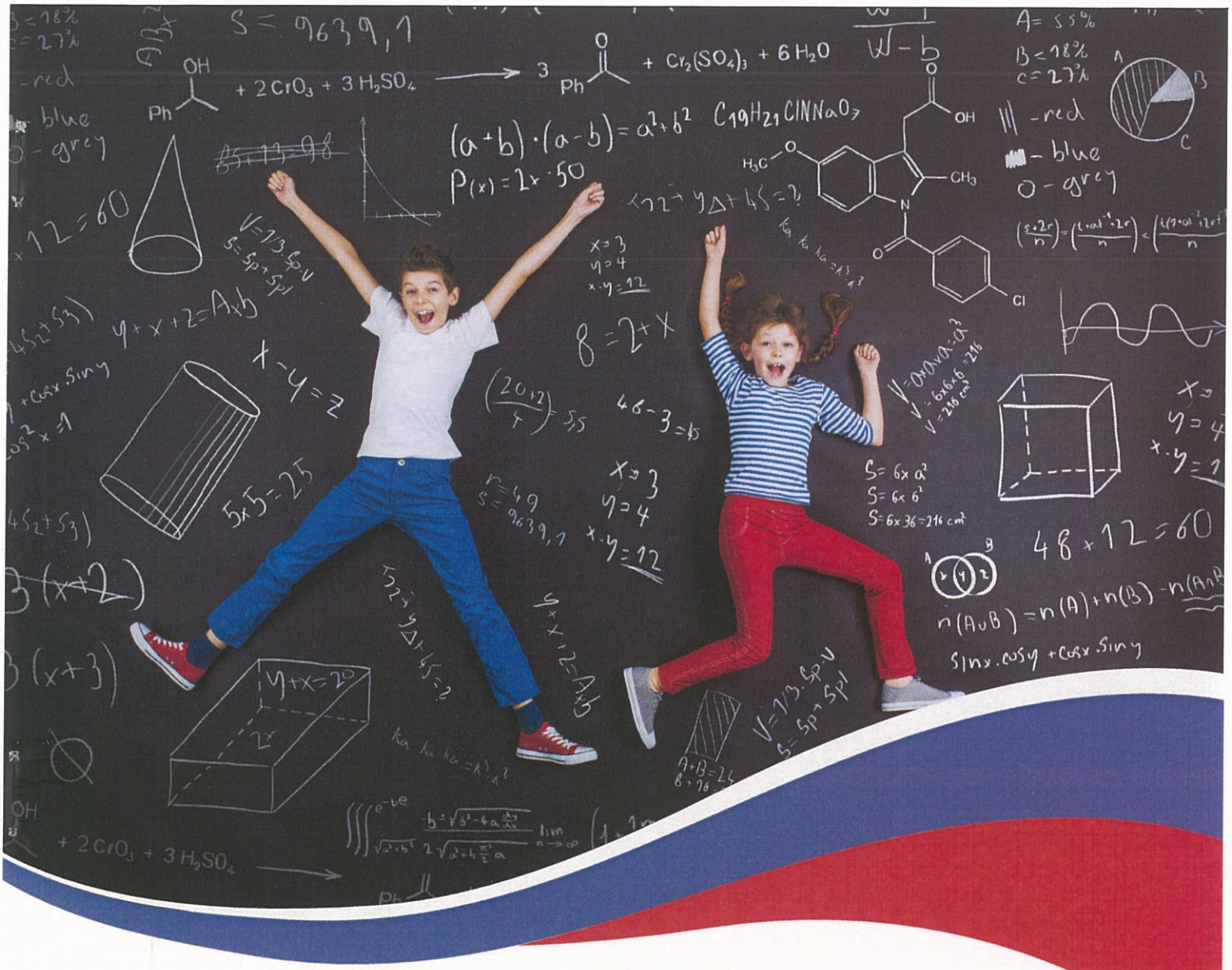

GRADE 10

MATHEMATICS OLYMPIAD

Official Guide

International Olympiad Foundation

International Olympiad Foundation

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1. Real Numbers

Key Points

Natural Numbers $N = \{1, 2, 3, 4, \ldots\ldots\}$

Whole Numbers $W = \{0, 1, 2, 3, 4, \ldots\ldots\}$

Integers $Z = \{\ldots\ldots -3, -2, -1, 0, 1, 2, 3, 4, \ldots\ldots\}$

Rational Numbers $Q = \{\frac{p}{q}, q \neq 0, p\ q \text{ are co-prime}\}$

These numbers when expressed as decimal representation are either finite or infinite recurring.

Irrational Numbers: Those which are not rational e.g. $\sqrt{3}, \sqrt{5}, \ldots\ldots \pi$, etc.

These numbers when expressed as decimal representation are infinite and non recurring.

Real Numbers: Collection of all above numbers i.e. N, W, Z, Q and irrational numbers.

All numbers can be represented on the number line.

Euclid's Division Algorithm: This is based on Euclid's division lemma.

Euclid's Division lemma: Given positive integers a and b, there exist unique integers q and r such that $a = bq + r$, $0 \leq r < b$

According to this the H.C.F. of any two positive integers a and b with $a > b$ is obtained as follows:

Step 1. Apply the division lemma to find q and r, where $a = bq + r$, $0 \leq r < b$

Step 2. If $r = 0$ then H.C.F. is b. If $r \neq 0$, apply Euclid's lemma to b and r.

Step 3. Continue the process till the remainder is zero. The divisor at this stage will be H.C.F. (a, b). Also H.C.F. (a, b) = H.C.F. (b, r).

Example: If $a = 117$, $b = 45$

By Euclid's division lemma,

$$117 = 45 \times 2 + 27$$

$$a = bd_1 + r_1,$$

where $d_1 = 2$, $r_1 = 27$

We observe that common divisor of $a = 117$ and $b = 45$ are also common divisor of $b = 45$ and $r_1 = 27$ and vice versa

$$45 = 27 \times 1 + 18$$

$b = d_2 r_1 + r_2$ where $d_2 = 1$ and $r_2 = 18$

We observe that common divisor of $r_1 = 27$ and $r_2 = 18$ are the common divisor of $b = 45$ and $r_1 = 27$ and vice versa

Applying Euclid's division lemma on

$r_1 = 27$ and $r_2 = 18$

$$27 = 18 \times 1 + 9$$

$r_1 = d_3 r_2 + r_3$, where $d_3 = 1$, $r_3 = 9$

$r_2 = 18$ and $r_3 = 9$ are common divisor of $a = 117$ and $b = 45$ and vice versa

$$18 = 9 \times 2 + 0$$

$r_3 = 9$ is a divisor of $r_2 = 18$ and $r_3 = 9$

$\therefore$ H.C.F. $= 9$

Example 1: Use Euclid's division algorithm to find H.C.F. of 4052 and 12576.

(a) 3 (b) 2 (c) 1 (d) 4

Solution: **(a)** $12576 = 4052 \times 3 + 400$

$4052 = 420 \times 9 + 272$

$420 = 272 \times 1 + 148$

$272 = 148 \times 1 + 124$

$148 = 124 \times 1 + 24$

$124 = 24 \times 5 + 4$

$24 = 4 \times 6 + 0$

Divisor at the last stage or remainder at the earlier stage is the H.C.F i.e, 4.

Example 2: Find the H.C.F. of 65 and 117 and express it in the form $65m + 117n$.

(a) 13 (b) 11 (c) 10 (d) 5

Solution: **(a)**

Here $117 = 65 \times 1 + 52$

$65 = 52 \times 1 + 13$

$52 = 13 \times 4 + 0$

H.C.F. of 65 and 117 = 13

Now $13 = 65 - 52 \times 1$

$13 = 65 - (117 - 65 \times 1)$

$13 = 65 - 117 + 65 \times 1$

$13 = 65 \times 2 + 117\,(-1)$

$13 = 65 \times 2 - 117$

$13 = 65m + 117n$, where $m = 2$, $n = -1$

Example 3: Find the largest number that divides 2053 and 967 and leaves a remainder of 5 and 7 respectively.

(a) 64 (b) 60 (c) 55 (d) 99

Solution: **(a)**

Here $2053 - 5 = 2048$

$967 - 7 = 960$

Required number = H.C.F. of 2048 and 960

960) 2048 (2
1920
128) 960 (7
896
64) 128 (2
128

The largest number = 64

If x and y are two numbers then the product of L.C.M. and H.C.F. is equal to product of these numbers i.e., xy

Example 4: If the H.C.F. of 592 and 252 is 7, then what is their L.C.M.?

(a) 21312 (b) 21000 (c) 21311 (d) 21310

Solution: **(a)**

$$\text{L.C.M.} \times \text{H.C.F} = 592 \times 252$$

$$\Rightarrow \quad \text{L.C.M.} \times 7 = 592 \times 252$$

$$\Rightarrow \quad \text{L.C.M.} = \frac{592 \times 252}{7} = 592 \times 36$$

$$\Rightarrow \quad = 21312$$

Fundamental Theorem of Arithmetic

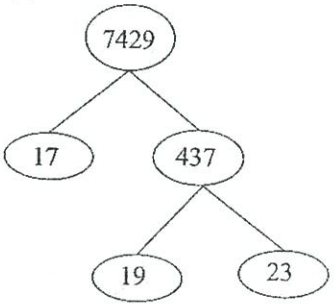
Every composite number can be expressed as a product of primes and this factorization is unique apart from the order in which the prime factors occur.

Revisiting Irrational Numbers: Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

Example 5: Express 7429 as the product of prime factors.

(a) $21 \times 17 \times 13$ (b) $17 \times 19 \times 23$ (c) $51 \times 19 \times 29$ (d) None of these

Solution: **(b)**



$7429 = 17 \times 19 \times 23$

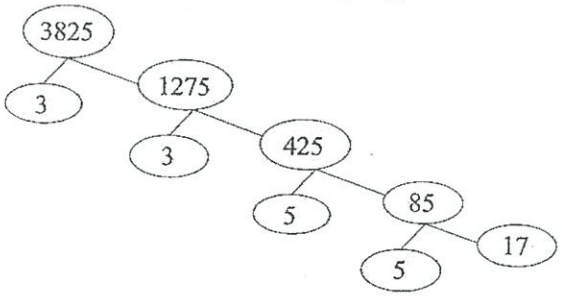
Example 6: Express 3825 as the product of prime factors.

(a) $3 \times 3 \times 5 \times 17$ (b) $3 \times 3 \times 5 \times 5 \times 17$

(c) $3 \times 5 \times 17$ (d) None of these

Solution: **(b)**

$3825 = 3 \times 3 \times 5 \times 5 \times 17$



Revisiting Rational Numbers and their Decimal Expansions

I. Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form $\frac{P}{d}$, where P and d are co–primes. The Prime factorization of d is of the form $2^m \times 5^n$, where m and n are non–negative integers.

Example 7: Find the decimal expansion of $\frac{189}{125}$.

(a) 1.5 (b) 1.4 (c) 1.1 (d) None of these

Solution: **(a)** $\frac{189}{125} = \frac{189}{5^3} = \frac{2^3 \times 189}{2^3 \times 5^3}$

$= \frac{8 \times 189}{(2 \times 5)^3} = \frac{1512}{10^3} = 1.512$

Example 8: Find decimal expansion of $\frac{7}{8}$.

(a) 8.5 (b) 0.875 (c) 8.4 (d) 0.878

Solution: **(b)** $\frac{7}{8} = \frac{7}{2^3} = \frac{7 \times 5^3}{2^3 \times 5^3}$

$= \frac{7 \times 125}{(2 \times 5)^3} = \frac{875}{10^3}$

$= 0.875$

II. Let $x = \frac{P}{d}$ be a rational number such that the prime factorization of d is of the form $2^m \times 5^n$ where m and n are non–negative integers then x has a decimal expansion which terminates after K places of decimals, where K is the larger of m and n.

Example 9: At how many places of decimal $\frac{13}{3125}$ terminates?

(a) 5 (b) 4 (c) 3 (d) 2

Solution: **(a)** $\frac{13}{3125} = \frac{13}{2^0 \times 5^5}$

So, it has terminating decimal expansion which terminates after 5 places of decimal.

III. If $x = \frac{P}{d}$ be a rational number such that the prime factorization of d is of the form $2^m \times 5^n$, where m and n are non-negative integers, then x has a decimal expansion which is non-terminating repeating.

Example 10: Is $\frac{64}{455}$ non-terminating repeating?

(a) yes (b) no (c) can't say (d) none of them

Solution: **(a)** $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

As 455 is not of the form $2^m \times 5^n$

So, the decimal expansion of $\frac{64}{455}$ is non-terminating repeating

Multiple Choice Questions

1. What is the largest number that divides 445, 572 and 699 having remainders 4, 5, 6 respectively?
   (a) 63 (b) 65
   (c) 62 (d) 64
2. If the H.C.F. of 408 and 1032 is expressed in the form of $1032m - 408 \times 5$. What is the value of m?
   (a) 4 (b) 2 (c) –2 (d) 3
3. What is the largest number which divides 615 and 963 leaving remainder 6 in each case?
   (a) 78 (b) 76
   (c) 87 (d) 83
4. If 5005 is expressed in the term of product of price factors, then which prime factor is the largest?
   (a) 5 (b) 11 (c) 13 (d) 17
5. What is the L.C.M. of 144, 180 and 192 by prime factorization method?
   (a) 2680 (b) 2780
   (c) 2880 (d) None of these
6. The H.C.F. of two numbers is 16 and their product is 3072. What is their L.C.M.?
   (a) 192 (b) 172
   (c) 152 (d) 186
7. What is the largest positive integer that will divide 398, 436 and 542 leaving remainder 7, 11 and 15 respectively?
   (a) 16 (b) 18
   (c) 17 (d) 14
8. The H.C.F of two numbers is 145, their L.C.M. is 2175. If one number is 725, then what is the other number?
   (a) 435 (b) 425
   (c) 415 (d) 465
9. What is the smallest number that when divided by 35, 56, and 91 leaves remainder 7 in each case?
   (a) 3847 (b) 3647
   (c) 3247 (d) 3547
10. If L.C.M. and H.C.F. of two rational numbers are equal then the numbers must be
    (a) Prime (b) Equal
    (c) Co–prime (d) Composite
11. $2 + \sqrt{2}$ is
    (a) Irrational (b) an integer
    (c) not real (d) rational
12. $1.2\overline{348}$ is
    (a) An integer
    (b) An irrational number
    (c) A rational number
    (d) None of these
13. Find the sum of the exponents of the prime factors in the prime factorization of 196.
    (a) 2 (b) 3 (c) 4 (d) 5
14. What is the H.C.F of 95 and 152?
    (a) 1 (b) 19 (c) 38 (d) 57
15. Find the smallest number by which $\sqrt{27}$ should be multiplied so as to get a rational number.
    (a) $\sqrt{27}$ (b) $3\sqrt{3}$
    (c) 3 (d) $\sqrt{3}$
16. If n is a natural number then $9^2n - 4^2n$ is always divisible by
    (a) 5 (b) 13
    (c) Both 5 and 13 (d) None of these
17. If n is any natural number then $6^n - 5^n$ always ends with
    (a) 3 (b) 1 (c) 7 (d) 5
18. Find L.C.M. of 42 and 63.
    (a) 120 (b) 126
    (c) 115 (d) 116
19. What is the sum of exponents of the prime factors in the prime factorization of 576?
    (a) 6 (b) 8 (c) 7 (d) 5
20. What is the difference of exponents of prime factors in prime factorization of 1225?
    (a) 1 (b) 2
    (c) 2 (d) None of these
21. Find the least number that is divisible by all the numbers between 1 and 10 both inclusive?
    (a) 2520 (b) 2320
    (c) 1920 (d) 2720

22. $5 - \sqrt{3}$ is
(a) Rational (b) Irrational
(c) Real (d) Integer

23. In the prime factorization of 13915 what is difference between largest factor and smallest factor?
(a) 18 (b) 23 (c) 17 (d) 15

24. If the H.C.F. of 210 and 55 is expressible in the form $210 \times 5 + 55y$ then what is the value of y?
(a) 19 (b) -19 (c) 15 (d) -15

25. The decimal expansion of the rational number $\frac{43}{2^4 \times 5^3}$ will terminate after
(a) 4 places (b) 2 places
(c) 3 places (d) 5 places

26. The product of H.C.F. and L.C.M. of the smallest prime number and smallest composite number is
(a) 2 (b) 6 (c) 4 (d) 8

27. Which of the following number has terminating decimal expansion?
(a) $\frac{17}{49}$ (b) $\frac{21}{2^3.5^6}$
(c) $\frac{89}{2^3.3^2}$ (d) $\frac{37}{45}$

28. What is product of H.C.F. and L.C.M. of the numbers 81 and 50?
(a) 900 (b) 4050
(c) 8100 (d) 2100

29. The decimal expansion of $\frac{147}{120}$ will terminate after how many places of decimal?
(a) 1 (b) 2
(c) 3 (d) None of these

30. If H.C.F. of 306 and 657 is 9, then what is the L.C.M. of 306 and 657?
(a) 22338 (b) 22318
(c) 22238 (d) 22118

31. If n is a natural number, then $9^{2n} - 4^{2n}$ is always divisible by _____.
(a) 5
(b) 13
(c) both (a) and (b)
(d) neither (a) nor (b)

32. N is a natural number such that when N^3 is divided by 9, it leaves remainder a. It can be concluded that
(a) a is a perfect square
(b) a is a perfect cube
(c) Both (a) and (b)
(d) Neither (a) nor (b)

33. If n is any natural number, then $6^n - 5^n$ always ends with _____.
(a) 1 (b) 3 (c) 5 (d) 7

34. Which of the following is always true?
(a) The rationalising factor of a number is unique
(b) The sum of two distinct irrational numbers is rational
(c) The product of two distinct irrational numbers is irrational
(d) None of these

35. Ashok has two vessels which contain 720 ml and 405 ml of milk respectively. Milk in each vessel is poured into glasses of equal capacity to their brim. Find the minimum number of glasses which can be filled with milk.
(a) 45 (b) 35 (c) 25 (d) 30

Answer Key

1. (a)	2. (b)	3. (c)	4. (c)	5. (c)	6. (a)	7. (c)	8. (a)	9. (b)	10 (b)
11. (a)	12. (c)	13. (c)	14. (b)	15. (d)	16. (c)	17. (b)	18. (b)	19. (b)	20. (c)
21. (a)	22. (b)	23. (a)	24. (b)	25. (a)	26. (d)	27. (b)	28. (b)	29. (c)	30. (a)
31. (c)	32. (c)	33. (a)	34. (d)	35. (c)					

Hints and Solutions

1. **(a)** Here $445 - 4 = 441$; $572 - 5 = 567$, $699 - 6 = 693$

Now we have to find H.C.F. of 441 , 567, 693.

