle DBC = 25°$. What is the measure of $\angle$BAC?



(a) 70° (b) 50°

(c) 90° (d) 130°

16. A wood log is cut first in the form of a cuboid of length 2.3 m. Width 0.75 m and of a certain thickness its volume is 1.104 m³. How many rectangular Planks of size 2.3 m × 0.75 m × 0.04 m can be cut from the cuboid?

(a) 12 (b) 14

(c) 16 (d) 24

17. If $\left(a+\frac{1}{a}\right)^2 = b$ then what is the value $a^3+\frac{1}{a^3}$?

(a) b^3 (b) $b^{3/2}$

(c) $b^{3/4}-3b^{1/2}$ (d) $b^{3/2}+3b^{1/2}$

18. The radius of the internal and external surface of a hollow spherical shell are 3 cm and 5 cm respectively. If it is melted and recast in to a solid cylinder of height $\frac{8}{3}$ cm. What is the diameter of cylinder?

(a) 7 cm (b) 10.5 cm

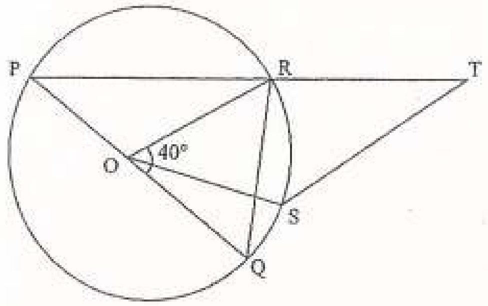
(c) 14 cm (d) 21 cm

19. If the diagonal of a cuboid is $\sqrt{251}$ cm. Its breadth is 9 cm and height is 7 cm. What is its length?

(a) 8 cm (b) 10 cm

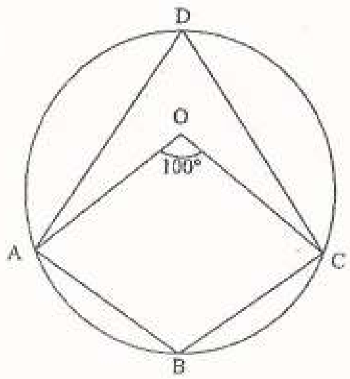
(c) 11 cm (d) 12 cm

20. In the above figure O is the centre and PQ is diameter. If $\angle ROS = 40°$. What is the measure of $\angle RTS$?



(a) 40° (b) 50°
(c) 60° (d) 70°

21. If O is the centre of the circle $\angle AOC = 100°$. What is the measure of $\angle ABC$?



(a) 50° (b) 100°
(c) 120° (d) 130°

22. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m. How much area of grass field will each cow be grazing?

(a) 24 m^2 (b) 36 m^2
(c) 48 m^2 (d) 96 m^2

23. If $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by $(x - 1)$ and $(x + 1)$, the remainders are 5 and 19 respectively. What is the remainder when $f(x)$ is divided by $(x - 2)$?

(a) 8 (b) 10
(c) 12 (d) 14

24. If $\frac{3+\sqrt{7}}{3-\sqrt{7}} = a + b\sqrt{7}$ then what is the difference between a and b?

(a) 3 (b) 5
(c) 8 (d) 11

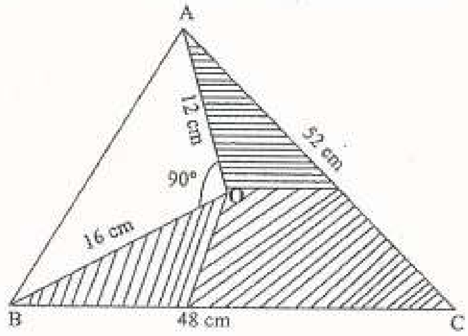
25. If $x^2 - 1$ is a factor of $ax^4 + bx^3 + ax^2 + dx + e$ then which of the following is correct?

(a) $a + c + e = b + d$
(b) $b + c + d = a + e$
(c) $a + b + c = d + e$
(d) $a + b + e = c + d$

26. If $2\angle A = 3\angle B = 6\angle C$. What is the difference between $\angle B$ and $\angle C$?

(a) 20° (b) 30°
(c) 40° (d) 60°

27. What is the area of the shaded region?



(a) 404 cm^2 (b) 392 cm^2
(c) 388 cm^2 (d) 384 cm^2

28. If $9^{x+2} = 240 + 9^x$ then what is the value of x?

(a) 0.1 (b) 0.2
(c) 0.4 (d) 0.5

29. If $a + b + c = 9$ and $a^2 + b^2 + c^2 = 35$ what is the value of $a^3 + b^3 + c^3 - 3abc$?

(a) 92 (b) 98
(c) 108 (d) 112

30. If $x^2+\frac{1}{a^2}=102$ then what is the value of $a-\frac{1}{a}$?

(a) 8 (b) 10

(c) 12 (d) 14

31. If $\frac{a}{b}+\frac{b}{a}=-1$. What is the value of a^3-b^3?

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) -1

32. If $a+b+c=0$ then what is the value of $\frac{(b+c)^2}{3bc}+\frac{(c+a)^2}{3ac}+\frac{(a+b)^2}{3ab}$?

(a) 0 (b) 1

(c) -1 (d) 3

33. If $x^2+x=5$ then what is the value of $(x+3)^3+\frac{1}{(x+3)^3}$?

(a) 110 (b) 120

(c) 130 (d) 105

34. What is the remainder when $x^{51}+51$ is divided by $x+1$?

(a) 50 (b) 51

(c) -51 (d) 52

35. A solid cylinder has total surface area of 462 square cm. Its curved surface area is one-third of total surface area. What is the volume of cylinder?