2. **(b)** Given H.C.F. of 408 and $1032 = 24$ and $1032m - 408 \times 5 = 24$

$\Rightarrow \quad 1032m = 24 + 2040$

$\Rightarrow \quad 1032m = 2064$

$\Rightarrow \quad m = \dfrac{2064}{1032} = 2$

3. **(c)** Here $615 - 6 = 609$; $963 - 6 = 957$

$\therefore$ H.C.F. of 609 and $957 = 87$

4. **(c)** $5005 = 5 \times 7 \times 11 \times 13$

5. **(c)** L.C.M. of 144, 180, 192

$144 = 2^4 \times 3^2$

$180 = 2^2 \times 3^2 \times 5$

$192 = 2^6 \times 3$

$\therefore$ L.C.M. $= 2^6 \times 3^2 \times 5 = 2880$

6. **(a)** L.C.M. $= \dfrac{\text{product of numbers}}{\text{H.C.F. of numbers}}$

$= \dfrac{3072}{16} = 192$

7. **(c)** $398 - 7 = 391$; $436 - 11 = 425$;

$542 - 15 = 527$

Required number = H.C.F. of 391, 425 and 527

$= 17$

8. **(a)** Other number $= \dfrac{\text{H.C.F.} \times \text{L.C.M.}}{\text{one numbers}}$

$= \dfrac{145 \times 2175}{725} = 435$

9. (b) L.C.M. of 35, 56, 91 = 3640

Remainder = 7

Required number = $3640 + 7 = 3647$

13. **(c)** We have $196 = 2^2 \times 7^2$

$2 + 2 = 4$

14. **(b)**. H.C.F. of 95 and 152 = 19

15. **(d)** $\sqrt{27} \times \sqrt{3} = \sqrt{81} = 9$

17. **(b)** $6^n - 5^n = 6^1 - 5^1 = 1$

$6^2 - 5^2 = 36 - 25 = 11$ and so on

18. **(b)** $324 = 2^2 \times 3^2$

19. **(b)** $576 = 2^6 \times 3^4$

$6 + 2 = 8$

20. **(c)** Here $1225 = 5^2 \times 7^2$

$\therefore$ Required difference $= 2 - 2 = 0$

21. **(a)** L.C.M. of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 = 2520

22. **(b)**

23. **(a)** $13915 = 5 \times 11 \times 11 \times 23$

Required difference $= 23 - 5 = 18$

24. **(b)** H.C.F. of 210 and 55 = 5

$210 \times 5 + 55y = 5$

$\Rightarrow \quad 55y = 5 - 1050$

$\Rightarrow \quad 55y = -1045$

$\Rightarrow \quad y = \dfrac{1045}{55} = -19$

25. **(a)** $\dfrac{43}{2^4 \times 5^3} = \dfrac{43}{16 \times 125}$

26. **(d)** Smallest prime number = 2

Smallest composite number = 4

$\therefore$ Required product $= 2 \times 4 = 8$

27. **(b)** $\dfrac{21}{2^3 \times 5^6} = \dfrac{21}{8 \times 125 \times 125}$

28. **(b)** Required product = H.C.F. $\times$ L.C.M.

$= 81 \times 50 = 4050$

30. **(a)** L.C.M. $\times$ H.C.F. $= 306 \times 657$

$\Rightarrow \quad$ L.C.M. $\times 9 = 306 \times 657$

$\Rightarrow \quad$ L.C.M. $= \dfrac{306 \times 657}{9} = 22338$

31. **(c)** Given expression is in the form $a^2 - b^2$

$9^{2n} - 4^{2n}$ is divisible by both $(9 - 4)$ and $(9 + 4)$ i.e. 5 and 13.

33. **(a)** For any natural number n, 6^n and 5^n end with 6 and 5 respectively.

35. **(c)** Here H.C.F. of 720 and 405 is 45

$$
\begin{array}{r}
405\overline{)720}(1 \\
405 \\
\hline
315\overline{)405}(1 \\
315 \\
\hline
90\overline{)315}(3 \\
270 \\
\hline
45\overline{)90}(2 \\
90 \\
\hline
\times
\end{array}
$$

$$\text{Hence required number} = \frac{720}{45} + \frac{405}{45}$$

$$= \frac{1125}{45} = 25$$

2. Polynomials

Polynomial

An expression of the form $P(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$ where $a_n \neq 0$ is called a **polynomial** in x of degree n. Here $a_0, a_1, ... a_n$ are real numbers and each power of x is a non-negative integer.

Example:

$2x + 3$ is a polynomial of degree 1.

$x^2 + x + 2$ is a polynomial of degree 2.

Types of Polynomial

There are many types of polynomials.

1. Linear Polynomial

A polynomial of degree one is called a linear polynomial. It is of the form $ax + b$, $a \neq 0$.

Example: $2x + 5, \sqrt{2x-3}, x - \frac{2}{5}$ etc.

2. Quadratic Polynomial

A polynomial having degree 2 is called a quadratic polynomial.

It is of the form $P(x) = ax^2 + bx + c$, $a \neq 0$.

Example: $x^2 + x + 2, 3x^2 - \sqrt{2x} + 5$ etc.

3. Cubic Polynomial

A polynomial of degree 3 is called a cubic polynomial.

It is of the form $ax^3 + bx^2 + cx + d$, $a \neq 0$

Example: $x^3 - 2x^2 + 3x + 1, \sqrt{2}z^3 - z^2 + 2z + 5$

Value of a Polynomial at a given Point

If $P(x)$ is a polynomial in x and if α is any real number then the value obtained by putting $x = \alpha$ in $P(x)$ is called the value of $P(x)$ at $x = \alpha$.

Example 1: If $P(x) = 2x^2 - 3x + 5$ then find $P(-5)$ and $P(3)$.

Solution: We have $P(x) = 2x^2 - 3x + 5$

then $P(-5) = 2(-5)^2 - 3(-5) + 5$

$= 50 + 15 + 5 = 70$

and $P(3) = 2(3)^2 - 3(3) + 5$

$= 18 - 9 + 5 = 23 - 9 = 14.$

Zeros of a Polynomial

A real number α is called a zero of the polynomial $P(x)$ if $P(\alpha) = 0$.

Example 2: If $P(x) = x^2 - 2x - 3$, find the zeros of polynomial.

Solution: Given $P(x) = x^2 - 2x - 3$

$= x^2 - 3x + x - 3$

$= x(x-3) + 1(x-3)$

$= (x+1)(x-3)$

then $P(3) = 3^2 - 2(3) - 3 = 9 - 6 - 3$

$= 9 - 9 = 0$

$P(-1) = (-1)^2 - 2(-1) - 3$

$= 1 + 2 - 3 = 0$

So, 3 and -1 are zeros of the polynomial.

Relation between Zeros and Coefficients of a Quadratic Polynomial

If α and β are the zeros of $P(x) = ax^2 + bx + c$, $a \neq 0$ then

$$\alpha + \beta = \frac{-b}{a}$$

and $$\alpha\beta = \frac{c}{a}$$

A quadratic polynomial whose zeros are α and β is given by $x^2 - (\alpha + \beta)x + \alpha\beta$.

Example 3: Find the zeros of the polynomial.

$f(x) = x^2 + 7x + 12$.

Solution: We have $f(x) = x^2 + 7x + 12$

$= x^2 + 4x + 3x + 12$

$= x(x+4) + 3(x+4)$

$= (x+4)(x+3)$

$f(x) = 0$

$\Rightarrow (x+4)(x+3) = 0$

$\Rightarrow x + 4 = 0 \Rightarrow x = -4$

or $x + 3 = 0 \Rightarrow x = -3$

Example 4: Find the sum and product of zeros of the polynomial $2x^2 + 5x - 12$.

Solution: We have $f(x) = 2x^2 + 5x - 12$

then sum of zeros $= \frac{-5}{2}$

and product of zeros $= \frac{-12}{2} = -6$.

Example 5: Find the quadratic polynomial the sum and product of whose zeros are -5 and 6 respectively.

Solution: Given sum of zeros $= -5$

and product of zeros $= 6$

then required polynomial $= x^2 - (-5)x + 6 = x^2 + 5x + 6$.

Example 6: Find the quadratic polynomial whose zeros are $\frac{2}{3}$ and $\frac{-1}{4}$

Solution: Here sum of zeros $= \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12}$

$$= \frac{5}{12}$$

and product of zeros $= \frac{2}{3} \times \frac{-1}{4} = \frac{-1}{6}$.

Required quadratic polynomial $= x^2 - \left(\frac{5}{12}\right)x + \left(-\frac{1}{6}\right)$

$$= x^2 - \frac{5}{12}x - \frac{1}{6} = 12x^2 - 5x - 2.$$

Relation between the Zeros and Co-efficients of a Cubic Polynomial

If $P(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$ is a cubic polynomial.

and α, β, γ are the zeros of the polynomial, then

1. $\alpha + \beta + \gamma = \frac{-b}{a}$,
2. $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$
3. $\alpha\beta\gamma = \frac{-d}{a}$

A cubic polynomial whose zeros are α, β, γ is given by

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$

Example 7: Find a cubic polynomial whose zeros are -2, -3, and -1.

Solution: The required polynomial is

$$[x - (-2)]\,[x - (-3)]\,[x - (-1)]$$
$$= (x + 2)\,(x + 3)\,(x + 1)$$
$$= (x^2 + 3x + 2x + 6)\,(x + 1)$$
$$= (x^2 + 5x + 6)\,(x + 1)$$
$$= x^3 + x^2 + 5x^2 + 5x + 6x + 6$$
$$= x^3 + 6x^2 + 11x + 6$$

Example 8: Find a cubic polynomial whose zeros are α, β, γ such that $\alpha + \beta + \gamma = 4$, $\alpha\beta + \beta\gamma + \gamma\alpha = 1$, $\alpha\beta\gamma = -6$.

Solution:

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$
$$= x^3 - 4x^2 + x - (-6)$$
$$= x^3 - 4x^2 + x + 6$$

Division Algorithm for Polynomials

Division Algorithm: It states that given any polynomial $p(x)$ and any non-zero polynomial $g(x)$, there are polynomials $q(x)$ and $r(x)$ such that

$$p(x) = g(x)\,q(x) + r(x)$$

where $r(x) = 0$, or degree $r(x)$ < degree $g(x)$.

Example 9: What is the quotient, when $2x^2 + x - 15$ is divided by $x + 3$?

Solution:

$$x+3\overline{)2x^2+x-15}\,(2x-5$$
$$\underline{2x^2+6x}$$
$$5x-15$$
$$\underline{5x-15}$$
$$-\quad+$$

$\therefore$ Quotient $= 2x - 5$

Example 10: What is the dividend if divisor is $x^2 - 2x + 3$, quotient is $5x - 3$ and remainder -5?

Solution:

Dividend = Divisor × quotient + Remainder
$= (x^2 - 2x + 3)(5x - 3) + (-5)$
$= 5x^3 - 3x^2 - 10x^2 + 6x + 15x - 9 - 5$
$= 5x^3 - 13x^2 + 21x - 14$

Mutiple Choice Questions

1. What are the zeros of $abx^2 + (b^2 - ac)x - bc$?

   (a) $-\frac{b}{a}, \frac{c}{b}$ (b) $\frac{b}{c}, \frac{a}{b}$

   (c) $-\frac{b}{c}, \frac{a}{b}$ (d) None of these

2. If a and b are the zeros of $ax^2 + bx + c$, find the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$.

   (a) $\frac{3abc - b^3}{a^2c}$ (b) $\frac{b^3 - abc}{ac}$

   (c) $\frac{3abc + b^3}{ac^2}$ (d) $\frac{3abc + b^3}{ac^2}$

3. If α, β are the zeros of the polynomial $x^2 - 5x + P$ such that $\alpha - \beta = 1$. What is the value of P?

   (a) $\frac{1}{6}$ (b) 6 (c) -6 (d) 3

4. If the sum of squares of zeros of the quadratic polynomial $P(x) = x^2 - 8x + k$ is 40. What is the value of k?

   (a) 12 (b) 16 (c) 18 (d) 8

5. If α, β are the zeros of quadratic polynomial $6x^2 + x - 2$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

   (a) $\frac{12}{25}$ (b) $\frac{-25}{12}$

   (c) $\frac{1}{12}$ (d) $\frac{1}{25}$

6. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$?

   (a) $4x - 10$ (b) $14x - 10$

   (c) $2x - 10$ (d) $14x + 10$

7. Find the other two zeros of the polynomial $2x^4 - 3x^3 - 3x^2 + 6x - 2$ if its two roots are $\sqrt{2}$ and $-\sqrt{2}$.

   (a) 1 and $\frac{1}{2}$ (b) -1 and $\frac{1}{2}$

   (c) 2 and 1 (d) -2 and 1

8. What is the cubic polynomial in which the sum, sum of the products of its zeros taken at a time and product of its zeros as 2, -7, -14 respectively?

   (a) $k(x^3 - 2x^2 - 7x + 14)$

   (b) $k(x^3 - 2x^2 + 8x - 14)$

   (c) $k(x^3 + 2x^2 - 7x - 4)$

   (d) $k(x^3 - x^2 - x - 14)$

9. Find the zeros of the polynomial $x^3-5x^2-16x+80$ if two zeros are equal in magnitude but opposite in sign.

(a) 4, –4, 5 (b) 5, –5, 4
(c) 4, –4, 7 (d) 5, –5, 6

10. If the product of two zeros of the polynomial $2x^3+6x^2-4x+9$ is 3 then what is its third zero?

(a) $\frac{3}{2}$ (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $-\frac{3}{2}$

11. Find a cubic polynomial whose zeros are α, β, γ such that $\alpha+\beta+\gamma=6$, $\alpha\beta+\beta\gamma+\gamma\alpha=-1$ and $\alpha\beta\gamma=-30$.

(a) x^3-6x^2-x+30

(b) x^3+6x^2+x-30

(c) $x^3-x^2-6x+30$

(d) None of these

12. If α, β, γ are the zeros of the polynomial $2x^3+x^2-13x+6$. What is the value of $\alpha\beta\gamma$?

(a) 3 (b) $-\frac{1}{2}$ (c) –3 (d) $-\frac{7}{2}$

13. Find the polynomial which when divided by $-x^2+x-1$ gives a quotient $x-2$ and remainder 3.

(a) $-x^3+3x^2-3x+5$

(b) $-x^3-3x^2-3x-5$

(c) x^3-3x^2+3x-5

(d) None of these

14. What must be added to $f(x) = 4x^4+2x^3-2x^2+x-1$ so that the resulting polynomial is divisible by $g(x)=x^2+2x-3$?

(a) $61x-65$ (b) $-61x+65$
(c) $-61x-65$ (d) None of these

15. If the polynomial $6x^4+8x^3+17x^2+21x+7$ is divided by another polynomial $3x^2+4x+1$, the remainder comes out to be $ax+b$. What is the value of a and b?

(a) $a=1, b=2$ (b) $a=2, b=1$
(c) $a=-2, b=1$ (d) $a=-1, b=-2$

16. What is the cubic polynomial whose zeros are α, β, γ such that $\alpha+\beta+\gamma=4$. $\alpha\beta\gamma=-6$ and $\alpha\beta+\beta\gamma+\gamma\alpha=1$?

(a) x^3-4x^2+x+6

(b) x^3-2x^2-x+6

(c) x^3-4x^2+4x-6

(d) x^3-4x^2-x-6

17. Find a cubic polynomial whose roots are –2, –3, and –1.

(a) x^3-6x^2+9x+6

(b) $x^3+6x^2+11x+6$

(c) $x^3+6x^2-11x-6$

(d) None of these

18. Which quadratic polynomial has sum of whose zeros is –5 and product of its zero is –12?

(a) $x^2+5x-12$ (b) $x^2-5x+12$
(c) x^2-5x+6 (d) $x^2-10x+12$

19. If divisor is $2-x+x^2$ and quotient is $(3x-1)$ then what is the dividend if dividend is completely divisible?

(a) $3x^3-2x^2-7x+2$

(b) $3x^3-4x^2+7x-2$

(c) $3x^3-4x^2-7x+2$

(d) None of these

20. What are the zeros of x^2-2x-3?

(a) 1, –3 (b) –3, –1
(c) 3, 1 (d) 3, –1

21. Find the quadratic polynomial whose zeros are $\frac{2}{3}$ and $-\frac{1}{4}$.

(a) $\frac{1}{12}(4x^2-5x-2)$

(b) $\frac{1}{12}(12x^2-5x-2)$

(c) $\frac{1}{12}(12x^2-2x+5)$

(d) $\frac{1}{12}(12x^2+5x-2)$

22. If α, β are zeros of $2x^2+5x-10$ then what is the value of $\alpha\beta$?

(a) -5 (b) 5

(c) $\frac{2}{5}$ (d) $\frac{-5}{2}$

23. The product of zeros of the polynomial x^3+4x^2+x-6 is

(a) -4 (b) 4 (c) -6 (d) 6

24. If l, m, n are the zeros of polynomial x^3-px^2+dx-r then what is the value of $\frac{1}{lm}+\frac{1}{mn}+\frac{1}{nl}$?

(a) $\frac{p}{r}$ (b) $\frac{r}{p}$

(c) $-\frac{p}{r}$ (d) $\frac{-r}{p}$

25. If one zero of the polynomial $(K^2+4)x^2+13x+4K$ is reciprocal of the other, what is the value of K?

(a) 2 (b) -2 (c) 4 (d) -4

26. If α, β are the zeros of a polynomial such $\alpha+\beta=-6$ and $\alpha\beta=-4$, then what is the polynomial?

(a) x^2+6x-4 (b) x^2-6x+4

(c) x^2-6x (d) None of these

27. If $f(x)=4\sqrt{3}x^2+5x-2\sqrt{3}$. If α and β be the zeros of the polynomial. What is the product of zeros?

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) $\frac{-5}{4}$ (d) $\frac{-5}{4\sqrt{3}}$

28. If divisor is $x-1-x^2$ and dividend is $3x^2-x^3-3x+5$, then what is the remainder?

(a) $x-3$ (b) 3

(c) -3 (d) $x+3$

29. Find the quotient if dividend is $30x^4+11x^3-82x^2-12x+48$ and divisor is $3x^2+2x-4$.

(a) $10x^2+3x-12$ (b) $10x^2-3x-12$

(c) $10x^2-6x-6$ (d) None of these

30. If α, β are zeros of the polynomial x^2-px+d, then what is the value of $\frac{1}{\alpha}+\frac{1}{\beta}$?

(a) $\frac{p}{d}$ (d) $\frac{d}{p}$

(c) 1 (d) $-\frac{p}{d}$

Answer Key

1. (a)	2. (a)	3. (b)	4. (a)	5. (b)	6. (b)	7. (a)	8. (a)	9. (a)	10 (d)
11. (a)	12. (c)	13. (a)	14. (b)	15. (a)	16. (a)	17. (b)	18. (a)	19. (b)	20. (d)
21. (b)	22. (a)	23. (d)	24. (a)	25. (a)	26. (a)	27. (b)	28. (b)	29. (b)	30. (a)

Hints and Solutions

1. (a) We have $abx^2+(b^2-ac)x-bc$

$$x=\frac{-(b^2-ac)\pm\sqrt{(b^2-ac)^2+4ab^2c}}{2ab}$$

$$x=\frac{ac-b^2\pm\sqrt{(b^2+ac)^2}}{2ab}$$

$$x=\frac{ac-b^2\pm b^2+ac}{2ab}$$

$$x=\frac{ac-b^2+b^2+ac}{2ab};\frac{ac-b^2-b^2-ac}{2ab}$$

$$x=\frac{2ac}{2ab};\frac{-2b^2}{2ab}$$

$$x=\frac{c}{b};\frac{-b}{a}$$

The zeros of the given polynomial are $\frac{c}{b}$ and $-\frac{b}{a}$

2. (a) Given α and β are the zeros of

ax^2+bx+c

$\therefore\ \alpha+\beta=\frac{-b}{a},\ \alpha\beta=\frac{c}{a}$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$=\frac{\left(\frac{-b}{a}\right)^3-\frac{3c}{a}\left(\frac{-b}{a}\right)}{\frac{c}{a}}$$

$$=\frac{-\frac{b^3}{a^3}+\frac{3bc}{a^2}}{\frac{c}{a}}=\frac{\frac{-b^3+3abc}{a^3}}{\frac{c}{a}}$$

$$=\frac{3abc-b^3}{a^3}\times\frac{a}{c}=\frac{3abc-b^3}{a^2c}$$

3. (b) Here α, β are the zeros of x^2-5x+P

then $\alpha+\beta=5$ ...(1)

and $\alpha\beta=P$

$\alpha-\beta=1$ ...(2)

Solving (1) and (2) we get

$2\alpha=6\ \Rightarrow\alpha=3$

and $\beta=5-3=2$

$\therefore\ \alpha\beta=P=(3)\,(2)=6$

4. (a) $P(x)=x^2-8x+k$

If α, and β are its zeros

then $\alpha+\beta=8,\ \alpha\beta=k$

and $\alpha^2+\beta^2=40$

$\Rightarrow(\alpha+\beta)^2-2\alpha\beta=8^2-2k$

$\Rightarrow\quad 40=64-2k$

$\Rightarrow\quad 2k=24$

$\Rightarrow\quad k=12$

5. (b) Given $6x^2+x-2$

If α, β are its zeros

then $\alpha+\beta=\frac{-1}{6},\ \alpha\beta=\frac{-2}{6}=-\frac{1}{3}$

Now $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$=\frac{\left(\frac{-1}{6}\right)^2-2\left(\frac{-1}{3}\right)}{-\frac{1}{3}}$$

$$=\frac{\frac{1}{36}+\frac{2}{3}}{\frac{-1}{3}}=\frac{\frac{1+24}{36}}{\frac{-1}{3}}$$

$$=\frac{25}{36}\times\frac{-3}{1}=-\frac{25}{12}$$

6. **(b)** Here

$$4x^2+3x-2\overline{)\,8x^4+14x^3-2x^2+7x-8}\,\big(2x^2+2x-1$$

$$\underline{-8x^4 \pm 6x^3 \mp 4x^2}$$

$$8x^3+2x^2+7x$$

$$\underline{-8x^3 \pm 6x^2 \mp 4x}$$

$$-4x^2+11x-8$$

$$\underline{\mp 4x^2 \mp 3x \pm 2}$$

$$14x-10$$

$\therefore$ $14x - 10$ must be subtracted.