(a) 529 cm^2 (b) 539 cm^3

(c) 549 cm^3 (d) 559 cm^3

36. The mean of 16 numbers is 8. If 2 is added to every number, then what will be the new mean?

(a) 9 (b) 10

(c) 12 (d) 14

37. A cone and a hemisphere have equal bases and equal volumes. What is the ratio of their heights?

(a) 1 : 2 (b) 2 : 1

(c) 1 : 3 (d) 3 : 1

38. The surface area of a sphere is 5544 cm^2. What is the volume of the sphere?

(a) 38808 cm^3 (b) 38208 cm^3

(c) 38608 cm^3 (d) 38818 cm^3

39. Five cubes each of side 5 cm are joined end to end. What is the surface area of the resulting cuboid?

(a) 475 cm^2 (b) 450 cm^2

(c) 550 cm^2 (d) 575 cm^2

40. If $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=a+\sqrt{15}\,b$. What is the value of b?

(a) 0 (b) 1

(c) 2 (d) 3

Answer Key

1. (b)	2. (d)	3. (b)	4. (b)	5. (c)	6. (c)	7. (c)	8. (c)	9. (a)	10 (b)
11. (a)	12. (a)	13. (c)	14. (c)	15. (b)	16. (c)	17. (c)	18. (c)	19. (c)	20. (d)
21. (d)	22. (c)	23. (b)	24. (b)	25. (a)	26. (b)	27. (d)	28. (d)	29. (c)	30. (b)
31. (a)	32. (b)	33. (a)	34. (a)	35. (b)	36. (b)	37. (b)	38. (a)	39. (c)	40. (b)

Hints and Solutions

1. **(b)**

$x+\frac{1}{x+1}=1$

$\Rightarrow x=1-\frac{1}{x+1}$

$\Rightarrow x=\frac{x+1-1}{x+1}$

$\Rightarrow x+1=1$

$\Rightarrow (x+1)^5+\frac{1}{(x+1)^5}$

$\Rightarrow \quad (1)^5+\frac{1}{(1)^5}$

$\Rightarrow \quad 1+1$

$\Rightarrow \quad 2$

2. **(d)**

$x=40°+30°+60°$

$x=130°$

3. **(b)**

Let BD : DC = $3x : 2x$

Area of ΔABC

$=\frac{1}{2}\times(3x+2x)\times h=40\text{ cm}^2$

$=5x\times \text{h}=80$

$=h=\frac{80}{5x}$

$=h=\frac{16}{x}$

Area of ΔABD

$=\frac{1}{2}\times\frac{16}{x}\times 3x=24\text{ cm}^2$

Area of ADC = $(40-24)\text{ cm}^2=16\text{ cm}^2$

4. **(b)**

$\frac{1}{x-5}=x$

$(x-5)=\frac{1}{x}$

$x-\frac{1}{x}=5$

$\Rightarrow\left(x-\frac{1}{x}\right)^2=(5)^2$

$\Rightarrow x^2+\frac{1}{x^2}-2=25$

$\Rightarrow x^2+\frac{1}{x^2}=23$

$\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2$

$\left(x+\frac{1}{x}\right)^2=23+2$

$x+\frac{1}{x}=\sqrt{25}=5$

$\because\left(x-\frac{1}{x}\right)^2=\left(x+\frac{1}{x}\right)^2=x^2-\frac{1}{x^2}$

$5\times5=x^2-\frac{1}{x^2}$

$\Rightarrow x^2-\frac{1}{x^2}=25$

5. **(c)**

$\sqrt{13-a\sqrt{10}}=\sqrt{8}+\sqrt{5}$

$\Rightarrow\left(\sqrt{13-a\sqrt{10}}\right)^2=\left(\sqrt{8}+\sqrt{5}\right)^2$

$\Rightarrow 13-a\sqrt{10}=13+2\sqrt{40}$

$\Rightarrow -a\sqrt{10} = 2\sqrt{40}$

$\Rightarrow -a\sqrt{10} = 2\sqrt{4} \times \sqrt{10}$

$\Rightarrow -a\sqrt{10} = 2 \times 2\sqrt{10}$

$\Rightarrow -a\sqrt{10} = 4\sqrt{10}$

$\Rightarrow -a = 4$

$\Rightarrow a = -4$

6. **(c)**

S – a = 8 cm ....(1)

S – b = 7 cm ....(2)

S – c = 5 cm ....(3)

(1) + (2) + (3)

3S – (a + b + c) = 20 cm

3S – 2S = 20 cm

S = 20 cm

Area $= \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{20 \times 8 \times 7 \times 5}$