7. **(a)** $2x^4-3x^3-3x^2+6x-2$

Let the other roots be α and β.

Sum of the roots $= \sqrt{2}+(-\sqrt{2})+\alpha+\beta = \dfrac{3}{2}$

$$\alpha+\beta = \frac{3}{2} \qquad \ldots(1)$$

Product of roots $= \dfrac{-2}{2} = -1$

$$\sqrt{2}+(-\sqrt{2})\alpha\beta = -1 \Rightarrow \alpha\beta = \frac{1}{2} \qquad \ldots(2)$$

$$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$$

$$= \left(\frac{3}{2}\right)^2 = 4\alpha\beta = \frac{9}{4}-\frac{4}{2} = \frac{1}{4}.$$

$$\alpha-\beta = \pm\frac{1}{2} \qquad \ldots(3)$$

From equation (1) and (3)

$$\alpha = 1, \beta = \frac{1}{2} \text{ or } \beta = 1, \alpha = \frac{1}{2}$$

8. **(a)** Given $\alpha+\beta+\gamma = 2 = \dfrac{-b}{a} \Rightarrow b = -2a$

$$\alpha\beta+\beta\gamma+\gamma\alpha = -7 = \frac{c}{a} \Rightarrow c = -7a$$

$$\alpha\beta\gamma = -14 = \frac{-d}{a} \Rightarrow d = 14a$$

$b : c : d = -2 : -7 : 14$

Cubic polynomial is

$k(x^3 - 2x^2 - 7x + 14)$

9. **(a)** Let the zeros are α, $-\alpha$, β.

$$\therefore \quad \alpha+(-\alpha)+\beta = -\left(\frac{-5}{1}\right) = 5$$

$$\Rightarrow \qquad \beta = 5$$

and $(\alpha)\,(-\alpha)\,(\beta) = \dfrac{-80}{1}$

$$\Rightarrow \qquad -\alpha^2\beta = -80 \Rightarrow \alpha^2\beta = 80$$

$$\Rightarrow \qquad \alpha^2 = \frac{80}{\beta} = \frac{80}{5} = 16$$

$$\Rightarrow \qquad \alpha = \pm 4$$

The zeros are 4, –4, 5.

10. **(d)** If α and β are two zeros then $\alpha\beta = 3$

Let third root be γ.

$$\alpha\beta\gamma = \frac{-9}{2}$$

$$\Rightarrow \quad 3\gamma = \frac{-9}{2}$$

$$\Rightarrow \quad \gamma = -\frac{9}{2\times 3} = \frac{-3}{2}$$

11. **(a)** Given $\alpha+\beta+\gamma = 6$

$\alpha\beta+\beta\gamma+\gamma\alpha = -1$

$\alpha\beta\gamma = -30$

The cubic polynomial is

$P(x) = x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma$

$= x^3 - 6x^2 - x - (-30)$

(substituting the given values)

$= x^3 - 6x^2 - x + 30$

12. **(c)** The given polynomial is

$2x^3+x^2-13x+6$

α, β, γ be its zeros

$$\therefore \quad \alpha\beta\gamma = \frac{-d}{a} = \frac{-6}{2} = -3$$

13. **(a)**

$$\text{Polynomial} = (-x^2+x-1)(x-2)+3$$

$$= -x^3+2x^2+x^2-2x-x+2+3$$

$$= -x^3+3x^2-3x+5$$

14. **(b)** Here

$$x^2+2x-3\overline{)4x^4+2x^3-2x^2+x-1}\,(4x^2-6x+22$$
$$\underline{\pm 4x^4 \pm 8x^3 \mp 12x^2}$$
$$-6x^3+10x^2+x$$
$$\underline{\mp 6x^3 \mp 12x^2 \pm 18x}$$
$$22x^2-17x-1$$
$$\underline{-22x^2 \pm 44x \mp 66}$$
$$-61x+65$$

Clearly $-61x + 65$ must be added.

15. **(a)**

$$3x^2+4x+1\overline{)6x^4+8x^3+17x^2+21x+7}\,(2x^2+5$$
$$\underline{-6x^4 \pm 8x^3 \pm 2x^2}$$
$$15x^2+21x+7$$
$$\underline{-15x^2 \pm 20x+5}$$
$$x+2$$

remainder $= x + 2$

$ax + b = x + 2$ (Given)

$\Rightarrow a = 1; b = 2$

16. **(a)** Required polynomial

$= P(x) = x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma$

$= x^3 - 4x^2 + x + 6$

17. **(b)** We have $\alpha + \beta + \gamma = -2 - 3 - 1 = -6$

$\alpha\beta + \beta\gamma + \gamma\alpha$

$= (-2)(-3) + (-3)(-1) + (-1)(-2)$

$= 6 + 3 + 2 = 11$

and $\alpha\beta\gamma = (-2)(-3)(-1) = -6$

$\therefore$ Required polynomial

$= x^3 - (-6)x^2 + 11x - (-6)$

$= x^3 + 6x^2 + 11x + 6$

18. **(a)** x^2 – (Sum of zeros)x + Product of zeros

$x^2 - (-5)x + (-12) = x^2 + 5x - 12$

19. **(b)** Here $(2 - x + x^2)(3x - 1)$

$= 6x - 2 - 3x^2 + x + 3x^3 - x^2$

$= 3x^3 - 4x^2 + 7x - 2$

20. **(d)** We have $x^2 - 2x - 3$

$$\therefore\ x = \frac{-(-2) \pm \sqrt{4+12}}{2}$$

$$= \frac{2 \pm 4}{2} = \frac{2+4}{2}, \frac{2-4}{2} = 3, -1$$

21. **(b)** Here $\alpha + \beta = \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12}$

$$\alpha\beta = \left(\frac{2}{3}\right)\left(-\frac{1}{4}\right) = -\frac{1}{6}$$

$\therefore$ Required polynomial $= x^2 - \frac{5}{12}x - \frac{1}{6}$

$$= \frac{12x^2 - 5x - 2}{12}$$

22. **(a)** α, β are zeros of $2x^2 + 5x - 10$

then $\alpha\beta = \frac{-10}{2} = -5$

23. **(d)** We have $x^3 + 4x^2 + x - 6$

Comparing above equation by

$x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma$

$\Rightarrow \alpha\beta\gamma = 6$

24. **(a)** Here $l + m + n = p$, $lmn = r$

Now $\frac{1}{lm} + \frac{1}{lm} + \frac{1}{nl} = \frac{n+l+m}{lmn} = \frac{p}{r}$

25. **(a)** Let one zero be α

other zero $= \frac{1}{\alpha}$

$$\therefore\ \alpha + \frac{1}{\alpha} = \frac{-13}{K^2+4}$$

$$\alpha \times \frac{1}{\alpha} = \frac{4K}{K^2+4}$$

$\Rightarrow K^2 + 4 = 4K$

$\Rightarrow K^2 - 4K + 4 = 0$

$\Rightarrow (K-2)^2 = 0$

$\Rightarrow K = 2$

26. **(a)** Required polynomial

$$= x^2 - (\alpha+\beta)x + \alpha\beta$$

$$\Rightarrow x^2 - (-6)x + (-4) = x^2 + 6x - 4$$

27. **(b)** The given polynomial is $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

$$\therefore \alpha\beta = \frac{-2\sqrt{3}}{4\sqrt{3}} = \frac{-1}{2}$$

28. **(b)**

$$\begin{array}{r|l} -x^2+x-1 & -x^3+3x^2-3x+5 \quad (x-2 \\ & \underline{\mp x^3 \pm x^2 \mp x} \\ & \quad 2x^2-2x+5 \\ & \quad \underline{\pm 2x^2 \mp 2x \pm 2} \\ & \quad\quad\quad 3 \end{array}$$

29. **(b)**

$$\begin{array}{r|l} 3x^2+2x-4 & 30x^4+11x^3-82x^2-12x+48 \quad (10x^2-3x-12 \\ & \underline{\pm 30x^4 \pm 20x^3 \mp 40x^2} \\ & \quad -9x^3-42x^2-12x \\ & \quad \underline{\mp 9x^3 \mp 6x^2 \pm 12x} \\ & \quad\quad -36x^2-24x+48 \\ & \quad\quad \underline{\mp 36x^2 \mp 24x \pm 48} \end{array}$$

30. **(a)** Given α and β are zeros of $x^2 - px + d$

then $\alpha+\beta = \frac{-(-p)}{1} = p$

and $\alpha\beta = \frac{d}{1} = d$

$$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{p}{d}$$

3.Linear Equations in Two Variables

Key Points

Linear Equations in Two Variables

An equation of the form $ax + by + c = 0$, where a, b, c are real numbers such that $a \neq 0$, $b \neq 0$ is termed as linear equation in two variables x and y.

a and b are also called coefficients of variables x and y.

Example: $3x + 5y = 9$; $2x - 3y + 7 = 0$ etc.

Solution of a Linear equation:

The values of variables which satisfy the given equation is called solution of the equation.

$x = \alpha, y = \beta$ is a solution of $ax + by + c = 0$ if $a\alpha + b\beta + c$.

Example 1: Show that $x = 4$, $y = 3$ is a solution of $3x - 2y = 6$

Solution: L.H.S $= 3x - 2y$

$= 3(4) - 2(3)$

$= 12 - 6$

$= 6 =$ R.H.S

Hence, $x = 4, y = 3$ is a solution of $3x - 2y = 6$.

Simultaneous Linear Equations in Two Variables

Two linear equations in two unknowns x and y are said to form a system of simultaneous linear equations if each of the equation be satisfied by the same pair of x and y.

The most general form of a pair of linear equation is $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers.

Solution of a System of Two Simultaneous Equations

A pair of values of x and y and satisfying each of the equation in a given system of two simultaneous equation in x and y is called a Solution of the system.

Example 2: Show that $x = 3, y = 1$ is a solution of system of linear equations $2x + 3y = 9$; $x - 2y = 1$.

Solution: $2x + 3y = 9$ ... (1)

$x - 2y = 1$ ... (2)

Putting $x = 3$. $y = 1$ in eqn (1) we get,

L.H.S. $= 2 \times 3 + 3 \times 1 = 9 =$ R.H.S.

Putting $x = 3, y = 1$ eqn (2), we get,

L.H.S. $= 3 - 2(1) = 1 =$ R.H.S.

So, $x = 3, y = 1$ is a solution of given system of linear equations.

Consistent System of Linear Equation

A system of two linear equations in two unknowns is said to be consistent if it has at least one solution.

Inconsistent System of Linear Equations

A system of two linear equations in two is said to be inconsistent if it has no solution at all.

Example: $x + y = 3$

$2x + 2y = 7$

For these linear equations we cannot find values of x and y which can satisfy both the given equations simultaneously.

Methods for Solving Simultaneous Linear Equations

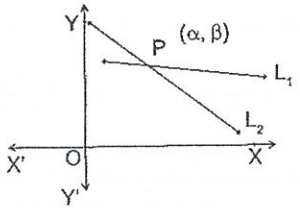
(1) **Graphical Method:** In this method points of intersection of lines give the solution.

(i) If the given lines are coincident → infinitely many solutions

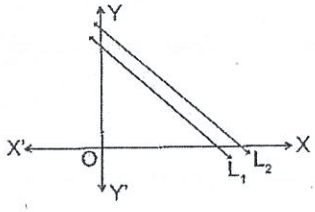
(ii) If the given lines are parallel → No solution

(iii) If the given lines intersect each other at one point, there is a unique solution.

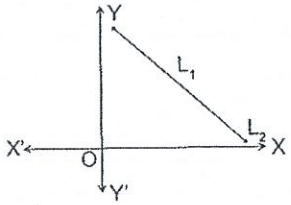
I. When the lines L_1 and L_2 intersect at a point, we get a unique solution for x and y.



II. When lines L_1 and L_2 are parallel, there is a no common point. In this case, there is no solution of the equations.



III. When the L_1 and L_2 coincide. In this case, there is only one line which represents both the equations. So the given system of equations has infinitely many solutions.



(2) **Algebraic Method:** There are two algebraic methods for solving Simultaneous linear equations.

(i) Elimination by Substitution

I. In this method, we express y in terms of x in one of the given condition.

II. Substitute this value of y in terms of x in the other equation. It gives a linear equation in x.

III. Solve the linear equation in x.

IV. Substitute this value of x to obtain a linear equation in y.

V. Solve the linear equation in y.

Example 3: Solve: $x + 2y = 8$; $3x - 4y + 6 = 0$.

(a) $x = 2, y = 3$ (b) $x = 3, y = 1$

(c) $x = 2, y = 0$ (d) None of these

Solution: (a) We have $x + 2y = 8$ ... (1)

$3x - 4y + 6 = 0$ ... (2)

From eqn (1), $y = \dfrac{8-x}{2}$

Putting $y = \dfrac{8-x}{2}$ in eqn (2)

$$3x - 4\left(\frac{8-x}{2}\right) + 6 = 0$$

$\Rightarrow \quad 3x - 16 + 2x + 6 = 0$

$\Rightarrow \quad 5x = 10 \Rightarrow x = 2$

Putting $x = 2$ in eqn (2)

$$3(2) - 4y + 6 = 0$$

$\Rightarrow \quad 4y = 12$

$\Rightarrow \quad y = 3$

Hence the solution is $x = 2, y = 3$.

(ii) Elimination by Equating the Coefficients

Example 4: Solve for x and y

$23x - 29y = 98$

$29x - 23y = 110$

(a) $x = 3; y = 1$ (b) $x = 3, y = -1$

(c) $x = 3, y = 2$ (d) None of these

Solution: (b) We have

$23x - 29y = 98$ ... (1)

$29x - 23y = 110$ ... (2)

Adding eqn (1) and (2)

$$23x - 29y + 29x - 23y = 98 + 110$$

$\Rightarrow \quad 52x - 52y = 208$

$\Rightarrow \quad 52(x - y) = 208$

$\Rightarrow \quad x - y = 4$ ... (3)

Subtracting eqn (1) from (2)

$$29x - 23y - 23x + 29y = 110 - 98$$

$$6x + 6y = 12$$

$$x + y = 2$$ ... (4)

From (3) and (4)

$x - y = 4$

$x + y = 2$

Adding eqn (3) and (4), we get $2x = 6 \Rightarrow x = 3$

From (4) $x + y = 2$

$\Rightarrow \quad y = 2 - x = 2 - 3 = -1$

Hence, $x = 3; y = -1$

(iii) Cross Multiplication Method: If $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

$$\frac{x}{b_1 \times c_1 \atop b_2 \times c_2} = \frac{y}{c_1 \times a_1 \atop c_2 \times a_2} = \frac{1}{a_1 \times b_1 \atop a_2 \times b_2}$$

$$\Rightarrow \frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - b_1a_2}$$

Hence, $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$, $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

I. The equation of a pair of linear equation
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
has no solution or system is inconsistent if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Example 5: Verify that the system of equation
$3x - 5y = 11$ and $6x - 10y = 7$ has no solution

Solution: Given
$3x - 5y - 11 = 0$
$6x - 10y - 7 = 0$

Here $a_1 = 3, b_1 = -5, c_1 = -11$
$a_2 = 6, b_2 = -10, c_2 = -7$

Now $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$,

$$\frac{b_1}{b_2} = \frac{-5}{-10} = \frac{5}{10} = \frac{1}{2}$$

and $\frac{c_1}{c_2} = \frac{-11}{-7} = \frac{11}{7}$

Clearly $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, so, the given system has no solution.

II. The system of equations
$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$
has an infinite number of solutions or lines are coincident
if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Example 6: Verify that the system of equation $4x + 6y = 7$; $12x + 18y = 21$ has infinitely many solutions.

Solution: Given system of equation
$4x + 6y - 7 = 0$
$12x + 18y - 21 = 0$

Here $a_1 = 4, b_1 = 6, c_1 = -7$

$a_2 = 12, b_2 = 18, c_2 = -21$

$$\therefore \quad \frac{a_1}{a_2} = \frac{4}{12} = \frac{1}{3},$$

$$\frac{b_1}{b_2} = \frac{6}{18} = \frac{1}{3}$$

$$\frac{c_1}{c_2} = \frac{-7}{-21} = \frac{1}{3}$$

Clearly $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, so, the given system of equation has infinitely many solutions.

The system of pair of linear equations

$a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$

has a unique solution or system is consistent if

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ and the solution is

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1};\ y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Example 7: Verify that the system of equation $x - 2y = 3$; $3x + ky = 1$ has a unique solution for $k \neq -6$.

Solution: Here

$x - 2y - 3 = 0$

$3x + ky - 1 = 0$

Now $a_1 = 1\ b_1 = -2, c_1 = -3$

$a_2 = 3, b_2 = k, c_2 = -1$

We see $\frac{1}{3} \neq \frac{-2}{k} \Rightarrow k \neq -6$

Multiple Choice Questions

1. What are the values of x and y if
$\frac{57}{x+y}+\frac{6}{x-y}=5; \frac{38}{x+y}+\frac{21}{x-y}=9$?
(a) $x=11, y=8$ (b) $x=8, y=11$
(c) $x=-8, y=11$ (d) None of these

2. The sum of the digits of a two digits number is 12. The number obtained by interchanging the digits exceeds the given number by 18. What is that number?
(a) 84 (b) 48
(c) 57 (d) 75

3. The sum of two numbers is 16 and the sum of their reciprocals is $\frac{1}{3}$. What are the numbers?
(a) 12, 4 (b) 8, 8
(c) 10, 6 (d) 14, 2

4. Five years ago, Ram was thrice as old as Mohan. Ten years later Ram shall be twice as old as Mohan. What are the present age of Ram and Mohan?
(a) 50 years, 20 years
(b) 56 years, 24 years
(c) 50 years, 24 years
(d) 56 years, 20 years

5. In a triangle $\Delta ABC, 3\angle B=\angle C=2(\angle A+\angle B)$. Which angle is the largest and what is its value?
(a) $\angle C, 120°$ (b) $\angle C, 80°$
(c) $\angle B, 40°$ (d) $\angle B, 100°$

6. What is the value of k for which the system of equations $3x+5y=0$ and $kx+10y=0$, has a non–zero solution?
(a) 2 (b) 4 (c) 6 (d) 8

7. What is the value of k for which the system of equations $3x+y=1$ and $(2k-1)x+(k-1)y=2k+1$ has no solution.
(a) 2 (b) –2 (c) 3 (d) 4

8. What are the values of x and y, if
$\frac{a}{x}-\frac{b}{y}=0, \frac{ab^2}{x}+\frac{a^2b}{y}=a^2+b^2,$
where $x\neq 0, y\neq 0$?
(a) $x=b, y=1$ (b) $x=a, y=b$
(c) $x=1, y=b$ (d) $x=a, y=1$

9. What are the values of x and y, if $2(ax-by)+(a+4b)=0$ and $2(bx+ay)+(b-4a)=0$
(a) $x=2, y=-1$ (b) $x=\frac{-1}{2}, y=2$
(c) $y=\frac{-1}{2}, x=1$ (d) $x=2, y=\frac{-1}{2}$

10. What are the values of m and n for which the system of linear equations has infinitely many solutions, $3x+4y=12, (m+n)x+2(m-n)y=(5m-1)$?
(a) $m=1, n=1$
(b) $m=5, n=1$
(c) $m=1, n=5$
(d) $m=5, n=2$

11. The length of a field exceeds its breadth by 3 meters. If the length is increased by 3 meters and breadth is decreased by 2 meters. The area remains the same. What are the length and breadth respectively of the field?
(a) 15 m, 12 m (b) 12 m, 15 m
(c) 18 m, 10 m (d) 10 m , 18 m

12. What are the values of x and y if
$x+y=a+b$
$ax-by=a^2-b^2$?
(a) a, b (b) $-a, -b$
(c) $a, 0$ (d) $0, b$

13. What is the value of k except which the given system of equations has a unique solution?
$2x+3y-5=0$ and $kx-6y-8=0$
(a) 4 (b) –4 (c) 2 (d) 3

14. What is the value of K for which the system has no solution?
$2x-ky+3=0; 3x+2y-1=0$
(a) $\frac{-4}{3}$ (b) $\frac{3}{4}$
(c) $\frac{-3}{4}$ (d) None of these

15. Five years ago, Ravi was thrice as old as Shashi. Ten years later Ravi will be twice as old as Shashi. What is the age of Shashi?
(a) 20 years (b) 30 years
(c) 35 years (d) 50 years

16. The sum of numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. What is the fraction?

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$

(c) $\frac{5}{7}$ (d) $\frac{1}{7}$

17. In a cyclic quadrilateral $ABCD$, $\angle A = 2x - 1$, $\angle B = y + 5$, $\angle C = 2y + 15$, $\angle D = 4x - 7$. Which is the greatest angle of quadrilateral.