$= \sqrt{10 \times 2 \times 4 \times 2 \times 5}$

$= \sqrt{5 \times 2 \times 2 \times 4 \times 2 \times 7 \times 5}$

$= 5 \times 2 \times 2\sqrt{14}$

$= 20\sqrt{14}\ \text{cm}^2$

7. **(c)**

$a = \dfrac{2^{x-1}}{2^{x-2}} = 2^{(x-1)-(x-2)}$

$b = \dfrac{2^{-x}}{2^{x+1}} = 2^{(-x)-(x+1)}$

$a = 2^{x-1-x+2}$

$b = 2^{-2x-x-1}$

$a = 2$

$b = 2^{-2x-1}$

$2 - 2^{-2x-1} = 0$

$[\because a - b = 0]$

$-2^{-2x-1} = -2$

$-2x - 1 = 1$

$\Rightarrow -2x = 2$

$\Rightarrow x = -1$

8. **(c)**

$1 + \dfrac{x}{y} = 1 + \dfrac{2}{3},\quad 1 - \dfrac{x}{y} = 1 - \dfrac{2}{3}$

$\dfrac{y+x}{y} = \dfrac{5}{3},\qquad \dfrac{y-x}{y} = \dfrac{1}{3}$

$$\frac{\frac{y+x}{y}}{\frac{y-x}{y}} = \frac{\frac{1}{3}}{\frac{5}{3}}$$

$\dfrac{4}{5} + \dfrac{y-x}{y+x} = \dfrac{4}{5} + \dfrac{1}{5}$

$\dfrac{4}{5} + \dfrac{y-x}{y+x} = 1$

9. **(a)**

$\left(x + \dfrac{1}{x}\right)^3 = x^3 + \dfrac{1}{x^3} + 3x\dfrac{1}{x}\left(x + \dfrac{1}{x}\right)$

$\left(x + \dfrac{1}{x}\right)^3 = 110 + 3\left(x + \dfrac{1}{x}\right)$

$\left(x + \dfrac{1}{x}\right)^3 - 3\left(x + \dfrac{1}{x}\right) = 110$

$\left(x + \dfrac{1}{x}\right)^3 - 3\left(x + \dfrac{1}{x}\right) = (5)^3 - 3(5)$

By comparing, we have

$x + \dfrac{1}{x} = 5$

10. **(b)**

AB || PQ

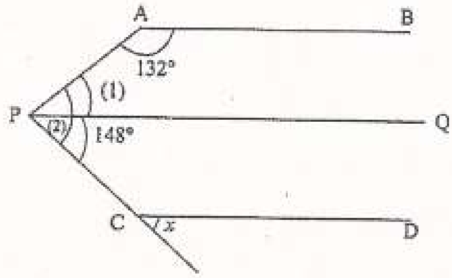
$\angle 1 = 180^\circ - 132^\circ$

$\angle 1 = 48^\circ$

$\angle 2 = 148° - 48° = 100°$

$\angle PCD = 180° - 100° = 80°$

$x = 180° - 80 = 100°$



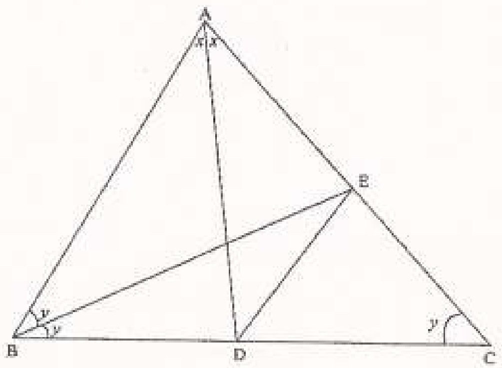
11. **(a)**

$f(1) = 0$

$4(1)^3 + 3(1)^2 - 4(1) + K = 0$

$4 + 3 - 4 + K = 0 \Rightarrow K = -3$

12. **(a)**



In $\angle ABC$

$\angle B = 2\angle C$

$\angle B = 2y$

Let $\angle BAD = \angle CAD = x$

$\Delta ABE \cong \Delta DCE$

$\angle ABE = \angle DCE = y$

$AB = CD$

$\angle CDE = 2x =$ and $\angle ADE = \angle DAE = x$

$x + 2x = 2y + x$

$x = y$

In ABC;

$\angle A + \angle B + \angle C = 180°$

$2x + 2y + y = 180°$

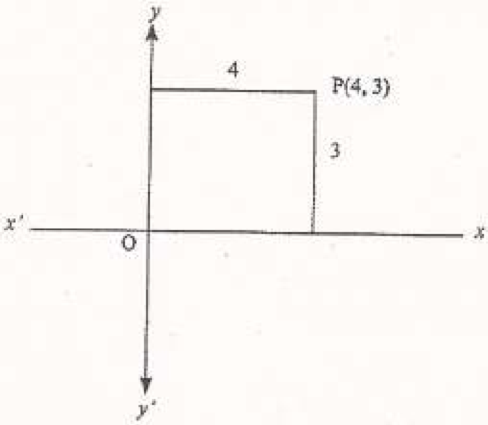
$2x + 2x + x = 180°$

$x = 36°$

$\angle BAC = 2x = 2 \times 36° = 72°$

13. **(c)**

Perpendicular distance from Y- axis of the



point $(4, 3) = 4$

14. **(c)**

$$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$$

$$S' = \frac{2(a+b+c)}{2} = a+b+c = 2S$$

$$\Delta' = \sqrt{2S(2S-2a)(2S-2b)(2S-2c)}$$

$$= \sqrt{2\times 2\times 2\times 2\ S(S-a)(S-b)(S-c)} = 4\Delta$$

$$\frac{4\Delta - \Delta}{\Delta} = \times 100 = 300\%$$

15.

In $\angle BCD$,

$BD = DC$

$\angle BCD = 25°$

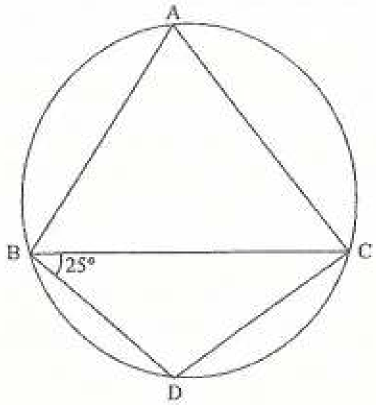
$\angle DCB + DBC + BDC = 180°$

$25° + 25° + \angle BDC = 180°$

$\angle BDC = 130°$

ABCD is a cyclic quadrilateral

$\angle BAC = 180° - 130° = 50°$



16. **(c)**

Number of rectangular planks

$$= \frac{\text{Volume of Cuboid}}{\text{Volume of a Plank}}$$

$$= \frac{1.104}{2.3 \times 0.75 \times 0.040} = 16$$

17. **(c)**

$$\left(a + \frac{1}{a}\right)^2 = b$$

$$a + \frac{1}{a} = \sqrt{b}$$

$$\left(a + \frac{1}{a}\right)^3 = \left(\sqrt{b}\right)^3$$

$$a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) = b^{3/2}$$

$$a^3 + \frac{1}{a^3} = b^{3/2} - 3\sqrt{b} = b^{3/2} - 3b^{1/2}$$

18. **(c)** Let r be the radius of the cylinder. Volume of spherical shell = volume of the cylinder

$$\frac{4}{3}\pi\,(5^3 - 3^3) = \pi r^2 \times \frac{8}{3}$$

$$125 - 27 = 2r^2$$

$$\frac{98}{2} = r^2 \Rightarrow r = 7 \text{ cm}$$

Diameter = $2 \times 7 = 14$ cm

19. **(c)**

Diagonal of the cuboid = $\sqrt{l^2 + b^2 + h^2}$

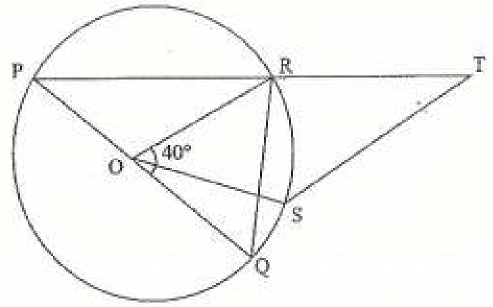
$$\sqrt{251} = \sqrt{l^2 + 9^2 + 7^2}$$

$$251 = l^2 + 81 + 49$$

$$l^2 = 251 - 130 = 121$$

$$l^2 = 11^2 \Rightarrow l = 11$$

20. **(d)**



$$\angle RQS = \frac{1}{2}\angle ROS$$

$$= \frac{1}{2} \times 40° = 20°$$

In ΔRQT,

$\angle QRT + \angle RQS + \angle RTQ = 180°$

$90° + 20° + \angle RTQ = 180°$

$110° + \angle RTQ = 180°$

$\angle RTQ = 180° - 110°$

$\angle RTQ = 70°$

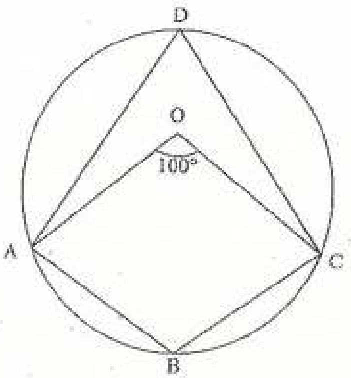
21. **(d)**

$$\angle ADC = \frac{1}{2} \times \angle AOC$$

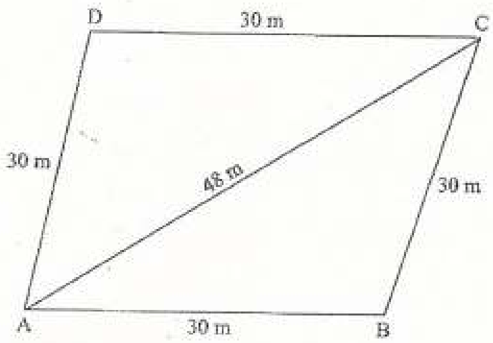
$$= \frac{1}{2} \times 100° = 50°$$

$\angle ABC = \frac{1}{2}(360° - 100°)$

$= \frac{1}{2} \times 260° = 130°$



22. **(c)**



$\Delta ABC \cong ADC$

Area ΔABC = area ΔADC

For ΔABC,

$5 = \frac{48+30+30}{2} = 54$

Area ΔABC =

$\sqrt{54(54-48)(54-30)(54-30)} = 432\ m^2$

Area of rhombus ABCD = 2 × 432 = 864 m^2

Area of grass field for each cow = $\frac{864}{18}$

= 48 m^2

23. **(b)**

$f(x) = x^4 - 2x^3 + 3x^2 - ax + b$

$f(1) = (1)^4 - 2(1)^3 + 3(1)^2 - a(1) + b = 5$

$f(1) = 1 - 2 + 3 - a + b = 5$

$2 - a + b = 5$

$-a + b = 3$ ....(1)

$f(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 19$

$1 + 2 + 3 + a + b = 19$

$a + b = 13$ ....(2)

From (1) & (2)

$2b = 16 \Rightarrow b = 8$

$a = 5$

$f(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$

$f(2) = 2^4 - 2(2)^3 + 3(2)^2 - 5(2) + 8$

$= 16 - 16 + 12 - 10 + 8 = 10$

24. **(b)**

$\frac{3+\sqrt{7}}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = a + b\sqrt{3}$