(a) 115° (b) 120° (c) 125° (d) 85°

18. The larger of two supplementary angles exceeds the smaller by 18°. What is the Value of larger angle?

(a) 81° (b) 99° (c) 109° (d) 89°

19. Rajesh scored 40 marks in a test getting 3 marks for each right answer and losing one mark of each wrong answer. If 4 marks have been awarded for each correct answer and 2 marks been deducted for each incorrect answer then Rajesh will score 50 marks. What is the number of questions in the test?

(a) 30 (b) 20 (c) 15 (d) 40

20. A railway half ticket cost half the full fare and the reservation charge is the same on half ticket as on full ticket. one reserved first class ticket form Delhi to Patna costs ₹ 216. One full and one half reserved first class ticket cost ₹ 328. What is the reservation charge?

(a) ₹ 16 (b) ₹ 26

(c) ₹ 6 (d) None of these

21. The student of a class are made to stand in rows. If 4 students are extra in each row, there would be 2 rows less. If 4 students are less to each row, there would be 4 rows more. What is the number of students in the class?

(a) 96 (b) 106 (c) 86 (d) 116

22. If $\sqrt{2}x - \sqrt{3}y = 0$ and $\sqrt{5}x + \sqrt{2}y = 0$ Then what is the sum of x and y?

(a) 0 (b) 2 (c) 4 (d) 3

23. If three times the larger of two numbers is divided by the smaller one, we get 4 as quotient and 3 as remainder. If seven times the smaller number is divided by the larger one, we get 5 as quotient and 1 as remainder. What is the smaller number?

(a) 15 (b) 20 (c) 18 (d) 25

24. The sum of two natural numbers be 8 and the sum of their reciprocal is $\frac{8}{15}$, which is the larger one?

(a) 5 (b) 7 (c) 3 (d) 6

25. A chemist has one solution containing 50% acid and a second one containing 25% acid. How much of each should be used respectively to make 10 litres of a 40% acid solution?

(a) 6 litres, 4 litres (b) 2 litres, 6 litres

(c) 4 litres, 6 litres (d) None of these

26. Taxi charges in a city consist of fixed charges and the remaining depending upon the distance traveled in kilometers. If a person travels 70 km and he pays ₹500, for traveling 100 km he pays ₹680. What is rate per kilometer?

(a) 6 (b) 8 (c) 10 (d) 7

27. What are the solutions of the equation $\frac{x}{a} + \frac{y}{b} = a + b; \frac{x}{a^2} + \frac{y}{b^2} = 2$?

(a) $x = a^2, y = b^2$

(b) $x = b^2, y = a^2$

(c) $x = 1, y = a$

(d) $x = a, y = b$

28. If $\frac{2}{x} + \frac{3}{y} = 13, \frac{5}{x} - \frac{4}{y} = -2$, given that $x \neq 0$, $y \neq 0$, then what is the value of y?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$

29. If $\frac{ax}{b} - \frac{by}{a} = a + b; ax - by = 2ab$, Then what is value of y?

(a) $-a$ (b) $-b$ (c) a (d) b

30. If $\frac{x+y}{xy} = 2; \frac{x-y}{xy} = 6$ $(x \neq 0, y \neq 0)$

What is the value of y?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{-1}{2}$ (d) $\frac{-1}{4}$

31. An examination consists of 100 questions. Two marks are awarded for every correct option. If one mark is deducted for every wrong option and half mark is deducted for every question left, then a person scores 135. Instead, if half mark is deducted for every wrong option and one mark is deducted for every question left, then the person scores 133. Find the number of questions left unattempted by the person.
(a) 14 (b) 16 (c) 10 (d) 12

32. Ram, Shyam, Tarun and Varun together had a total amount of ₹ 240 with them. Ram had half of the total amount with the others. Shyam had one-third of the total amount with the others. Tarun had one-fourth of the total amount with the others. Find the amount in rupees with Varun.
(a) 64 (b) 70 (c) 52 (d) 58

33. A two-digit number is formed by either subtracting 17 from nine times the sum of the digits or by adding 21 to 13 times the difference of the digits. Find the number.
(a) 37
(b) 73
(c) 71
(d) Cannot be determined

34. Swathi starts her job with certain monthly salary and earns a fixed increment every year. If her salary was ₹ 22500 per month after 6 years of service and ₹ 30000 per month after 11 years of service. Find her salary in rupees after 8 years of service.
(a) 24000 (b) 25500
(c) 26000 (d) 24500

35. A three digit number abc is 459 more than the sum of its digits. What is the sum of the 2 digit number *ab* and the 1-digit number *a*?
(a) 37
(b) 73
(c) 51
(d) Cannot be determined

Answer Key

1. (a)	2. (c)	3. (a)	4. (a)	5. (a)	6. (c)	7. (a)	8. (b)	9. (b)	10 (b)
11. (b)	12. (a)	13. (b)	14. (a)	15. (a)	16. (c)	17. (c)	18. (b)	19. (b)	20. (c)
21. (a)	22. (a)	23. (c)	24. (a)	25. (a)	26. (a)	27. (a)	28. (b)	29. (a)	30. (b)
31. (a)	32. (c)	33. (b)	34. (b)	35. (c)					

Hints and Solutions

1. (a) Let $\frac{1}{x+y}=a, \frac{1}{x-y}=b$

then $57a+6b=5$ ...(1)

$38a+21b=9$ ...(2)

Solving (1) and (2) we get $a=\frac{1}{19}, b=\frac{1}{3}$

Hence

$x+y=19$ ...(4)

$x-y=3$ ...(5)

Solving (4) and (5), we get $x=11, y=8$

2. (c) Let unit's digit be x and ten's digit be y.

Then number $=10y+x$

$x+y=12$ ...(1)

$10x+y-(10y+x)=18$

$9x-9y=18 \Rightarrow x-y=2$ ...(2)

Solving (1) and (2), we get $x=7, y=5$.

$\therefore$ number $=10\times5+7=50+7=57$

3. (a) Let the numbers be x and y.

$x+y=16$ ...(1)

and $\frac{1}{x}+\frac{1}{y}=\frac{1}{3} \Rightarrow \frac{y+x}{xy}=\frac{1}{3}$

Now $x-y=\sqrt{(x+y)^2-4xy}$

$=\sqrt{16^2-4\times48}$

$=\sqrt{256-192}=\sqrt{64}=8$

$\Rightarrow x-y=8$ ...(2)

From (1) and (2), we get $x=12, y=4$.

4. (a) Let present age of Ram be x years and present age of Mohan be y years.

$x-5=3(y-5)$

$\Rightarrow x-3y=-10$ ...(1)

and $x+10=2(y+10)$

$x-2y=10$ ...(2)

$x-3y=-10$

$x-2y=10$

Subtracting (1) and (2)

$-y=-20 \Rightarrow y=20$

Now $x=10+2y=10+2\times20=50$

5. (a) Let $\angle A=x^\circ, \angle B=y^\circ, \angle C=3\angle B=3y^\circ$

In ΔABC, $\angle A+\angle B+\angle C=180^\circ$

$x^\circ+y^\circ+3y^\circ=180^\circ$

$x^\circ+4y^\circ=180$ ...(1)

$\angle C=2(\angle A+\angle B) \Rightarrow 3y=2(x^\circ+y^\circ)$

$2x^\circ-y^\circ=0$ ...(2)

From (1) and (2)

$x=20, y=40$

$\therefore \angle A=20^\circ, \angle B=40^\circ, \angle C=3\times40=120^\circ$

6. (c) For a non–zero solution,

$\frac{a_1}{a_2}=\frac{b_1}{b_2} \Rightarrow \frac{3}{k}=\frac{5}{10} \Rightarrow k=\frac{10\times3}{5}=6$

7. (a) For no solution, it must have

$\frac{a_1}{a_2}=\frac{b_1}{b_2} \Rightarrow \frac{c_1}{c_2}$

$\frac{3}{2k-1}=\frac{1}{k-1} \Rightarrow \frac{-1}{-(2k+1)}$

$3k-3=2k-1 \Rightarrow k=2$

8. (b) Let $\frac{1}{x}=m, \frac{1}{y}=n$

Then $am-bn=0$ ...(1)

$ab^2m+a^2bn=a^2+b^2$ ...(2)

Now $am-bn+0=0$

$ab^2m+a^2bn-(a^2+b^2)=0$

By cross multiplication,

$\frac{m}{b(a^2+b^2)}=\frac{n}{a(a^2+b^2)}=\frac{1}{(a^3b+ab^3)}$

$m=\frac{b(a^2+b^2)}{ab(a^2+b^2)}=\frac{1}{a} \Rightarrow \frac{1}{x}=\frac{1}{a} \Rightarrow x=a$

$n=\frac{a(a^2+b^2)}{ab(a^2+b^2)}=\frac{1}{b} \Rightarrow \frac{1}{y}=\frac{1}{b} \Rightarrow y=b$

9. (b) We have

$2ax-2by=-a-4b$ ...(1)

and $2bx+2ay=4a-b$ ...(2)

$2a^2x-2aby=-a^2-4ab$ ...(3)

Now $2b^2x+2aby=4ab-b^2$

$2x(a^2+b^2)=-(a^2+b^2) \Rightarrow x=\frac{-1}{2}$

Now putting the value of $2aby$ from eqn. (3), we get

$$2b\left(\frac{-1}{2}\right) + 2ay = 4a - b$$

$$\Rightarrow -b + 2ay = 4a - b \Rightarrow 2ay = 4a$$

$$\Rightarrow y = \frac{4a}{2a} = 2$$

10. **(b)** For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{3}{m+n} = \frac{4}{2(m-n)} = \frac{-12}{1-5m}$$

On solving these equations, we get

$m = 5, n = +1$

11. **(b)** Let the breadth of field be x m and length be y m.

Then, area $= xy$ m^2

Now $y = 3 + x$

$\Rightarrow y - x = 3$ ...(1)

New length $= y + 3$

breadth $= x - 2$

$\therefore$ area $= (x - 2)(y + 3) = xy$

$\Rightarrow xy + 3x - 2y - 6 = xy$

$\Rightarrow 3x - 2y = 6$ ...(2)

From (1) and (2), we get

$x = 12$ m, $y = 15$ m

12. **(a)** Here

$bx + by = ab + b^2$ ...(1)

and $ax - by = a^2 - b^2$ ...(2)

Adding, $x(a + b) = a(b + a) \Rightarrow x = a$

Putting $x = a$ in eqn (2), we have

$a^2 - by = a^2 - b^2$

$\Rightarrow by = -b^2 \Rightarrow y = b$

Hence $x = a, y = b$

13. **(b)** Here

$2x + 3y - 5 = 0$

and $kx - 6y - 8 = 0$

For a unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\frac{2}{k} \neq \frac{3}{-6} \Rightarrow k \neq -4$$

14. **(a)** Given

$2x - ky + 3 = 0$

$3x + 2y - 1 = 0$

For no solution, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{c_1}{c_2}$$

$$\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{-k}{2}, \frac{c_1}{c_2} = \frac{3}{-1}$$

Now $\frac{a_1}{a_2} = \frac{2}{3} \neq \frac{c_1}{c_2} = \frac{3}{-1}$

So, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{2}{3} = \frac{-k}{2} \neq k = \frac{-4}{3}$

15. **(a)** Let Shashi's present age be x and Ravi's present age be y.

$y - 5 = 3(x - 5) \Rightarrow 3x - y = 10$ ...(1)

$y + 10 = 2(x + 10) \Rightarrow 2x - y = 10$ ...(2)

Solving eqns (1) and (2), we get

$x = 20, y = 50$

16. **(c)** Let the fraction be $\frac{x}{y}$

then $x + y = 12$ ...(1)

and $\frac{x}{y+3} = \frac{1}{2} \Rightarrow 2x - y = 3$ ...(2)

By solving eqn (1) and (2), we get $x = 5, y = 7$

$$\frac{x}{y} = \frac{5}{7}$$

17. **(c)** In a cyclic quadrilateral,

$\angle A + \angle C = 180°, \angle B + \angle D = 180°$

Here $2x - 1 + 2y + 15 = 180°$

$\Rightarrow 2x + 2y = 166$

$x + y = 83$ ...(1)

and $4x - 7 + y + 5 = 180°$

$\Rightarrow 4x + y = 182$ ...(2)

Solving eqns (1) and (2), we get $x = 33, y = 50$

$\angle A = 2x - 1 = 2 \times 33 - 1 = 65°$

$\angle B = y + 5 = 50 + 5 = 55°$

$\angle C = 2y + 15 = 2 \times 50 + 15 = 115°$

$\angle D = 4x - 7 = 4 \times 33 - 7 = 125°$

18. **(b)** Let x be the larger angle and y be smaller angle.

then $x + y = 180°$ ...(1)

and $x - y = 18°$

Solving eqns (1) and (2), we get

$x = 99°, y = 81°$

19. **(b)** Let number of right answer be x and number of wrong answer be y.

$3x - y = 40$ ...(1)

$4x - 2y = 50 \Rightarrow 2x - y = 25$ ...(2)

Solving (1) and (2) $x = 15, y = 5$

No. of questions = 15 + 5= 20

20. **(c)** Let ₹x be the charge of full first class ticket and ₹y be reservation charge.

$x + y = 216$ ...(1)

$x + y + \frac{x}{2} + y = 327$

$\Rightarrow 3x + 4y = 654$ ...(2)

By solving eqns (1) and (2), we get

$x = 210, y = 6$

21. **(a)** Let the number of rows be x and the number of students in each row be y.

Total number of students = xy

$(x - 2)(y + 4) = xy$

$\Rightarrow xy + 4x - 2y - 8 = xy$

$\Rightarrow 4x - 2y = 8$ ...(1)

and $(x + 4)(y - 4) = xy$

$xy - 4x + 4y - 16 = xy$

$\Rightarrow -4x + 4y = 16$ ...(2)

From eqn. (1) and (2)

$4x - 2y - 4x + 4y = 24$

$2y = 24 \Rightarrow y = 12$

and $4x - 2 \times 12 = 8$

$\Rightarrow 4x = 8 + 24 = 32 \Rightarrow x = 8$

Total no. of students = $12 \times 8 = 96$.

22. **(a)** Here $\sqrt{2}x - \sqrt{3}y = 0$ ...(1) $\times \sqrt{2}$

$\sqrt{5}x + \sqrt{2}y = 0$ ...(2) $\times \sqrt{3}$

then $2x - \sqrt{6}y = 0$

$\sqrt{15}x + \sqrt{6}y = 0$

Adding both equations $(2 + \sqrt{15})x = 0 \Rightarrow x = 0$

Now $\sqrt{2} \times 0 - \sqrt{3}y = 0$

$\sqrt{3}y = 0 \Rightarrow y = 0$

Hence $x + y = 0 + 0 = 0$

23. **(c)** Let the larger number be x and the smaller number be y.

Now according to question, $3x = 4y + 3$

$3x - 4y = 3$ ...(1)

and $7y = 5x + 1$

$5x - 7y = 1$ ...(2)

$21x - 28y = 21$

$20x - 28y = -4$

$- \quad + \quad +$

$x = 25$

and $3 \times 25 - 4y = 3$

$\Rightarrow 4y = 72 \Rightarrow y = 18$

$\therefore$ Smaller number = 18

24. **(a)** Let one number be x

other number = $8 - x$

$\therefore \frac{1}{x} + \frac{1}{8-x} = \frac{8}{15}$

$\Rightarrow \frac{8 - x + x}{x(8-x)} = \frac{8}{15}$

$\Rightarrow \frac{8}{8x - x^2} = \frac{8}{15}$

$\Rightarrow 8x - x^2 = 15$

$\Rightarrow x^2 - 8x + 15 = 0$

$\Rightarrow x^2 - 5x - 3x + 15 = 0$

$\Rightarrow x(x - 5) - 3(x - 5) = 0$

$\Rightarrow (x - 5)(x - 3) = 0$

$\Rightarrow x = 5$ or $x = 3$

25. **(a)** Let x litres of the 40% solution be mixed with y litres of 25% solution.

then $x + y = 10$ ...(1)

50% of x + 25% of y = 40% of 10

$\Rightarrow \frac{50x}{100} + \frac{25y}{100} = \frac{40 \times 10}{100}$

$\Rightarrow 50x + 25y = 400$

$\Rightarrow 2x + y = 16$ ...(2)

From (1) and (2)

$2x + 2y = 20$

$2x + y = 16$

$x = 25$

$\therefore \quad x = 10 - 4 = 6$

26. **(a)** Let the fixed charges be ₹ x and other charges be ₹ y per km.

$x + 70y = 500$ ...(1)

and $x + 100y = 680$ ...(2)

Solving $30y = 180$

$\Rightarrow \quad y = 6$

and $x + 70 \times 6 = 500 \Rightarrow x = 500 - 420 = 80.$

27. (a) We have $\frac{1}{a}x + \frac{1}{b}y - (a+b) = 0$ ...(1)

$\frac{1}{a^2}x + \frac{1}{b^2}y - 2 = 0$ ...(2)

By cross multiplication

$$\frac{x}{\frac{a-b}{b^2}} = \frac{y}{\frac{a-b}{a^2}} = \frac{1}{\frac{a-b}{a^2b^2}}$$

$$x = \frac{a-b}{b^2} \times \frac{a^2b^2}{a-b} = a^2$$

and $y = \frac{a-b}{a^2} \times \frac{a^2b^2}{a-b} = b^2$

28. (b) Let $\frac{1}{x} = u;\ \frac{1}{y} = v$

then $2u + 3v = 13$ ...(1)

and $5u - 4v = -2$ ...(2)

Solving $\frac{u}{-46} = \frac{v}{-69} = \frac{1}{-23}$

$\Rightarrow u = 5;\ v = 3$

$\therefore \ \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2};\ \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$

29. (a) $\frac{ax}{b} - \frac{by}{a} = a + b$ ...(1)

and $ax - by = 2ab$ ...(2)

Now $ax - \frac{b^2y}{a} = ab + b^2$

Solving $ax - by = 2ab$

$$\Rightarrow y\left(b - \frac{b^2}{a}\right) = b^2 - ab$$

$$\Rightarrow y\left(\frac{ab - b^2}{a}\right) = b^2 - ab$$

$$\Rightarrow y = \frac{(b^2 - ab)a}{ab - b^2}$$

$y = -a$

30. (b) Given $\frac{x+y}{xy} = 2$

$\Rightarrow \frac{1}{y} + \frac{1}{x} = 2$ ...(1)

and $\frac{x-y}{xy} = 6 \Rightarrow \frac{1}{y} - \frac{1}{x} = 6$ ...(2)

Adding (1) and (2), we get

$\frac{1}{x} + \frac{1}{y} + \frac{1}{y} - \frac{1}{x} = 2 + 6$

$\Rightarrow \frac{2}{y} = 8 \Rightarrow y = \frac{2}{8} = \frac{1}{4}$

31. (a) If x, y, z are the number of correct answers, wrong answers and unattempted questions respectively then

$x + y + z = 100$ ...(i)

$2x - y - \frac{z}{2} = 135$ ...(ii)

$2x - \frac{y}{2} - z = 133$ ...(iii)

Now solve above equations.