$\frac{9+7+2\times3\times\sqrt{7}}{9-7} = a + b\sqrt{3}$

$\frac{16+6\sqrt{7}}{2} = a + b\sqrt{3}$

$8 + 3\sqrt{7} = a + b\sqrt{3}$

$\Rightarrow a = 8 \quad b = 3$

Difference of a & $b = 8 - 3 = 5$

28. **(d)**

$9^{x+2} = 240 + 9^x$

$9^x . 9^2 = 240 + 9x$

$81 \times 9^x - 9^x = 240$

$9^x(81-1) = 240 \Rightarrow 9^x = \frac{240}{80} = 3$

$(3)^x = 3 \Rightarrow 3^{2x} = 3$

$2x = 1 \Rightarrow x = \frac{1}{2}$

29. **(c)**

$a + b + c = 9$

$a^2 + b^2 + c^2 = 35$

$a^3 + b^3 + c^3 - 3abc = ?$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$9^2 = 35 + 2(ab + bc + ca)$

$(ab + bc + ca) = \frac{81-35}{2} = \frac{46}{2} = 23$

$a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)\,[a^2 + b^2 + c^2 - (ab + bc + ca)]$

$= 9 \times [35 - 23] = 9 \times 12 = 108$

30. **(b)**

$a^2 + \frac{1}{a^2} = 102$

$\left(a - \frac{1}{2}\right)^2 = a^2 + \frac{1}{a^2} - 2 = 102 - 2$

$a - \frac{1}{a} = \sqrt{100} = 10$

31. **(a)**

$\frac{a}{b} + \frac{b}{a} = -1 \Rightarrow \frac{a^2 + b^2}{ab} = -1$

$a^2 + b^2 + ab = 0$

$a^3 - b^3 = (a-b)\,(a^2 + b^2 + ab)$
$= (a-b) \times 0 = 0$

32. **(b)**

$a + b + c = 0$

$\frac{(b+c)^2}{3bc} + \frac{(c+a)^2}{3ac} + \frac{(a+b)^2}{3ab}$

$\frac{(-a)^2}{3bc} + \frac{(-b)^2}{3ac} + \frac{(-c)^2}{3ab}$

$= \frac{a^3 + b^3 + c^3}{3abc} = \frac{3abc}{3abc} = 1$

$[a + b + c = 0 \text{ then } a^3 + b^3 + c^3 = 3abc]$

33. **(a)**

$x^2 + x = 5 \Rightarrow x^2 + 2 \times x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 5$

$\left(x + \frac{1}{2}\right)^2 = 5 + \frac{1}{4} = \frac{21}{4}$

$x + \frac{1}{2} = \frac{\sqrt{21}}{2} = x = \frac{\sqrt{21}-1}{4}$

$x + 3 = \frac{\sqrt{21}-1}{2} + 3 = \frac{\sqrt{21}-1+6}{2}$

$= \frac{\sqrt{21}+5}{2}$

$(x+3)^3 = \left(\frac{\sqrt{21}+5}{2}\right)^3$

$= \frac{(\sqrt{21})^3 + 5^3 + 3 \times \sqrt{21} \times 5\,(\sqrt{21}+5}{8}$

$= 55 + 12\sqrt{21}$

$\frac{1}{(x+3)^3} = 55 - 12\sqrt{21}$

$(x+3)^3 + \frac{1}{(x+3)^3}$

$= 55 + 12\sqrt{21} + 55 - 12\sqrt{2}$
$= 110$

34. **(a)**

Let $g(x) = x + 1 = 0 \Rightarrow x = -1$

$f(x) = x^{51} + 51$

$f(-1) = (-1)^{51} + 51 = -1 + 51 = 50$

35. **(b)**

Curved surface area of cylinder

$= \frac{1}{3} \times$ total surface area of cylinder

$2\varpi rh = \frac{1}{3} \times 462$

$2\varpi rh = 154$ .... (1)

Total surface area = 462

$2\varpi rh + 2\varpi r^2 = 462$

$154 + 2\pi r^2 = 462 \Rightarrow 2\pi r^2 = 462 - 154$

$r^2 = \dfrac{308 \times 7}{2 \times 22} = 49$

$r = 7$ cm

$2\pi r h = 154 \Rightarrow h = \dfrac{154 \times 7}{2 \times 22 \times 7} = \dfrac{7}{2}$

Volume of cylinder $= \pi r^2 h$

$= \dfrac{22}{7} \times 7^2 \times \dfrac{7}{2}$

$= 11 \times 49 = 539 \text{ cm}^3$

36. **(b)**

$\dfrac{x_1 + x_2 + \ldots\ldots\ldots\ldots x_{16}}{16} = 8$

$x_1 + x_2 + x_3 \ldots\ldots\ldots\ldots\ldots + x_{16} = 16 \times 8 = 128$

New Mean =

$\dfrac{(x_1+2)+(x_2+2)+\ldots\ldots\ldots+(x_{16}+2)}{16}$

$= \dfrac{(x_1 + x_2 + \ldots\ldots\ldots\ldots x_{16}) + 2 \times 16}{16}$

$= \dfrac{128 + 32}{16} = \dfrac{160}{16} = 10$

37. **(b)**

Let r be the radius of the base of the cone and h be the height.

r = radius of hemisphere

Volume of cone = volume of hemisphere

$\dfrac{1}{3}\pi r^2 h = \dfrac{2}{3}\pi r^3$

$h = 2r$

$h : r = \dfrac{2r}{r} = 2 : 1$

38. **(a)**

Surface area of sphere = 5544

$4\pi r^2 = 5544 \Rightarrow r^2 = \dfrac{5544 \times 7}{4 \times 22} = 441$

$r = 21$

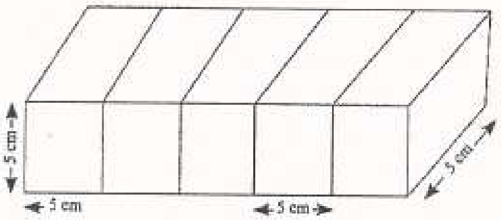
Volume of the sphere $= \dfrac{4}{3}\pi r^3$

$= \dfrac{4}{3} \times \dfrac{22}{7} \times 21^3$

$= \dfrac{4}{3} \times \dfrac{22}{7} \times 21 \times 21 \times 21$

$= 88 \times 441 = 38808 \text{ cm}^2$

39. **(c)**



Length of cuboid = 5 × 5 = 25 cm

Breadth = 5 cm; height = 5 cm

Surface area of cuboid = $2(lb + bh + lh)$

$= 2(25 \times 5 + 5 \times 25 \times 5) = 2(125 + 25 + 125)$

$= 2 \times 275 = 550 \text{ cm}^2$

40. **(b)**

$\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \dfrac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})} \times \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

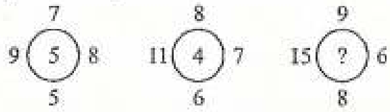
$= \dfrac{5+3+2\sqrt{15}}{5-3} = \dfrac{8+2\sqrt{15}}{2}$

$= 4 + \sqrt{15} = a + \sqrt{15}\, b$

So, $a = 4$, b = 1

Model Test Paper - 2

1. In the given question, which number will replace the question mark?



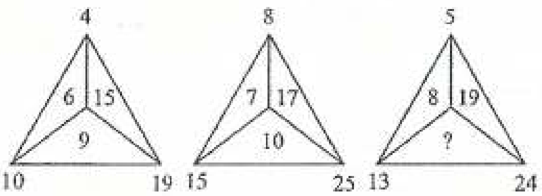
(a) 3 (b) 4
(c) 5 (d) 6

2. In the given matrix the value of A , B, C respectively are

9	A	12
B	10	7
8	C	11

(a) $A = 13, B = 14, C = 6$
(b) $A = 14, B = 18, C = 16$
(c) $A = 16, B = 13, C = 15$
(d) $A = 14, B = 16, C = 18$

3. Which number will replace the question mark?

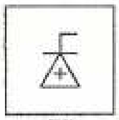


(a) 10 (b) 12
(c) 11 (d) 13

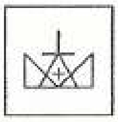
4. A flatoxin is related to food poisoning in the same way as histamine is related to _____?

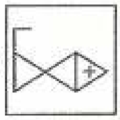
(a) Head ache (b) Inhabited
(c) Anthrax (d) Allergy

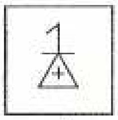
5. Find the figure which contains the figure (X) as its embedded part

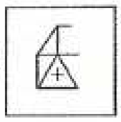


(X)

(a) 

(b) 

(c) 

(d) 

6. If $8^{x-1} = 64$ then what is the value of 3^{2x+1}?

(a) 1 (b) 3 (c) 9 (d) 27

7. If $\frac{a}{b}+\frac{b}{a} = -1$ then what is the value of $a^3 - b^3$?

(a) 0 (b) $\frac{1}{2}$ (c) -1 (d) 1

8. In the given figure if $\frac{y}{x} = 5$ and $\frac{z}{x} = 4$ then what is the value of x?

(a) 12° (b) 15° (c) 8° (d) 18°

9. The abscissa of a point is positive in the

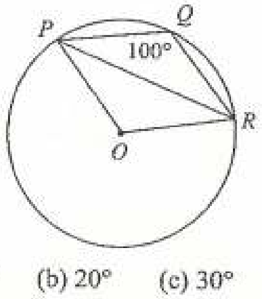
(a) First & fourth quadrant
(b) First & second quadrant
(c) Second & Third quadrant
(d) Third & fourth quadrant

10. Diagonals of a quadrilateral $ABCD$ bisect each other if $\angle A = 45°$ what is the value of $\angle B$?

(a) 135° (b) 120° (c)115° (d) 125°

11. $ABCD$ is rectangle with O as any point in its interior. If area(ΔAOD) = 3 cm^2, area

12. In the given figure what is the $\angle POR$?



(a) 10° (b) 20° (c) 30° (d) 80°

13. If the mean of a, b, c, d, e is 28 the mean of b and d is 34, then what is the mean of a, c and e ?

(a) 22 (b) 24 (c) 32 (d) 28

14. In a football match, a player makes 4 goals from 10 penalty kicks. The probability of converting a penalty kick in to a goal by player is

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

15. The ratio of the volume of a right circular cylinder and a right circular cone of the same height and base is

(a) 1 : 3 (b) 3 : 4 (c) 1 : 2 (d) 3 : 1

16. If $a + b + c = 0$ then what is the value of $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{cd}$?

(a) 2 (b) 3 (c) 4 (d) 5

17. An angle is 14° more than its complementing angle, then what is its measure?

(a) 48° (b) 38° (c) 58° (d) 28°

18. The surface area of sphere of radius 5 cm is five times the area of the curved surface of a
18. cone of radius 4 cm. What is the height of the cone?