No. of questions left unattempted = 14.

32. (c) If r, s, t, and v are the amounts with Ram, Shyam, Tarun and Varun respectively then

$r = \frac{s+t+v}{2}, \quad s = \frac{r+t+v}{3}, \quad t = \frac{r+s+v}{4},$

$r + s + t + v = 240$

Now solve the above equation to get v.

4 Quadratic Equations

Quadratic Equation

The equation of the form $ax^2 + bx + c = 0$ $(a \neq 0)$ is called a quadratic equation in x.

If α is the zero of the quadratic expression, $ax^2 + bx + c$ then $a\alpha^2 + b\alpha + c = 0$.

A quadratic equation may have two roots or solutions.

Solution of a Quadratic Equation

There are following methods of solving quadratic equation:

Factorisation

If α and β are roots of the equation then $(x - \alpha)(x - \beta) = 0$

Example 1: Solve the quadratic equation

$$6x^2 + 7x - 10 = 0.$$

(a) $-2, \frac{5}{6}$ (b) $-2, \frac{-5}{6}$ (c) $-2, -\frac{1}{5}$ (d) $-2, -4$

Solution: **(a)** We have

$$6x^2 + 7x - 10 = 0$$

$$\Rightarrow 6x^2 + 12x - 5x - 10 = 0$$

or, $6x(x + 2) - 5(x + 2) = 0$

or $(x + 2)(6x - 5) = 0$

$$x + 2 = 0 \Rightarrow x = -2$$

or $6x - 5 = 0 \Rightarrow x = \frac{5}{6}$

Hence the roots are -2 and $\frac{5}{6}$.

Example 2: Solve $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

(a) $\pm\frac{2}{\sqrt{5}}$ (b) $\pm\frac{3}{\sqrt{5}}$ (c) $\pm\frac{\sqrt{3}}{4}$ (d) $\frac{-2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

Solution: **(d)** We have

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

$$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$$

$$\Rightarrow (\sqrt{3}x + 2)(4x - \sqrt{3}) = 0$$

$$(\sqrt{3}x + 2) = 0 \Rightarrow x = \frac{-2}{\sqrt{3}}$$

and $4x - \sqrt{3} = 0 \Rightarrow x = \frac{\sqrt{3}}{4}$

Thus the roots are $\frac{-2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$.

Quadratic Formula Method

If $ax^2 + bx + c = 0$ is a quadratic equation, then the solution is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \; b^2 - 4ac = D$$

is called discriminant of the quadratic equation.

If the roots are α and β, then

$$\alpha = \frac{-b + \sqrt{D}}{2a}, \; \beta = \frac{b - \sqrt{D}}{2a}$$

Example 3: Solve the equation $x^2 + 6x + 6 = 0$.

(a) $3 \pm \sqrt{3}$ (b) $-3 \pm \sqrt{3}$ (c) $2 \pm \sqrt{3}$ (d) None of these

Solution: **(b)** We have

$a = 1, b = 6. c = 6$

$D = b^2 - 4ac = 6^2 - 4 \times 1 \times 6$

$\therefore \quad = 36 - 24 = 12$

The given equation has real roots.

$$x = \frac{-b \pm \sqrt{D}}{2a} \Rightarrow x = \frac{-6 \pm \sqrt{12}}{2 \times 1}$$

$$x = \frac{-6 \pm 2\sqrt{3}}{2}$$

$$x = \frac{2(-3 \pm \sqrt{3})}{2}$$

$$x = -3 \pm \sqrt{3}$$

Example 4: Solve the equation $2x^2 + 5\sqrt{3}x + 6 = 0$.

(a) $\pm 2\sqrt{3}$ (b) $\pm \frac{\sqrt{3}}{2}$ (c) $\pm \frac{\sqrt{3}}{4}$ (d) $-2\sqrt{3}, -\frac{\sqrt{3}}{2}$

Solution: **(d)** Comparing the given equation by $ax^2 + bx + c = 0$, we get

$a = 2, b = 5\sqrt{3}, c = 6$

$\therefore \quad D = b^2 - 4ac = (5\sqrt{3})^2 - 4 \times 2 \times 6$

$= 75 - 48 = 27$

$$x = \frac{-b \pm \sqrt{D}}{2a} \Rightarrow x = \frac{-5\sqrt{3} \pm \sqrt{27}}{2 \times 2}$$

$$x = \frac{-5\sqrt{3} \pm 3\sqrt{3}}{4}$$

$$x = \frac{-5\sqrt{3} + 3\sqrt{3}}{4} = \frac{-2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2}$$

$$x = \frac{-5\sqrt{3} - 3\sqrt{3}}{4} = \frac{-8\sqrt{3}}{4} = -2\sqrt{3}$$

If $ax^2 + bx + c = 0$ is a quadratic equation
α and β are its roots then

sum of roots $= \alpha + \beta = \frac{-b}{a}$, and product of roots $= \alpha\beta = \frac{c}{a}$

Example 5: Find the sum and product of the roots of equation $16x^2 - 24x + 1 = 0$.

(a) $\frac{3}{2}, \frac{1}{16}$ (b) $\frac{3}{2}, \frac{1}{15}$ (c) $\frac{5}{2}, \frac{1}{9}$ (d) None of these

Solution: **(a)** The given equation is $16x^2 - 24x + 1 = 0$

Here $a = 16$, $b = -24$, $c = 1$

Sum of roots $= \frac{-b}{a} = \frac{-(-24)}{16} = \frac{24}{16} = \frac{3}{2}$

Product of roots $= \frac{c}{a} = \frac{1}{16}$

By method of completing square, we can find whether the given quadratic equation has roots and if they exist, we can find them.

Nature of Roots of Quadratic Equation

If $ax^2 + bx + c = 0$ $(a \neq 0)$ is a quadratic equation
then Discriminant, $(D) = b^2 - 4ac$

Case I: When $D > 0$

Then the roots are real and distinct
These roots are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a}, \beta = \frac{-b - \sqrt{D}}{2a}$$

Case II: When $D = 0$

In this case, the roots are real and equal

Each root is given by $\frac{-b}{2a}$.

Case III: When $D < 0$

In this case, the roots are imaginary and unequal or complex conjugate.
There are no real roots.

Case IV: If $D > 0$ and D is perfect square, then roots are unequal and rational.

Case V: If $D > 0$ and D is not perfect square, then roots are irrational and unequal.

Example 6: Find the nature of the roots of the equation $4x^2 - 5x + 3 = 0$.

(a) real roots (b) imaginary

(c) unequal and rational (d) None of these

Solution: **(a)** The given equation is

$4x^2 - 5x + 3 = 0$

Here $a = 4, b = -5, c = 3$

Now $D = b^2 - 4ac = (-5)^2 - 4 \times 4 \times 3$

$= 25 - 48 = -23 < 0$

As $D < 0$, then the equation has no real roots

Example 7: Find the value of α for which the equation $4x^2 - 3\alpha x + 1 = 0$ has real and equal roots.

(a) $\pm\frac{3}{4}$ (b) $\pm\frac{4}{3}$ (c) ± 5 (d) None of these

Solution: **(b)** We have

$a = 4, b = -3\alpha, c = 1$

For real and equal roots $D = 0$

$\Rightarrow \quad b^2 - 4ac = 0$

$\Rightarrow \quad (-3\alpha)^2 - 4 \times 4 \times 1 = 0 \Rightarrow 9\alpha^2 = 16$

$\alpha^2 = \frac{16}{9} \Rightarrow \alpha = \pm\frac{4}{3}$

Formation of a Quadratic Equation

If α and β are roots of a quadratic equation, then the equation can be formed by

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Example 8: If the roots of a quadratic equation are $2+\sqrt{3}$ and $2-\sqrt{3}$. Find the quadratic equation.

(a) $x^2 - 4x + 5 = 0$ (b) $x^2 - 4x + 3 = 0$

(c) $x^2 - 4x + 1 = 0$ (d) None of these

Solution: **(c)** Here

$\alpha + \beta = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

and $\alpha\beta = (2+\sqrt{3})(2-\sqrt{3})$

$= (2)^2 - (\sqrt{3})^2$

$= 4 - 3 = 1$

The required equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow \quad x^2 - 4x + 1 = 0$

Example 9: If $(a+b)$ and $(a-b)$ are the roots of a quadratic equation then what is the quadratic equation?

(a) $x^2 - 2ax + (a^2 - b^2) = 0$ (b) $x^2 + 2ax + a^2 - b^2 = 0$

(c) $x^2 - 2ax + a^2 - b^2 = 0$ (d) none of them

Solution: **(a)** Let

$\alpha = a + b, \beta = a - b$

then $\alpha + \beta = a + b + a - b = 2a$

and $\alpha\beta = (a+b)(a-b) = a^2 - b^2$

Hence the required equation is $x^2 - 2ax + (a^2 - b^2) = 0$

Multiple Choice Questions

1. An aeroplane traveled a distance of 400 km at an average speed. On the return journey the speed was increased by 40 km/h. If the return journey took 30 minutes less than the onward journey. What is the average speed of the train?
   (a) 160 km/h (b) 180 km/h
   (c) 100 km/h (d) 200 km/h

2. If the roots of a quadratic equation $(a-b)x^2+(b-c)x+(c-a)=0$ are equal then which of the following is correct?
   (a) $b+c=a$ (b) $b-2a=c$
   (c) $a+c=b$ (d) $a=2c$

3. Which of the following is a root of quadratic equation $a^2b^2x^2-(4b^4-3a^4)x-12a^2-b^2=0$?
   (a) $\frac{-3a^2}{b^2}$ (b) $\frac{4a^2}{b^2}$
   (c) $\frac{a^2}{b^2}$ (d) $\frac{4a^2}{b}$

4. Find the value of x if $5^{x+1}+5^{2-x}=126$.
   (a) 2 or –1 (b) 2 or –2
   (c) 4 or – 4 (d) –2 or –4

5. What are the roots of the equation $\frac{1}{a+b+c}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$?
   (a) a or b (b) $-a$ or $-b$
   (c) $2a$ or $-2b$ (d) None of these

6. An aeroplane left 30 minutes later than its scheduled time and in order to reach its destination 1500 km away in time, it had to increase its speed by 250 km/h from its average speed. What is the average speed?
   (a) 1000 km/h (b) 1200 km/h
   (c) 750 km/h (d) 800 km/h

7. Which of the following equation has no real roots?
   (a) $2x^2-5x-4=0$
   (b) $9x^2-6x+1=0$
   (c) $15x^2-11x+3=0$
   (d) $9x^2-12x+4=0$

8. Which of the following equation have both roots equal?
   (a) $12x^2-4\sqrt{15}x+5=0$
   (b) $x^2+8x+16=0$
   (c) $x^2+4x+4=0$
   (d) $3x^2-2\sqrt{6}x+2=0$

9. Which of the two digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places?
   (a) 63 (b) 36 (c) 92 (d) 29

10. The speed of motorboat in still water is 15 km/h it goes 30 km downstream and returns back to the starting point in a total time of $4\frac{1}{2}$ hours. What is the speed of the stream?
    (a) 5 km/h (b) 6 km/h
    (c) 4 km/h (d) 3 km/h

11. The diagonal of a rectangular field is 60 meter more than the shorter side. If the longer side is 30 meter more than the shorter side what are the sides of the field?
    (a) 90m, 120m (b) 20m, 60m
    (c) 120m, 150m (d) 60m, 90m

12. What is the integral solution of the equation $3x^2+x-14=0$?
    (a) $x=2$ (b) $x=-2$
    (c) $x=\frac{-7}{3}$ (d) $x=\frac{7}{3}$

13. Write the solution of the equation $\frac{1}{x+6}+\frac{1}{x-10}=\frac{3}{x-4}$, $x\neq 10,-6$.
    (a) (14, – 14) (b) (14, 12)
    (c) (–12, 14) (d) (–14, –12)

14. If the roots of a quadratic equation are $\left(\frac{m}{n}-\frac{-n}{m}\right)$ then the equation is
    (a) $nx^2-(n^2+m^2)x-mn=0$
    (b) $mnx^2-(2m-n^2)x-mn=0$
    (c) $mx^2-(m^2+1)x+m=0$
    (d) $m^2x^2-(m^2-n^2)x-mn=0$

15. If -5, is a root of the quadratic equation $2x^2 + Px - 15 = 0$ and the quadratic equation $P(x^2 + x) + K = 0$ has equal roots. What is the value of K?

(a) $\frac{7}{4}$ (b) $\frac{7}{8}$

(c) $\frac{4}{7}$ (d) $\frac{8}{7}$

16. If α and β are the roots of $3x^2 + 8x + 2 = 0$ then what is the value $\alpha^2 + \beta^2$?

(a) 48 (b) $\frac{52}{9}$

(c) 42 (d) $\frac{9}{52}$

17. If one root of $3x^2 + 11x + K = 0$ be reciprocal of the other then what is the value of K?

(a) 3 (b) 5 (c) -3 (d) $\frac{-11}{3}$

18. If the sum of roots of the equation $Kx^2 + 2x + 3K = 0$ is equal to their product, then the value of K is

(a) $\frac{1}{3}$ (b) $\frac{-2}{3}$

(c) $\frac{4}{3}$ (d) $\frac{-3}{4}$

19. If α, β are the roots of the equation $3x^2 + 8x + 2 = 0$ then the value of $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ is

(a) 4 (b) -4

(c) $\frac{3}{2}$ (d) $\frac{2}{3}$

20. The roots of a quadratic equation are 7 and -3, then what is the equation?

(a) $x^2 + 10x - 21 = 0$

(b) $x^2 - 4x - 21 = 0$

(c) $x^2 - 4x + 21 = 0$

(d) $x^2 - 7x + 21 = 0$

21. If the equation $mx^2 + nx + p = 0$ has equal roots then what is the value of p?

(a) $\frac{n^2}{4m}$ (b) $\frac{m}{4n^2}$

(c) $\frac{n}{2m}$ (d) $\frac{-n}{2m}$

22. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots then the value of q is

(a) $\frac{4}{49}$ (b) 4

(c) $\frac{49}{4}$ (d) $\frac{7}{4}$

23. If α, β are the roots of the equation $x^2 + px + q = 0$ then which equation has the roots $-\frac{1}{\alpha}$ and $-\frac{1}{\beta}$?

(a) $x^2 - px + q = 0$

(b) $qx^2 - px + 1 = 0$

(c) $qx^2 - px + 1 = 0$

(d) $x^2 + px + q = 0$

24. If α, β are the roots of the equation $x^2 - p(x + 1) - c = 0$ then $(\alpha + 1)(\beta + 1)$ is equal to

(a) $1 + c$ (b) $1 - c$

(c) $c - 1$ (d) c

25. If $(a\alpha + b)^{-2} + (a\beta + b)^{-2} = 1$, where α, β are the roots of $ax^2 + bx + c = 0$ then $ac = ?$

(a) $2b$ (b) $\frac{b^2}{3}$

(c) b^3 (d) b^2

26. If the difference of the root $x^2 - px + q = 0$ is unity then

(a) $p^2 - 4a = 1$

(b) $p^2 + 4q = 1$

(c) $p^2 + 4q2 = (1 + 2q)^2$

(d) $4p^2 + q^2 = (1 + 2p)^2$

27. If α, β are the roots of the equation $ax^2 + bx + c = 0$ then $\frac{1}{a\alpha + b} + \frac{1}{a\beta + b} = ?$

(a) $\frac{c}{ab}$ (b) $\frac{a}{bc}$

(c) $\frac{b}{ac}$ (d) None of these

28. If a, b are the roots of the equation $x^2 + x + 1 = 0$ then $a^2 + b^2 = ?$

(a) 1 (b) 2 (c) 3 (d) -1

29. If the equation $x^2 + 2x + 3K = 0$ and $2x^2 + 3x + 5K = 0$ have a non–zero common roots then $K = ?$

(a) $K = 1$ (b) $K = -1$
(c) $K = -2$ (d) $K = 2$

30. If A and B are the roots of the quadratic equation $x^2 - 12x + 27 = 0$, then $A^3 + B^3$ is

(a) 27 (b) 729
(c) 756 (d) 64

31. By drawing which of the following graphs can the quadratic equation $4x^2 + 6x - 5 = 0$ be solved by graphical method?

(a) $y = x^2$, $3x - 2y - 5 = 0$
(b) $y = 4x^2$, $6x - 2y - 5 = 0$
(c) $y = x^2$, $6x - y - 5 = 0$
(d) $y = 2x^2$, $6x + 2y - 5 = 0$

32. If the quadratic equation $(a^2 - b^2)x^2 + (b^2 - c^2)x + (c^2 - a^2) = 0$ has equal roots, then which of the following is true?