(a) 3 cm (b) 4 cm (c) 2cm (d) 5cm

19. The sides of a triangle are 11 am, 60 cm and 61 cm. What is the length of altitude to the smallest side?

(a) 66 cm (b) 60 cm
(c) 11cm (d) 50 cm

20. If $x^{140} + 2x^{151} + k$ is divisible by $x + 1$, then what is the value of k?

(a) 2 (b) 3
(c) 1 (d) 4

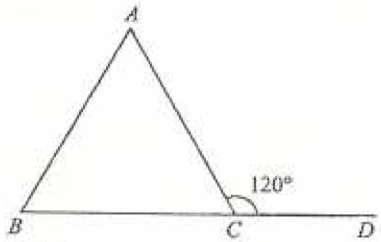
21. If $(a^2 + b^2 + ab - a + b + 1)$ is the one factor of $a^3 - b^3 + 1 + 3ab$, then what is the other factor ?

(a) $a - b + 1$ (b) $a + b - 1$
(c) $b - a - 1$ (d) None of these

22. If $x^2 + \frac{1}{x^2} = 23$ then what is the value of $x + \frac{1}{x}$?

(a) 4 (b) 5 (c) 6 (d) 3

23. In ΔABC, $AB = AC$ and $\angle ACD = 120°$ what is the value of $\angle A$?



(a) 90° (b) 60°
(c) 70° (d) 50°

24. Probability of an event can be

(a) $\frac{11}{9}$ (b) – 0.7

(c) 1.001 (d) 0.6

25. The perimeter of a circle is equal to the perimeter of a square. Then what is the ratio of their areas respectively is

(a) 4 : 1 (b) 22 : 7
(c) 11 : 7 (d) 14 : 11

26 A group of students decided to collect as many paisa from each member of the group as the number of members. If the total collection amounts to 59.29 what is the number of members in the group?

(a) 77 (b) 87
(c) 67 (d) 57.

27. In how many years will a sum of ₹ 800 at 10% per annum compounded half yearly becomes ₹ 926.10?

(a) $1\frac{1}{3}$ (b) $1\frac{1}{2}$
(c) $2\frac{1}{3}$ (d) $2\frac{1}{2}$

28. What is the remainder when $9x^3 - 3x^2 + x - 5$ is divided by $x - \frac{2}{3}$?

(a) 3 (b) 2 (c) – 3 (d) –2

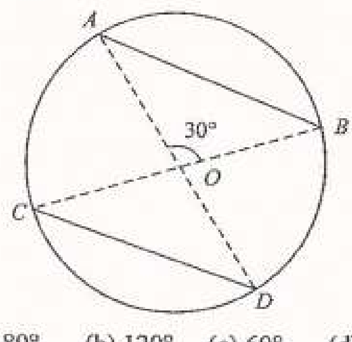
29. If $4^{44} + 4^{44} + 4^{44} + 4^{44} = 4^x$, then what is the value of x?

(a) 45 (b) 44
(c) 172 (d) 11

Model Test Paper - 2

1. Pride is related to Humility in the same way as Desire is related to ________ ?
   (a) Hate (b) Wish
   (c) Eagerness (d) Indifference
2. 12 years old Manoj is three times as old as his brother Saroj. How old will Manoj be when he is twice as old as Saroj ?
   (a) 20 years (b) 18 years
   (c) 16 years (d) 14 years
3. Fill in the blanks by proper number. 13,32,24,43,35,54,46-----?------
   (a) 65 (b) 63
   (c) 62 (d) 64

   If DELHI is coded as 73541 and CALCUTTA as 82589662. How can CALICUT be coded ?
   (a) 8251896 (b) 8543691
   (c) 5978213 (d) 5279431
4. Find the missing number.
   I. 3(30)7 II. 8(51)9
   III. 12(?) 17 IV. 15 (99) 18
   (a) 57 (b) 87
   (c) 78 (d) 93
5. AB & CD are two equal chords of O circle with centre O such that $\angle AOB = 80°$, Find $\angle COD$.

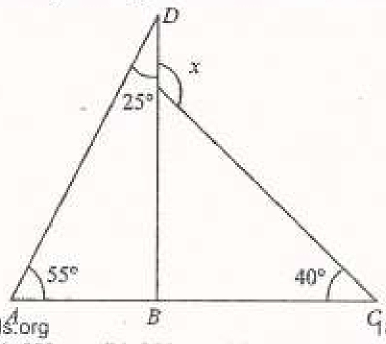


   (a) 80° (b) 120° (c) 60° (d) 100°

6. An equilateral triangle of side 9 cm is inscribed in a circle. What is the radius of the circle?
   (a) $3\sqrt{2}$ cm (b) $3\sqrt{3}$ cm
   (c) $4\sqrt{3}$ cm (d) $\sqrt{3}$ cm
7. The height of a cylinder is 14 cm and its curved surface area is 264 cm^2. What is the volume of the cylinder?
   (a) 396 cm^3 (b) 496 cm^3
   (c) 1848 cm^3 (d) 1232 cm^3
8. The mean of the following data is 8, then find the value of P .

X	3	5	7	9	11	13
Y	6	8	15	P	8	4

   (a) 23 (b) 24 (c) 25 (d) 21
9. In a group of 60 persons 35 like coffee. Out of this group if one person is chosen at random. What is the probability that he or she does not like coffee?
   (a) $\frac{7}{12}$ (b) $\frac{5}{12}$ (c) $\frac{3}{7}$ (d) $\frac{5}{7}$
10. The angles of a quadrilateral are in the ratio 1:3:5:6. What is the different between smallest and largest angle?
   (a) 90° (b) 110° (c) 120° (d) 100°
11. If the point A (3, 5) and B (1, 4) lie on the graph of the line $ax + by = 7$, then what is the sum of a and b?
   (a) 0 (b) 1 (c) 2 (d) 3
12. In the given figure, find the value of x.



   (a) 65° (b) 80° (c) 120° (d) 100°

13. If $(x + a)$ is the factor of $x^3 + ax^2 - 2x + a + 4$ then what is the value of a?

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $-\frac{4}{3}$ (d) $-\frac{1}{3}$

14. When $x^3 - ax^2 + x$ is divided by $x - a$, then what is the remainder ?

(a) 0 (b) $2a$ (c) a (d) $3a$

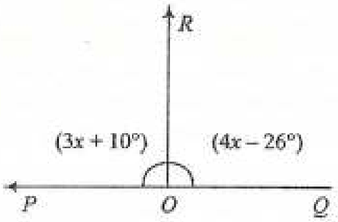
15. What is the remainder when $x^{31} + 31$ is divided by $x + 1$?

(a) 0 (b) 1 (c) 30 (d) 31

16. $\sqrt{7} = 2.646$ then what is the value of $\frac{1}{\sqrt{7}}$?

(a) 0.375 (b) 0.441
(c) 0.378 (d) 0.384

17. In the given figure POQ is a straight line. If $\angle POR = 3x + 10$, $\angle QOR = 4x - 26$ then what is the value of $\angle POR$?



(a) 94° (b) 86° (c) 84° (d) 76°

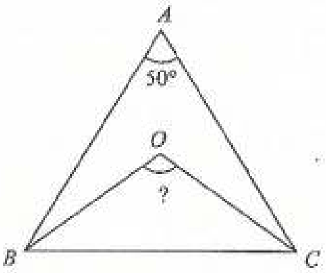
18. An angle is one fifth of its supplement. What is the value of that angle?

(a) 15° (b) 30°
(c) 150° (d) 75°

19. If $3\angle A = 4\angle B = 6\angle C$ then what is the $A : B : C$?

(a) 6 : 4 : 3 (b) 2 : 3 : 4
(c) 4 : 3 : 2 (d) 3 : 4 : 6

20. In the given figure BO & CO are the bisectors of angles $\angle B$ & $\angle C$ respectively. If $\angle A = 50°$, then what is the value of $\angle BOC$?



(a) 100° (b) 120°
(c) 115° (d) 130°

21. In ΔABC, $\angle A = 40°$, $\angle B = 60°$, then which is the longest side of ΔABC ?

(a) AB
(b) AC
(c) BC
(d) Cannot be determined

22. If O is any point in the interior of ΔABC, then which of the following option is correct?

(a) $(OA + OB + OC) > (AB + BC + CA)$

(b) $(OA + OB + OC) > \frac{1}{2}(AB + BC + CA)$

(c) $(OA + OB + OC) < \frac{1}{2}(AB + BC + CA)$

(d) None of these

23. Which of the following points does not lie on the line $y = 3x + 4$?

(a) (1, 7) (b) (2, 10) (c) (–1, 1) (d) (4, 12)

24. The perpendicular distance of the point (4, 3) from the y-axis is

(a) 3units (b) 4 units
(c) 5 units (d) 7 units

25. The difference between the semi– perimeter and the sides of ΔABC are 8 cm, 7 cm and 5 cm respectively. What is the area of triangle?

(a) $20\sqrt{7}$ (b) $20\sqrt{4}$ cm^2
(c) $10\sqrt{14}$ cm^2 (d) None of these

26. The probability of guessing the correct answer to a certain test question is $\frac{p}{2}$ if the probability of not guessing the correct answer

27. to this question is $\frac{2}{3}$, what is the value of p?

(a) $\frac{2}{3}$ (b) 2 (c) $\frac{1}{3}$ (d) 3

28. If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ then find the value of $x^2 + y^2$?

(a) 88 (b) 68
(c) 32 (d) 40

29. A library has an average of 510 visitors on Sundays and 240 on other days. The best estimate average number of visitors per day in a month of 30 days beginning with a Sunday is

(a) 280 (b) 285
(c) 276 (d) 270

30. If the radius of a circle is decreased by 50%, what is percentage decrease in its area?

(a) 70% (b) 75%
(c) 80% (d) 60%

If $5^{55} + 5^{55} + 5^{55} + 5^{55} + 5^{55} = 5^x$ then x is

(a) 54 (b) 55
(c) 56 (d) 176

ANSWER KEY TEST 1

1.(b)	2.(b)	3.(c)	4.(d)	S.(d)	6.(d)
7.(a)	8.(d)	9.(a)	IO.(a)	II.(c)	12(a)
13.(b)	14.(b)	IS.(a)	16.(b)	17.(b)	IB.(a)
19.(b)	20.(c)	21.(a)	22.(b)	23.(b)	24.(d)
25.(d)	26.(a)	27.(b)	28.(c)	29.(a)	

ANSWER KEY TEST 2

I.(a)	2.(c)	3.(a)	4.(a)	S.(b)	6.(a)
7.(b)	B.(a)	9.(c)	IO.(b)	Il.(c)	12(b)
13.(c)	14.(c)	IS.(c)	16.(c)	17.(c)	18.(a)
19.(c)	20.(	21.(c)	22.(a)	23.(b)	24.(d)
2S.(d)	26.()	27.(a)	28.(b)	29.(b)	30{b)