(a) $b^2 + c^2 = a^2$ (b) $b^2 + c^2 = 2a^2$
(c) $b^2 - c^2 = 2a^2$ (d) $a^2 = b^2 + 2c^2$

33. Which of the following are the roots of the equation $|x|^2 + |x| - 6 = 0$?

(1) 2 (2) –2 (3) 3 (4) –3
(a) Both (1) and (4)
(b) Both (3) and (4)
(c) (1), (2), (3) and (4)
(d) None of the above

34. What are the values of x which satisfy the equation, $\sqrt{5x-6} + \frac{1}{\sqrt{5x-6}} = \frac{10}{3}$?

(a) 3 (b) 4, $\frac{11}{9}$
(c) $\frac{11}{9}$ (d) 3, $\frac{11}{9}$

Answers Key

				15. (a)	16. (b)	17. (a)	18. (b)	19. (b)	20. (b)
21. (a)	22. (c)	23. (c)	24. (b)	25. (c)	26. (a)	27. (c)	28. (d)	29. (b)	30. (b)
31. (d)	32. (b)	33. (a)	34. (d)						

Hints and Solutions

15. **(a)** The given equation is

$2x^2 + Px - 15 = 0$

$2(-5)^2 + P(-5) - 15 = 0$

$\Rightarrow 50 - 5P - 15 = 0$

$\Rightarrow -5P = -35$

$\Rightarrow P = 7$

$P(x^2 + x) + K = 0$

$\Rightarrow 7(x^2 + x) + K = 0 \Rightarrow 7x^2 + 7x + K = 0$

$\Rightarrow a = 7, b = 7, c = K$

$\therefore D = b^2 - 4ac = (7)^2 - 4(7)K = 49 - 28K$

The equation has equal roots

$D = 0 \Rightarrow 49 - 28K = 0 \Rightarrow K = \frac{49}{28} = \frac{7}{4}$

16. **(b)** Given equation is

$3x^2 + 8x + 2 = 0$

and α, β are its roots then

$\alpha + \beta = \frac{-8}{3}, \alpha\beta = \frac{2}{3}$

$\therefore\ \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\left(\frac{-8}{3}\right)^2-2\left(\frac{2}{3}\right)$

$=\frac{64}{9}-\frac{4}{3}=\frac{64-12}{9}$

$=\frac{52}{9}$

17. **(a)** The given equation is

$3x^2+11x+K=0$

Let one root be α then other root is $\frac{1}{\alpha}$

$\therefore\ \alpha\cdot\frac{1}{\alpha}=\frac{-11}{3},\ \alpha\cdot\frac{1}{\alpha}=\frac{K}{3}$

$\Rightarrow 1=\frac{K}{3}\Rightarrow K=3$

18. **(b)** Here $Kx^2+2x+3K=0$

then given Sum of roots = Product of roots

$\frac{-2}{K}=\frac{3K}{K}\Rightarrow\frac{-2}{K}=3\Rightarrow K=\frac{-2}{3}$

19. **(b)** We have $3x^2+8x+2=0$

and $\alpha+\beta=\frac{-8}{3},\ \alpha\beta=\frac{2}{3}$

Now $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}=\frac{\frac{-8}{3}}{\frac{2}{3}}=\frac{-8}{2}=-4$

20. **(b)** The required equation is

$x^2-(\text{Sum of roots})x+\text{Product of roots}=0$

$\Rightarrow x^2-[7+(-3)]x+(7)(-3)=0$

$\Rightarrow x^2-4x-21=0$

21. **(a)** The equation $mx^2+nx+p=0$ has equal roots

then $n^2-4mp=0\Rightarrow 4mp=n^2$

$\Rightarrow p=\frac{n^2}{4m}$

22. **(c)** Here $x^2+px+q=0$ has one root $a=4$

$\therefore\ 4^2+b4+12=0$

$\Rightarrow b=-7$

Now the equation $x^2-7x+q=0$ has equal roots

$(-7)^2-4(1)(q)=0$ then

$\Rightarrow 4q=49\Rightarrow q=\frac{49}{4}$

23. **(c)** Here $x^2+px+q=0$

then $\alpha+\beta=p,\ \alpha\beta=q$

$\left(-\frac{1}{\alpha}\right)-\left(-\frac{1}{\beta}\right)=\frac{-\alpha-\beta}{\alpha\beta}=\frac{-(\alpha+\beta)}{\alpha\beta}=\frac{-p}{q}$

$\therefore\ \left(-\frac{1}{\alpha}\right)\left(-\frac{1}{\beta}\right)=\frac{1}{\alpha\beta}=\frac{1}{q}$

$\therefore$ Required equation is $x^2+\left(\frac{p}{q}\right)x+\frac{1}{q}=0$

24. **(b)** The given equation is

$x^2-p(x+1)-c=0$

$\Rightarrow x^2-px+(-c-p)=0$

$\therefore\ \alpha+\beta=p$

and $\alpha\beta=\frac{-(c+p)}{1}=-(c+p)$

Now $(\alpha+1)(\beta+1)=\alpha\beta+\alpha+\beta+1$

$=-c-p+p+1=1-c$

25. **(c)** Here $ax^2+bx+c=0$

then $\alpha+\beta=\frac{-b}{a},\ \alpha\beta=\frac{c}{a}$

Now $(a\alpha+b)^{-2}+(ab+b)^{-2}=1$

$\Rightarrow \frac{1}{(a\alpha+b)^2}+\frac{1}{(a\beta+b)^2}=1$

$\Rightarrow \frac{(a\beta+b)^2+(a\alpha+b)^2}{(a\alpha+b)^2(a\beta+b)^2}=1$

$\Rightarrow \frac{a^2\beta^2+b^2+2ab\beta+a^2\alpha^2+b^2+2ab\alpha}{[(a\alpha+b)(a\beta+b)]^2}$

$\Rightarrow \frac{a^2(\alpha^2+\beta^2)+2b^2+2ab(\alpha+\beta)}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}=1$

$\Rightarrow \frac{a^2[(\alpha+\beta)^2-2\alpha\beta]+2b^2+2ab(\alpha+\beta)}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}=1$

$\Rightarrow \frac{a^2\left[\left(\frac{-b}{a}\right)^2-\frac{2c}{a}\right]+2b^2+2ab\times\frac{-b}{a}}{a^2\times\frac{c}{a}+ab\times\frac{-b}{a}+b^2}=1$

$\Rightarrow \frac{a^2\left[\frac{b^2-2ac}{a^2}\right]+2b^2-2b^2}{ac-b^2+b^2}=1$

$$\Rightarrow \frac{b^2-2ac}{ac}=1$$

$$\Rightarrow b^2-2ac=ac$$

$$\Rightarrow b^2=3ac$$

$$\Rightarrow ac=\frac{b^2}{3}$$

26. **(a)** We have $x^2-px+q=0$

Now $\alpha+\beta=p, \alpha\beta=q$

$\therefore \alpha-\beta=1$

$2\alpha=p+1$

$\alpha=\frac{p+1}{2}$

Now $\beta=p-\alpha=p-\frac{p+1}{2}=\frac{2p-p-1}{2}$

$=\frac{p-1}{2}$

and $\alpha\beta=q$

$\left(\frac{p+1}{2}\right)\left(\frac{p-1}{2}\right)=q$

$p^2-1=4q \Rightarrow p^2-4q=1$

27. **(c)** The given equation is

$ax^2+bx+c=0$

$\alpha+\beta=-\frac{b}{a}, \alpha\beta=\frac{c}{a}$

$\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}$

$=\frac{a\beta+b+a\alpha+b}{(a\alpha+b)(a\beta+b)}=\frac{a(\beta+\alpha)+2b}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$

$=\frac{a\times\frac{-b}{a}+2b}{a^2\times\frac{c}{a}+ab\times\frac{-b}{a}+b^2}=\frac{-b+2b}{ac-b^2+b^2}=\frac{b}{ac}$

28. **(d)** Given a and b are roots of $x^2+x+1=0$

then $a+b=-1$ and $ab=1$

$\therefore a^2+b^2=(a+b)^2-2ab$

$=(-1)^2-2(1)=1-2=-1$

29. **(b)** Let α be the non-zero common root.

then $\alpha^2+2\alpha+3K=0$

$2\alpha^2+3\alpha+5K=0$

$\therefore K=\frac{-(\alpha^2+2\alpha)}{3}$

and $K=\frac{-(2\alpha^2+3\alpha)}{5}$

$\Rightarrow \frac{(\alpha^2+2\alpha)}{3}=\frac{(2\alpha^2+3\alpha)}{5}$

$\Rightarrow 5\alpha^2+10\alpha=6\alpha^2+9\alpha$

$\Rightarrow 6\alpha^2+9\alpha-5\alpha^2-10\alpha=0$

$\Rightarrow \alpha^2-\alpha=0$

$\Rightarrow \alpha(\alpha-1)=0 \Rightarrow \alpha=0$ or $\alpha=1$

when $\alpha=1$ then $K=\frac{-(1^2+2\times1)}{3}=\frac{-(3)}{3}=-1$

30. **(b)** Given A and B are the roots of

$x^2-12x+27=0$

$\Rightarrow (x-3)(x-9)=0$

$\therefore A=3, B=9$

Hence $A^3+B^3=3^3+9^3=27+729=756$

31. **(d)**

32. **(b)** Observe the coefficient of each term and guess one root.

33. **(a)** Let $|x|=4$ then

$y^2+y-6=0$

$\Rightarrow y^2+3y-2y-6=0$

$\Rightarrow y(y+3)-2(y+3)=0$

$\Rightarrow (y-2)(y+3)=0$

$\Rightarrow y=2,-3$

$\therefore |x|=2, |x|=-3$

Hence both (a) and (d)

34. **(d)** Find the LCM and then square on both sides.

5. Arithmetic Progression

Progression: Sequence which follow a definite pattern is called progression.

Example:

1. 4, 7, 10, 13, 16...
2. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

Arithmetic Progression

A sequence in which each term differs from its preceding terms by a constant is called an arithmetic progression (A.P.)

Common difference: The constant term in arithmetic progression, is called a common difference. It is denoted by d.

$$d = a_n - a_{n-1}$$

Example: 5, 8, 11, 14, 17,...

Common difference $= 11 - 8 = 3 = 14 - 11$

If a is the first term and d is the common difference then AP can be written as a, $a + d$, $a + 2d$, $a + 3d$, . . .

General Term of an Arithmetic Progression

Let a be the first term, d be the common difference of an Arithmetic Progression then nth term of the Arithmetic Progression is given by

$$t_n = a + (n - 1)\, d$$

Example 1: Find the n^{th} term of the Arithmetic Progression 2, 5, 8, 11, . . .

(a) $3n - 1$ (b) $3n + 1$ (c) $2n - 1$ (d) $2n + 1$

Solution: **(a)** Here

$a = 2, d = 5 - 2 = 3$

then $t_n = 2 + (n - 1)3 = 2 + 3n - 3 = 3n - 1$

Example 2: If nth term of an Arithmetic Progression is $5n - 2$. What is the common difference?

(a) 4 (b) 5 (c) 6 (d) 7

Solution: **(b)**

Given $t_n = 5n - 2$

then $t_1 = 5(1) - 2 = 3$

$t_2 = 5(2) - 2 = 8$

$t_3 = 5(3) - 2 = 13$

$\therefore$ AP is 3, 8, 13, ...

then $d = t_2 - t_1 = t_3 - t_2 = 8 - 3 = 13 - 8 = 5$

Example 3: Which term of the Arithmetic Progressive is 81?

(a) 20 (b) 19 (c) 21 (d) 18

5, 9, 13, 17,...81

Solution: **(a)** Here

$a = 5, d = 9 - 5 = 4, t_n = 81; n = ?$

We know $t_n = a + (n-1)d$

$\Rightarrow 81 = 5 + (n-1)4 \Rightarrow 76 = (n-1)4$

$\Rightarrow n - 1 = 19 \Rightarrow n = 20$

Example 4: What is the 17^{th} term of the Arithmetic Progression 16, 9, 2, – 5, . . .?

(a) 95 (b) 96 (c) 85 (d) 90

Solution: **(b)** Here

$a = 16, d = 9 - 16 = -7, n = 17$

$t_{17} = 16 + (17 - 1)(-7)$

$= 16 + 16(-7)$

$= 16 - 112 = -96$

Sum of *n* terms of an Arithmetic Progression

If a be the first term, d be the common difference, l be the last term. Then the sum of n terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}[a + a + (n-1)d]$$

$$S_n = \frac{n}{2}[a + l] \qquad (\because l = a + (n-1)d)$$

If sum of n terms S_n is given then general term $T_n = S_n - S_{n-1}$, where S_{n-1} is sum of $(n-1)$ terms of AP.

Example 5: Find the sum of first 20 terms of the Arithmetic Progression 5, 8, 11, 14, . . .

(a) 650 (b) 670 (c) 500 (d) 100

Solution: **(b)** We have

$a = 5, d = 8 - 5 = 3, n = 20$

$S_{20} = \frac{20}{2}[2 \times 5 + (20-1)3]$

$= 10[10 + 57] = 10 \times 67 = 670$

Example 6: What is the sum of first 21 terms of the Arithmetic Progression whose 2^{nd} term is 8 and 4^{th} term is 14.

(a) 535 (b) 635 (c) 735 (d) 750

Solution: **(c)**

Let a be the first term and d be the common difference

then $a + d = 8$

and $a + 3d = 14$

Subtracting $-2d = -6 \Rightarrow d = 3$

$\therefore \quad a = 8 - 3 = 5$

Thus $S_{21} = \frac{21}{2}[2 \times 5 + (21 - 1)\,3]$

$= \frac{21}{2} \times 70 = 735$

Arithmetic Mean

If $a, A, b,$ are in arithmetic progression, then A is called arithmetic mean.

$A = \frac{a+b}{2}$

$\Rightarrow \quad 2A = a + b$

Example 7: Find the arithmetic mean of $a - b$ and $a + b$.

(a) a (b) b (c) $a + b$ (d) $a - b$

Solution: **(a)**

Arithmetic Mean $= \frac{a-b+a+b}{2} = \frac{2a}{2} = a$

If three numbers are in arithmetic progression then the numbers are $a - d, a, a + d$.

Example 8: Find three numbers in Arithmetic Progression whose sum is 15 and Product is 80.

(a) 3, 5, 8 (b) 2, 5, 8 (c) 2, 5, 7 (d) 2, 5, 9

Solution: **(b)** Let the three numbers be $a - d, a, a + d$

then $a - d + a + a + d = 15$

$\Rightarrow \quad 3a = 15 \Rightarrow a = 5$

and Product = 80

then $(a - d)\,a(a + d) = 80$

$\Rightarrow \quad a(a^2 - d^2) = 80$

$\Rightarrow \quad 5(5^2 - d^2) = 80$

$\Rightarrow \quad 25 - d^2 = 16$

$\Rightarrow \quad d^2 = 25 - 16$

$\Rightarrow \quad d^2 = 9 \Rightarrow d = \pm 3$

Thus numbers are $(5 - 3), 5, (5 + 3)$

$= 2, 5, 8$ or, $8, 5, 2$.

Multiple Choice Questions

1. How many terms are there in the Arithmetic Progression?
   7, 11, 15, ...139?
   (a) 32 (b) 34
   (c) 31 (d) 30

2. Find the sum of all two-digits odd positive integers.
   (a) 2475 (b) 2375
   (c) 2175 (d) 1835

3. If the sum of n terms of an Arithmetic Progression is given by $S_n = 3n^2 + 2n$. Find its first term and common difference.
   (a) 2, 3 (b) 4, 5
   (c) 5, 6 (d) 5, 7

4. If $a_1, a_2, a_3 \ldots, a_n$ be an Arithmetic Progression of non-zero terms, then what is the value of $\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \ldots + \frac{1}{a_{n-1}a_n}$?
   (a) $\frac{n-1}{a_1 + a_n}$ (b) $\frac{n-1}{a_1a_n}$
   (c) $\frac{n-1}{a_1^2a_n^2}$ (d) None of these

5. What is the n^{th} term of the sequence $\log a$, $\log ab$, $\log ab^2$, $\log ab^3$, . . .
   (a) $\log b$ (b) $\log a^n b^n$
   (c) $\log (ab^{n-1})$ (d) $\log b^n$

6. If p^{th}, q^{th} and r^{th} terms of an arithmetic progression are a, b, c respectively, then what is the value of
   $a(q-r) + b(r-p) + c(p-q)$?
   (a) 0 (b) 1
   (c) $a + b + c$ (d) $p + d - r$

7. The sum of the three numbers in arithmetic progression is 12. The sum of their cubes is 288. What are the numbers?
   (a) 2, 4, 6 (b) 4, 6, 8
   (c) 6, 8, 10 (d) None of these

8. What is the sum of first 20 terms of an arithmetic progression in which third term is 7 and 7^{th} term is two more than thrice of its third term?
   (a) 740 (b) 800
   (c) 840 (d) 640

9. If in an arithmetic progression the sum of m terms is n and the sum of n terms is m then what is the sum of $(m + n)$ terms?
   (a) $m + n$ (b) $2m$
   (c) $-(m + n)$ (d) $2n$

10. The first term of an arithmetic progression is 2 and the last term is 50. The sum of all these terms is 442. What is the common difference?
    (a) 3 (b) –3 (c) 6 (d) –6

11. If the sum of first 4 terms is 24. 12^{th} term of this arithmetic progression is –13. What is the sum of first 20 terms?
    (a) 200 (b) –200
    (c) 0 (d) –100

12. The sum of first seven terms of an arithmetic progression is 10 and the sum of next seven terms is 17, then what is the common difference of this arithmetic progression?
    (a) 1 (b) 7 (c) $\frac{1}{7}$ (d) –7

13. If $\frac{b+c-a}{a}$, $\frac{c+a-b}{b}$, $\frac{a+b-c}{c}$ are in arithmetic progression then which of the following is correct?
    (a) a, b, c are in arithmetic progression
    (b) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in arithmetic progression
    (c) $\frac{1}{b}$, $\frac{1}{c}$, $\frac{1}{a}$ are in arithmetic progression
    (d) None of these

14. Find the sum of first 25 terms of an arithmetic progression whose n^{th} term is given by $7 - 3n$.
    (a) 800 (b) –800
    (c) 400 (d) –400

15. What is the sum of all three digit natural numbers which are multiples of 7?
    (a) 72336 (b) 70336
    (c) 72036 (d) 72396

16. The first, second and last term of an arithmetic progression are respectively 4, 7 and 31. How many terms are there in the given arithmetic progression?

(a) 14 (b) 10 (c) 9 (d) 11

17. What is the sum of first 30 terms of the arithmetic progression whose 2^{nd} term is 8 and 4^{th} term is 14?

(a) 1435 (b) 1495

(c) 1455 (d) 1465

18. If the m^{th} term of an arithmetic is $\frac{1}{n}$ and its n^{th} term $\frac{1}{m}$ then what is its $(mn)^{th}$ term?

(a) 0 (b) $m+n$

(c) 1 (d) -1

19. In an arithmetic progression if p^{th} term is q and q^{th} term is p. then what is its n^{th} term?

(a) $p+q$ (b) $p-q$

(c) $p+d-n$ (d) None of these

20. If 8^{th} term of an arithmetic progression is 31 and its 15^{th} term is 16 more than 11^{th} term of the arithmetic progression. What is the first term of that arithmetic progression?

(a) 4 (b) 3 (c) -4 (d) -3

21. What is the value of k for which $5k+2$, $4k-1$ and $k+2$ are in arithmetic progression?

(a) 2 (b) 3 (c) 4 (d) 6

22. The sum of three numbers in a rithmetic is 27 and their product is 405. What are the numbers?

(a) 3, 9, 15 (b) 1, 9, 17

(c) 5, 9, 13 (d) None of these

23. If four numbers in arithmetic progression are such that their sum is 50 and the greatest number is 4 times the least. Which number is the least one?

(a) 10 (b) 15 (c) 5 (d) 20

24. How many terms of an arithmetic progression 3, 7, 11, 15, ... are taken so that the sum is 406?

(a) 14 (b) 10 (c) 12 (d) 8

25. The sum of n terms of the three arithmetic progression are S_1, S_2, S_3. The first term of each arithmetic progression is unity. The common differences are 1, 2, 3 respectively, then which of the following options is correct?

(a) $S_1 + S_3 = 2S_2$

(b) $S_1 - S_3 = S_2$

(c) $S_1 + S_2 = S_3$

(d) $S_1 + S_3 + = S_2$

26. If the 3^{rd} and 7^{th} terms of an arithmetic progression are 17 and 27 respectively. Find the first term of arithmetic progression.

(a) 12 (b) 14

(c) 10 (d) 8

27. Find the sum of n terms of an arithmetic progression whose k^{th} term is $5k+1$

(a) $\frac{n}{2}(5n+1)$ (b) $\frac{n}{2}(5n+6)$

(c) $\frac{n}{2}(5n+7)$ (d) None of these

28. If the sum of n terms of an arithmetic progression is $nP + \frac{1}{2}n(n-1)Q$, where P and Q are constants. What is the common difference?

(a) P (b) $P+Q$

(c) $2Q$ (d) Q

29. If $S_1, S_2, S_3, \ldots S_m$ are the sum of n terms of m arithmetic progression whose first terms are 1, 2, 3, ... m and common differences are 1, 3, 5, ... $(2m-1)$ respectively. What is the value of $S_1 + S_2 + \ldots S_m$?

(a) $\frac{mn}{2}(mn+1)$

(b) $\frac{mn}{2}(mn-1)$

(c) $\frac{m+n}{2}(mn+1)$

(d) None of these

Answer Key

1. (b)	2. (b)	3. (c)	4. (b)	5. (c)	6. (a)	7. (a)	8. (a)	9. (c)	10 (a)
11. (b)	12. (c)	13. (b)	14. (b)	15. (b)	16. (b)	17. (c)	18. (c)	19. (c)	20. (b)
21. (b)	22. (a)	23. (c)	24. (a)	25. (a)	26. (a)	27. (c)	28. (d)	29. (a)	

Hints and Solution

1. (b) Given AP is 7, 11, 15, ...139

Here $a = 7, d = 11 - 7 = 4, l = 139$

$l = a + (n-1)\,d$

$\Rightarrow 139 = 7 + (n-1)\,4 \Rightarrow n - 1 = \dfrac{139-7}{4}$

$\Rightarrow n - 1 = 33$

$\Rightarrow \quad n = 34$

2. (b) Given AP is $11 + 13 + 15 + \ldots\ldots + 99$

Here $a = 11, d = 13 - 11 = 2, l = 99$

$l = a + (n-1)d$

$\Rightarrow 99 = 11 + (n-1)2 \Rightarrow n - 1 = \dfrac{99-11}{2}$

$\Rightarrow n - 1 = 44$

$\Rightarrow \quad n = 45$

Now $S_n = \dfrac{n}{2}[2a + (n-1)\,d]$

$= \dfrac{45}{2}[2 \times 11 + (45-1)2]$

$= \dfrac{45}{2}[22 + 88] = \dfrac{45}{2} \times 110$

$= 45 \times 55 = 2475$

3. (c)

Given $S_n = 3n^2 + 2n$

$\therefore \; S_1 = 3 \times 1^2 + 2 \times 1 = 5$

$S_2 = 3 \times 2^2 + 2 \times 2 = 16$

The sequence is 5, 11, ...

$a = 5, d = 11 - 5 = 6$

5. (c)

$\log a, \log ab, \log ab^2, \log ab^3, \ldots.$

$\log a, \log a + \log b, \log a + 2\log b, \log a + 3\log b$

$t_1 = \log a, d = \log a + \log b - \log a = \log b$

$t_n = t_1 + (n-1)d = \log a + (n-1)\log b$

$= \log a + \log b^{n-1}$

$= \log ab^{n-1}$

6. (a) Let x be the first term and d be the common difference.

p^{th} term $= a$

$x\,(p-1)d = a \qquad \ldots(1)$

q^{th} term $= b \Rightarrow x + (q-1)d = b \qquad \ldots(2)$

r^{th} term $= c \Rightarrow x + (r-1)d = c \qquad \ldots(3)$

$\therefore \; a(q-r) + b(r-p) + c(p-q)$

$= x[q - r + r - p + p - q] + d[(p-1)\,(q-r) + (q-1)\,(r-p) + (r-1)\,(q-q)]$

$\Rightarrow a(q-r) + b(r-p) + c(p-q) = 0$

7. (a) Let the numbers be $a - d, a, a + d$.

$a - d + a + a + d = 12 \Rightarrow 3a = 12 \Rightarrow a = 4$

and $(a-d)^3 + a^3 + (a+d)^3 = 288$

$a^3 - d^3 - 3ad(a-d) + a^3 + a^3 + d^3 + 3ad(a+d) = 288$

$a^3 - d^3 - 3ad(a-d) + a^3 + a^3 + a^3 + 3ad + 3ad^2 = 288$

$\Rightarrow 3a^3 + 6ad^2 = 288 \Rightarrow 3 \times 4^3 + 6 \times 4 \times d^2 = 288$

$\Rightarrow \quad 24d^2 = 288 - 192$

$\Rightarrow \quad d^2 = \dfrac{95}{24} = 4$

$\Rightarrow \quad d = \pm 2$

The numbers are, $4 - 2, 4, 4 + 2 = 2, 4, 6$

8. (a) Let a be the first term and d be the common difference.

$a + 2d = 7$

$a + 6d = 3(a + 2d) + 2$

$a + 6d = 3a + 6d + 2$

$a - 3a = 2 \Rightarrow -2a = 2 \Rightarrow a = -1$

and $a+2d=7 \Rightarrow 2d-7-a=7-(-1)=8$

$d=4$

$\therefore\ S_{20}=\frac{20}{2}[2\times -1+(20-1)4]$

$=10[-2+76]=10\times 74=740$

9. (c) Let a be the first term and d be the common difference.

$S_m=\frac{m}{2}[2a+(m-1)d]=n$

$2am+m(m-1)d=2n \quad (1)$

and $S_n=\frac{n}{2}[2a+(n-1)d]=m$

$2an+n(n-1)d=2m \quad (2)$

Subtracting (2) from eqn (1), we get

$2a(m-n)+[(m^2-n^2)-(m-n)]d=2(n-m)$

$\Rightarrow (m-n)[2a+(m+n-1)d]=2(n-m)$

$2a+(m+n-1)d=-2$

$S_{m+n}=\frac{m+n}{2}[2a+(m+n-1)d]$

$=\frac{m+n}{2}\times -2=-(m+n)$

10. (a) $a=2$, $l=50$

Now $S_n=\frac{n}{2}(a+l)\Rightarrow 442=\frac{n}{2}(2+50)$

$\Rightarrow n=\frac{2\times 442}{52}=17$

$\therefore\ l=2+(17-1)d\Rightarrow \frac{50-2}{16}=d\Rightarrow d=3$

12. (c) $S_7=10$, $S_{14}=10+17=27$

13. (b)

Given $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in arithmetic progression

$\Rightarrow \frac{b+c-a}{a}+2, \frac{c+a-b}{b}+2, \frac{a+b-c}{c}+2$ are in arithmetic progression

$\Rightarrow \frac{b+c-a}{a}, \frac{c+a+b}{b}, \frac{a+b+c}{c}$ are in arithmetic progression

$\Rightarrow \frac{b+c+a}{a(a+b+c)}, \frac{c+a+b}{b(a+b+c)}, \frac{a+b+c}{c(a+b+c)}$

are in arithmetic progression

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$, are in arithmetic progression

14. (b)

$t_n=7-3n$

$t_1=7-3-1=4\Rightarrow a=4$

$t_2=7-3-2=1$

$d=1-4=-3$

$S_{25}=\frac{25}{2}[2\times 4+(25-1)(-3)]$

$=\frac{25}{2}[8-72]=-800$

15. (b) Hint: The series is $105+112+119\ldots+994$

16. (b)

$a=4$, $d=7-4=3$, $t_n=31$

$\therefore\ t_n=a+(n-1)d$

$\Rightarrow 31=4+(n-1)3\Rightarrow n-1=\frac{31-4}{3}$

$\Rightarrow n-1=9\Rightarrow n=10$

17. (c) Let a be the first term and d be common difference.

$\therefore\ a+d=8;\ a+3d=14$

$S_{30}=\frac{30}{2}[2\times 5+(30-1)3]$

$=15[10+87]=15\times 97=1455$

18. (c) Let a be the first term and d be the common difference

$t_m=\frac{1}{n}$

$\Rightarrow a+(m-1)d=\frac{1}{n} \quad (1)$

and $t_n=\frac{1}{m}$

$\Rightarrow a+(n-1)d=\frac{1}{m} \quad (2)$

Subtracting eqn (2) from eqn (1)

$a+(m-1)d-a-(n-1)d=\frac{1}{n}-\frac{1}{m}$

$\Rightarrow d[m-1-n+1]=\frac{m-n}{mn}$

$$\Rightarrow d(m-n)=\frac{m-n}{mn}\Rightarrow d=\frac{1}{mn}$$

and $a=\frac{1}{mn}$

$$\therefore\ t_{mn}=\frac{1}{mn}+(mn-1)\frac{1}{mn}$$

$$=\frac{1}{mn}+\frac{mn}{mn}-\frac{1}{mn}=1$$

20. **(b)** Let a be the first term and d be the common difference

$a+7d=31$

$a+14d=a+10d+16$

$a+14d-a-10d=16\Rightarrow d=4$

$\therefore\ a=31-7d=31-28=3$

21. **(b)** Here

$4k-1-(5k+2)=(k+2)-(4k-1)$

$\Rightarrow\ 4k-1-5k-2=k+2-4k+1$

$\Rightarrow\ -k-3=-3k+3$

$\Rightarrow\ 2k=6\Rightarrow k=3$

22. **(a)** Let the numbers be $a-d, a, a+d$

Now $a-d+a+a+d=27\Rightarrow a=9$

and $(a-d)\,a\,(a+d)=405$

$a(a^2-d^2)=405$

$9(9^2-d^2)=405$

$\Rightarrow\ 81-a^2=45$

$\Rightarrow\ d^2=36\Rightarrow d=\pm 6$

The numbers are 3, 9, 15.

23. **(c)** Let the numbers are $a-3d$, $a-d$, $a+d$, $a+3d$

$a-3d+a-d+a+d+a+3d=50$

$\Rightarrow 4a=50\Rightarrow a=\frac{50}{4}=\frac{25}{2}$

$a+3d=4(a-3d)$

$\Rightarrow a+3d=4a-12d\Rightarrow 3a=15d$

$\Rightarrow\ a=5d$

$\Rightarrow\ \frac{25}{2}=5d$

$\Rightarrow\ d=\frac{5}{2}$

$$a-3d=\frac{25}{2}-\frac{15}{2}=\frac{10}{2}=5$$

24. **(a)** 3, 7, 11, 15.....

$a=3, d=4, S_n=406$

$S_n=\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow 406=\frac{n}{2}[2\times 3+(n-1)\,4]$

$\Rightarrow n[6+4n-4]=812$

$\Rightarrow 4n^2+2n=812$

$\Rightarrow 2n^2+n=406$

$\Rightarrow 2n^2+29n-28n-406=0$

$\Rightarrow n(2n+29)\times 14(2n+29)=0$

$\Rightarrow (n-14)\,(2n+29)=0$

$n=14,\ n=-\frac{-29}{2}$ (Not possible)

25. **(a)**

$$S_1=\frac{n}{2}[2\times 1+(n-1)\,1]$$

$$=\frac{n}{2}[2+n-1]$$

$$S_1=\frac{n}{2}(n+1)$$

$$S_2=\frac{n}{2}[2\times 1+(n-1)2]$$

$$=\frac{n}{2}[2+2n-2]=n^2$$

$$S_3=\frac{n}{2}[2\times 1+(n-1)\,3]$$

$$=\frac{n}{2}[2+3n-3]=\frac{n}{2}[3n-1]$$

Now $S_1+S_3=\frac{n}{2}[n+1+3n-1]$

$$=\frac{n}{2}\times 4n=2n^2=2S_2$$

26. **(a)** Let a be the first term and d be the common difference

$a+2d=17$

$a+6d=27$

Solving these eqns we get $a=12$

27. **(c)**

$t_k=5k+1$

$t_1=5\times 1+1=6$

$t_n=5n+1$

$\therefore\ S_n = \frac{n}{2}[a + l] = \frac{n}{2}[6 + 5_n + 1]$

$= \frac{n}{2}[5n + 7]$

28. (d)

$S_n = nP + \frac{1}{2}n(n-1)Q$

$S_1 = P + \frac{1}{2} \times 1(1-1)Q = P$

$S_2 = 2p + \frac{1}{2} \times 2(2-0)Q = 2P + Q$

$S_2 - S_1 = 2P + Q - P = P + Q$

$P, P + Q \ldots.$

Common difference $= Q$

29. (a)

$S_1 = \frac{n}{2}[2 \times 1 + (n-1)1]$

$S_1 = \frac{n}{2}[2 + n - 1] = \frac{n}{2}(n+1)$

$S_2 = \frac{n}{2}[2 \times 2 + (n-1)\,3]$

$= \frac{n}{2}[4 + 3n - 3] = \frac{n}{2}[3n + 1]$

$S_m = \frac{n}{2}[2m + (n-1)(2m-1)]$

$= \frac{n}{2}[2m + 2mn - n - 2m + 1]$

$= \frac{n}{2}[2mn - n + 1]$

$S_1 + S_2 + \ldots\ldots + S_m$

$a = S_1 = \frac{n}{2}(n+1)$

$d = S_2 - S_1 = \frac{n}{2}[3n + 1 - n - 1]$

$= \frac{n}{2}[2n] = n^2$

$S_1 + S_2 + \ldots. + S_m = \frac{m}{2}[S_1 + S_m]$

$= \frac{m}{2} \times \frac{n}{2}[n + 1 + 2mn - n + 1]$

$= \frac{mn}{2 \times 2}[2mn + 2]$

$= \frac{mn}{2}(mn + 1)$

6. Triangles

Triangle

It is a polygon with three edges and three vertices. It is one of the basic shapes in geometry.

Similar Triangles

Two triangles are said to be similar triangles to each other if

(*i*) their corresponding angles are equal

(*ii*) their corresponding sides are proportional.

Basic Proportionality Theorem (BPT) or Thales Theorem

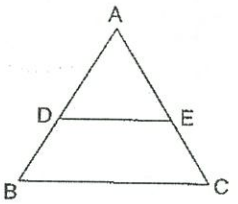
If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points then the other two sides are divided in the same ratio.

In ΔABC,

$DE \parallel BC$

DE intersects AB at D and AC at E, then

$$\frac{AD}{DB} = \frac{AE}{EC}$$



Corollary: In a ΔABC a line drawn parallel to BC intersects other two sides AB and AC at D and E respectively.

In ΔABC, $DE \parallel BC$

$$\frac{AB}{DB} = \frac{AC}{EC}$$

$$\therefore \quad \frac{AD}{AB} = \frac{AE}{AC}$$

Converse of Thales Theorem

If a line divides any two sides of a triangle in the same ratio then the line must be parallel to third side.

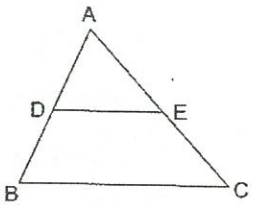
Example 1: In the given figure, in ΔABC, $DE \parallel BC$, so that $AD = 2.4$ cm, $AE = 3.2$ cm, and $EC = 4.8$ cm, find AB.

(a) 3.5 cm (b) 3.6 cm (c) 2.5 cm (d) 1 cm

Solution: **(b)** In ΔABC, $DE \parallel BC$

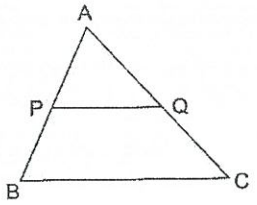
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{2.4}{DB} = \frac{3.2}{4.8} \Rightarrow DB = \frac{4.8 \times 2.4}{3.2} = 3.6 \text{ cm.}$$



Example 2: In the given figure $PQ \parallel BC$. If $AP = 1.7$ cm, $AB = 6.8$ cm, $AC = 9$ cm. Find AQ.

(a) 2.25 cm (b) 2.20 cm (c) 1.20 cm (d) 1.25 cm



Solution: (a) Given $PQ \parallel BC$,

$$\therefore \quad \frac{AP}{AB} = \frac{AQ}{AC}$$

$$\Rightarrow \frac{1.7}{6.8} = \frac{AQ}{9}$$

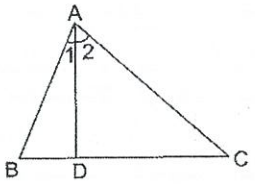
$$\Rightarrow AQ = \frac{9 \times 1.7}{6.8} = \frac{9}{4}$$

$$= 2.25 \text{ cm.}$$

Angle Bisector Theorem

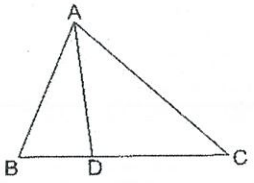
The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angles.

In ΔABC, AD is the angle bisector of $\angle A$.



Then $$\frac{BD}{DC} = \frac{AB}{AC}$$

Example 3: In the given figure AD is the bisector of $\angle BAC$, if $AB = 10$ cm, $AC = 6$ cm. $BC = 12$ cm, Find DC.



(a) 4 cm (b) 5 cm (c) 4.5 cm (d) 5.5 cm

Solution: (c) Let $BD = x$

$BC = 12$ cm

$DC = 12 - x$

By angle bisector theorem;

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{x}{12-x} = \frac{10}{6}$$

$$\Rightarrow \quad 6x = 120 - 10x$$

$$\Rightarrow \quad 16x = 120 \Rightarrow x = \frac{120}{16} = 7.5 \text{ cm}$$

$$\therefore \quad BD = 7.5 \text{ cm}$$

and $DC = 12 - 7.5 = 4.5$ cm

Example 4: In ΔABC, AD is bisector of $\angle BAC$, if $AB = 5.6$ cm, $BD = 3.2$ cm, $BC = 6$ cm. Find AC.

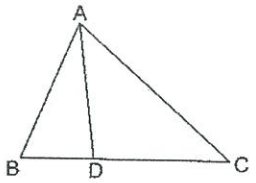
(a) 4.8 cm (b) 4.9 cm (c) 5 cm (d) 4 cm

Solution: **(b)**

Here $BD = 3.2$ cm

$BC = 6$ cm

$DC = 6 - 3.2 = 2.8$ cm



By angle bisector theorem,

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{3.2}{2.8} = \frac{5.6}{AC} \Rightarrow AC = \frac{2.8 \times 5.6}{3.2} = 4.9 \text{ cm}$$

Criteria for Similarity of Triangles

AAA Similarity: If in two triangles, the corresponding angles are equal then their corresponding sides are proportional and hence the two triangles are similar.

SSS Similarity: If the corresponding sides of two triangles are proportional then their corresponding angles are equal and hence triangles are similar.

SAS Similarity: If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional then the two triangles are similar.

Theorem: If a perpendicular is drawn from the vertex of a right angle to the hypotenuse the triangles on both sides of perpendicular are similar to the whole triangle and also to each other.

Example 5: In the given figure $\angle ABC = 90°$ and $BD \perp AC$ if $BD = 8$ cm, $AD = 4$ cm, find CD.

(a) 15 cm (b) 16 cm (c) 14 cm (d) 5 cm

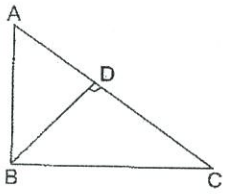
Solution: **(b)** In ΔABD and ΔBCD

$$\angle BDA = \angle BDC$$

$$\angle DBA = \angle DCB = 90° - \angle A$$

$$\therefore \quad \Delta ABD \sim \Delta BCD,$$

$$\therefore \quad \frac{BD}{CD} = \frac{AD}{BD}$$



$$\Rightarrow \quad CD = \frac{BD \times BD}{AD}$$

$$= \frac{8 \times 8}{4}$$

$$= 16 \text{ cm}$$

Areas of Similar Triangles: The ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides.

$$\Delta ABC \sim \Delta DEF$$

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$$

Example 6: $\Delta ABC \sim \Delta DEF$ and their areas are 64 cm^2 and 121 cm^2 respectively. If $EF = 15.4$ cm, find BC.

(a) 11.1 cm (b) 12 cm (c) 13 cm (d) 11.2 cm

Solution: **(d)**

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$\Rightarrow BC^2 = \frac{64 \times (15.4)^2}{121}$$

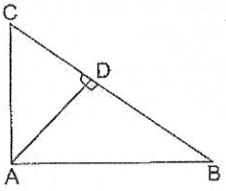
$$\Rightarrow BC = \frac{8 \times 15.4}{11}$$

$$= 8 \times 1.4 = 11.2 \text{ cm.}$$

Pythagoras Theorem

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$AB^2 + AC^2 = BC^2$$



Example 7: A ladder 25 m long just reaches the top of a building 24 m high from the ground. Find the distance of the foot of the ladder from the building.

(a) 5 m (b) 7 m (c) 6 m (d) 8 m

Solution: **(b)**

By Pythagoras's theorem,

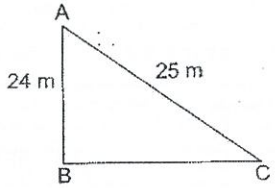
$$BC^2 = AC^2 - AB^2$$

$$BC = \sqrt{25^2 - 24^2}$$

$$BC = \sqrt{625 - 576}$$

$$BC = \sqrt{49}$$

$$= 7\text{m}$$



Example 8: Find the height of an equilateral triangle of side 12 cm.

(a) $6\sqrt{3}$ cm (b) $5\sqrt{3}$ cm (c) 4 cm (d) 6 cm

Solution: **(b)** ΔABC is an equilateral triangle

In ΔACD,

$$AD^2 + DC^2 = AC^2$$

$$\Rightarrow AD^2 + 6^2 = 12^2$$

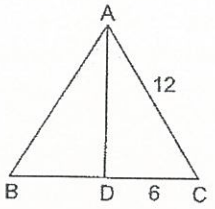
$\Rightarrow \quad AD^2 = 12^2 - 6^2$

$\Rightarrow \quad AD = \sqrt{144-36}$

$= \sqrt{108}$

$= \sqrt{36 \times 3}$

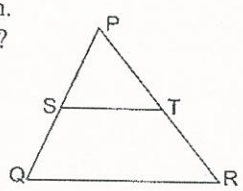
$= 6\sqrt{3}$ cm



Multiple Choice Questions

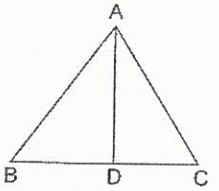
1 In the given figure in ΔPQR, $ST \parallel QR$, so that $PS = (7x - 4)$ cm, $PT = (5x - 2)$ cm, $QS = (3x + 4)$ cm, $RT = 3x$ cm. What is the value of x?

(a) 4
(b) 3
(c) 5
(d) 2



2. In ΔABC, $\frac{AB}{AC} = \frac{BD}{DC}$, $\angle B = 70°$, $\angle C = 50°$ then $\angle BAD = ?$

(a) 40°
(b) 30°
(c) 5°
(d) 50°

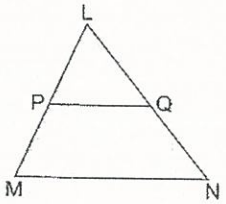


3. A ladder 15 m long reaches a window which is 9 m above the ground on one side of a street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window 12 m high. What is the width of the street?

(a) 21 m (b) 20 m
(c) 12 m (d) 15 m

4. In ΔLMN, P and Q are the mid-points of LM and LN respectively. What is the ratio of the area of ΔLPQ, and ΔLMN?

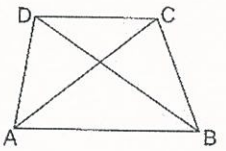
(a) 4:1
(b) 1:4
(c) 2:3
(d) 3:2



5. The corresponding altitude of two similar triangle are 6 cm and 9 cm respectively. What is ratio of their areas?

(a) 16 : 9
(b) 4 : 9
(c) 1 : 4
(d) 16 : 25

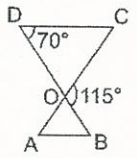
6. $ABCD$ is a trapezium in which $AB \parallel DC$ and $AB = 2CD$. If the diagonals of the trapezium intersect each other at point O. What is the ratio of the area of ΔAOB and ΔCOD?



(a) 1:4 (b) 4:1
(c) 1:9 (d) 9:1

7. In the given figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 115°$, $\angle CDO = 70°$. What is the value of $\angle OAB$?

(a) 45°
(b) 65°
(c) 35°
(d) 55°



8. Two poles of heights 6 m, and 11 m stand vertically on a plane ground. If the distance between their feet is 12 m. What is the distance between their tops?

(a) 13 m (b) 12 m
(c) 15 m (d) None of these

9. A tree is broken of 6 m from the ground and its top touches the ground at a distance of 8 m from the base of the tree. What is the original height of the tree?

(a) 20 m (b) 12 m
(c) 16 m (d) 14 m

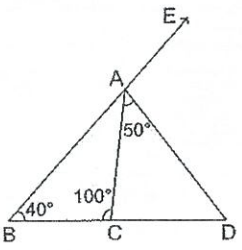
10. In a triangle ABC, $\angle B = 55°$, $\angle C = 35°$ then which of the following option is true?

(a) $BC^2 = AB^2 + AC^2$
(b) $AC^2 = AB^2 + BC^2$
(c) $AB^2 = BC^2 + AC^2$
(d) None of these

11. An exterior angle of a triangle measures 110° and its interior opposite angles are in the ratio 2:3. Which is the largest angle of the triangle?

(a) 66° (b) 70°
(c) 44° (d) 64°

12. In the figure given below, what is the measures of angle $\angle DAE$?



(a) 60°
(b) 50°
(c) 90°
(d) 80°

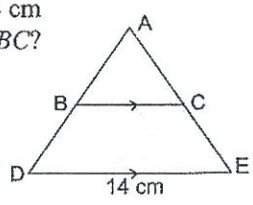
13. What is the perimeter of a rhombus the length of whose diagonal are 16 cm and 30 cm?

(a) 64 cm (b) 68 cm
(c) 72 cm (d) 76 cm.

14. Two isosceles triangles with equal vertical angles have their areas in the ratio 225:289. What is the ratio between corresponding altitudes?

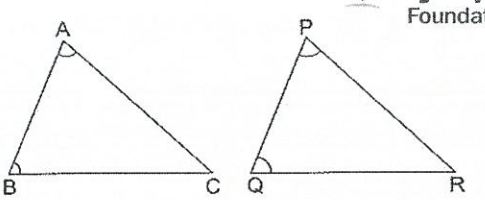
(a) 17:15 (b) 15:17
(c) 15:13 (d) 13:15

15. In the given figure BC is parallel to DE. Area of ΔABC is 25 cm^2. Area of trapezium $BCED$ is 24 cm^2, if $DE = 14$ cm
What is the length of BC?



(a) 15 cm
(b) 10 cm
(c) 28 cm
(d) 20 cm

16.



$\Delta ABC \sim \Delta PQR$ such that $\angle A = \angle P$ and $\angle B = \angle Q$
What is $\frac{ar(\Delta ABC)}{ar(\Delta PQR)}$ if $BC : QR = 9:7$?

(a) $\frac{81}{49}$ (b) $\frac{49}{81}$
(c) $\frac{81}{7}$ (d) $\frac{49}{9}$

17. A man goes 10 m due south and then 24 m due west. How far is he from the starting point?

(a) 26 m (b) 25 cm
(c) 27 cm (d) None of these

18. $\Delta ABC \sim \Delta APQ$, if $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm, $AP = 2.8$ cm. What is length of AC?

(a) 5.6 cm (b) 3.5 cm
(c) 6.5 cm (d) 2.8 cm

19. A vertical pole 12 m long casts a shadow of 8 m long on the ground. At the same time, a tower casts the shadow 40 m long on the ground. What is the height of the tower?

(a) 80 m
(b) 20 m
(c) 60 m
(d) None of these

20. ΔABC is a right angled triangle in which $\angle C = 90°$ $AC = \sqrt{3}BC$. What is the measure of $\angle B$?

(a) 30°
(b) 45°
(c) 60°
(d) None of these

21. In an isosceles triangle ΔABC if $AB = AC =$ 13 cm and altitude from A on BC is 5 cm. What is the length of BC?

(a) 12 cm
(b) 24 cm
(c) 6 cm
(d) None of these.

22. In an isosceles triangle $AB = AC = 25$ cm $BC = 14$ cm. What is the length of altitude from A on BC?
(a) 24 cm (b) 26 cm
(c) 12 cm (d) 13 cm

23. The Perimeter of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm. Then what is the corresponding side of the second circle?
(a) 5.4 cm (b) 4.5 cm
(c) 3.5 cm (d) 6.4 cm

24. The area of two equilateral triangle are in the ratio 196:169. What is the ratio between their perimeter?
(a) 13:14 (b) 14:13
(c) 1:14 (d) 14:1

25. In ΔABC, AD is bisector of $\angle A$. If $AB = 5.6$ cm, $AC = 4$ cm, $DC = 3$ cm. What is length of BC?
(a) 7.2 cm (b) 4.9 cm
(c) 6.2 cm (d) 5.8 cm

26. D and E are points on the sides AB and AC respectively of a ΔABC such that $DE \parallel BC$ find value of x if $AD = (7x - 4)$ cm, $AE = (5x - 2)$ cm, $DB = (3x - 4)$ cm, $EC = 3x$ cm
(a) 11 (b) 5 (c) 4 (d) 2

Anwer Key

1. (a)	2. (b)	3. (a)	4. (b)	5. (b)	6. (b)	7. (a)	8. (a)	9. (c)	10 (a)
11. (b)	12. (c)	13. (b)	14. (b)	15. (b)	16. (a)	17. (a)	18. ( )	19. (c)	20. (c)
21. (b)	22. (a)	23. (a)	24. (b)	25. (a)	26. (c)				

Hints and Solutions

1. (a) In ΔPQR and ΔPST

$\angle PST = \angle PQR$ $\left[\because ST \parallel QR, \therefore \angle PST \text{ and } \angle PQR \text{ are corresponding angles}\right]$

$\angle PTS = \angle PRQ$

$\therefore \Delta PST \sim \Delta PQR$

$\Rightarrow$ According to Thales' theorem

$$\frac{PS}{PQ} = \frac{PT}{PR}$$

Subtracting unity from both sides, we have,

$$\frac{PS}{QS} = \frac{PT}{RT} \Rightarrow \frac{7x-4}{3x+4} = \frac{5x-2}{3x}$$

$\Rightarrow 21x^2 - 12x = 15x^2 - 6x + 20x - 8$

$\Rightarrow 6x^2 - 26x + 8 = 0$

$\Rightarrow 3x^2 - 13x + 4 = 0$

$\Rightarrow 3x^2 - 12x - x + 4 = 0$

$\Rightarrow 3x(x-4) - 1(x-4) = 0$

$\Rightarrow (x-4)(3x-1) = 0$

$\Rightarrow x = 4, \frac{1}{3}$ $\left\{x \neq \frac{1}{3} \text{ because } PS < 0\right\}$

$\therefore x = 4$

2. (b) In ΔABD and ΔACD

$$\frac{AB}{AC} = \frac{BD}{CD}$$

By Angle Bisector Theorem

$\frac{AB}{AC} = \frac{BD}{CD}$, it is given,

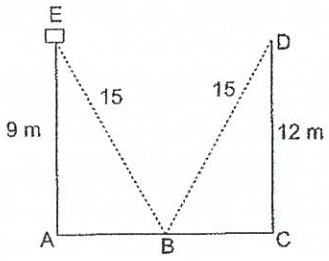
and $\angle BAC = 180 - 70 - 50$

$= 180 - 120 = 60°$

so, $\angle BAD = \frac{1}{2} \angle BAC$

$= \frac{1}{2} \times 60° = 30°$

3. (a)



In ΔABE,

$AE^2 + AB^2 = BE^2$

$\Rightarrow 9^2 + AB^2 = 15^2$

$\Rightarrow AB^2 = 225 - 81$

$= 144$

$\Rightarrow AB = 12$ m

Similarly,

In ΔBCD

$\Rightarrow BC^2 = BD^2 - DC^2 = 15^2 - 12^2 = 9^2$

$BC = 9$m

$\therefore$ width of street $= (AB + BC) = 12 + 9 = 21$ m

4. (b) $\because$ P and Q are mid-points of $\angle M$ and $\angle N$ respectively

$\therefore$ $PQ \| MN$ (according to midpoint theorem) and, also,

$$\frac{PQ}{MN} = \frac{LP}{LM} = \frac{LQ}{LN} = \frac{1}{2}$$

$$\therefore \frac{\text{Area}(\Delta LPQ)}{\text{Area}(\Delta LMN)} = \frac{\frac{1}{2} \times PQ \times LQ}{\frac{1}{2} \times MN \times LH}$$

$$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$, where G and H are feets of perpendicular dropped from L on PQ and MN respectively.

5. (b) Ratio of areas = (ratio of altitudes)^2

$$= \left(\frac{6}{9}\right)^2 = \frac{4}{9}$$

6. (b) $\angle CAB = \angle ACD$

$\{$*Alternate opposite* int*erior angles*$\}$

$\angle BDC = \angle DBA$

$\therefore$ $\Delta AOB \sim \Delta COD$ (SS similarity)

$$\therefore \frac{\text{Area}(\Delta AOB)}{\text{Area}(\Delta COD)} = \left(\frac{AB}{CD}\right)^2 = \left(\frac{2CD}{CD}\right)^2 = 4:1$$

7. (a) $\angle CDO + \angle DCO = 115°$

$\Rightarrow \angle DCO = 115° - 70° = 45°$

$\angle OAB = 45°$ ($\because \Delta ODC \sim \Delta OBA$)

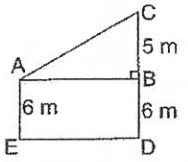
8. (a) $CB = CD - BD = 11\text{m} - 6\text{m} = 5\text{m}$

$ED = AB = 12$ m

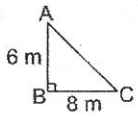
$\therefore AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{169} = 13$ m



9. (c)



A is the point of cleavage
C is the point where top of three touches the ground

In ΔABC

$AB^2 + BC^2 = AC^2$ (Pythagoras' Theorem)

$6^2 + 8^2 = AC^2$

$= AC = 10$ m

$\therefore$ Total (original) height $= 10\text{ m} + 6\text{ m} = 16\text{ m}$

10. (a) $\angle A = 180° - (\angle B + \angle C)$

$= 180° - (55° + 35°) = 90°$

$\therefore$ $\angle A$ is 90° or the triangle is right angled at A

$\therefore$ $AB^2 + AC^2 = BC^2$ (pythagoras' theorem)

11. (b) Let the angles be $2x$ and $3x$ respectively

$\therefore 2x + 3x = 110°$

$\Rightarrow 5x = 110°$

$\Rightarrow x = 22°$

$\therefore$ Angles are 44°, 66°, and 70°

12. (c) In ΔABC

$\angle BAC = 180° - (100° + 40°) = 40°$

$\angle DAE = 180° - (50° + 40°) = 90°$ [linear pair]

13. (b) Let the side of rhombus be x cm. Now, we know that the diagonals of rhombus bisect each other at right angles.

$\therefore$ By using Pythagoras' Theorem

$$x^2 = \left(\frac{16}{2}\right)^2 + \left(\frac{30}{2}\right)^2 = (8)^2 + (15)^2$$

$$= 64 + 225 = 289$$

$\Rightarrow x = 17$ cm

$\therefore$ Perimeter $= 4 \times x = 4 \times 17 = 68$ cm

14. (b) $\because$ Vertical angles of isosceles Δs are identical

$\therefore$ Other two equal angles of both the Δs will be identical.

$\therefore$ By using S-S-S similarity, we can say, that the two triangles are similar.

$\therefore$ Ratio of altitudes $= \sqrt{\text{Ratio of areas}}$

$$= \sqrt{\frac{225}{289}} = 15:17$$

15. (b) $\because BC \| DE$

$\therefore \Delta ABC \sim \Delta ADE \left[\because \begin{matrix}\angle B = \angle D \\ \angle C = \angle E\end{matrix} \left(\begin{matrix}\text{corresponding} \\ \text{angle}\end{matrix}\right)\right]$

$$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADE} = \left(\frac{BC}{DE}\right)^2$$

$$\Rightarrow \left(\frac{25}{25+24}\right) = \left(\frac{BC}{14}\right)^2$$

$[\text{Ar}(\Delta ADE) = \text{Ar}(\Delta ABC) + \text{Ar}(\square BCED)$

$$\Rightarrow \frac{BC}{14} = \frac{5}{7}$$

$\Rightarrow BC = 10$ cm

16. (a) $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\dfrac{BC}{QR}\right)^2 = \left(\dfrac{9}{7}\right)^2$

$$= \frac{81}{49} = 81:49$$

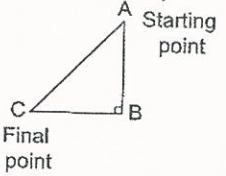
17. (a) In ΔABC

$AC^2 = AB^2 + BC^2$ (Pythagoras' theorem)

$\Rightarrow AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{(10)^2 + (24)^2}$

$= \sqrt{676} = 26$ m



18. $\because \Delta ABC \sim \Delta APQ$

$$\therefore \frac{BC}{PQ} = \frac{AC}{AQ} = \frac{BA}{AP}$$

$$\Rightarrow \frac{8}{4} = \frac{AC}{AQ} = \frac{6.5}{2.8}$$

19. (c) Both Δs are right angled, and also the elevation of coming sunlight will be same.

$\therefore$ All the angles of the ΔABC and ΔPQR will be correspondingly identical.

$\therefore \Delta ABC \sim \Delta PQR$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR}$$

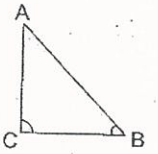
$$\Rightarrow \frac{12}{PQ} = \frac{8}{40}$$

$\Rightarrow PQ = 12 \times 5 = 60$m

20. (c) $\dfrac{AC}{BC} = \tan B$

$\Rightarrow \sqrt{3} = \tan B$

$\Rightarrow \angle B = 60°$



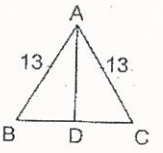
21. (b) In ΔABD

$AB^2 = BD^2 + AD^2$

$\Rightarrow BD = \sqrt{AB^2 - AD^2}$

$= \sqrt{13^2 - 5^2}$

$= 12$ cm



$\because D$ bisects BC

$\therefore BC = 2 \times BD$

$= 2 \times 12 = 24$ cm

22. (a) $BD = \dfrac{BC}{2} = \dfrac{14}{2} = 7$ cm

[fig. is in the previous ques]

$\therefore AD^2 = C^2 - D^2$

$= (25)^2 - (7)^2 = 576$

$\Rightarrow AD = 24$ cm

23. (a) We know that if, $\Delta ABC \sim \Delta PQR$, then

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

or, Applying componendo and dividendo, it can be clearly seen that,

$$\frac{\text{Perimeter}(\Delta ABC)}{\text{Perimeter}(\Delta PQR)} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\Rightarrow \frac{25}{15} = \frac{9}{PQ}$$

$$\Rightarrow PQ = \frac{9}{25} \times 15 = 5.4 \text{ cm}$$

24. (b) Ratio of perimeters = ratio of sides

$$= \sqrt{\text{ratio of areas}}$$

$$= \sqrt{\frac{196}{169}}$$

$$= \frac{14}{13} = 14:13$$

25. (a) By angle bisector theorem,

$$\frac{BD}{DC} = \frac{AB}{AC} \Rightarrow \frac{BD}{3} = \frac{5.6}{4}$$

$$\Rightarrow BD = \frac{3 \times 5.6}{4} = 4.2$$

$$BC = BD + DC = 4.2 + 3 = 7.2 \text{ cm}$$

26. (c) $\because DE \| BE$

$\therefore \Delta ADE \sim \Delta ABC$

$$\frac{AD}{AB} = \frac{AF}{EC}$$

$$\Rightarrow \frac{AD}{AB} - 1 = \frac{AE}{EC} - 1$$

$$\Rightarrow \frac{-DB}{AD} = \frac{-FC}{AE}$$ adjusting this expression, we have

$$\Rightarrow \frac{AD}{AB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{7x-4}{3x+4} = \frac{5x-2}{3x}$$

$$\Rightarrow 21x^2 - 12x = 15x^2 - 6x - 8 + 20x$$

$$\Rightarrow 6x^2 - 26x + 8 = 0$$

$$\Rightarrow 6x^2 - 24x - 2x + 8 = 0$$

$$\Rightarrow 6x(x-4) - 2(x-4) = 0$$

$$\Rightarrow x = \frac{2}{6}, 4 = \frac{1}{3}, 4$$

When, $x = \frac{1}{3}$, $AD = 7 \times \frac{1}{3} - 4 < 0$

$\therefore x \neq \frac{1}{3}$, so, $x = 4$

7. Co-Ordinate Geometry

Cartesian Co-ordinate

The concept of co-ordinate system in a plane enables us to represent points by ordered pairs of real numbers called cartesian co-ordinate.

Distance between Two Points

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance of P(x, y) from O$(0, 0)$ origin is given by

$$OP = \sqrt{(x_1 - 0_1)^2 + (y_1 - 0)^2} = \sqrt{x_1^2 + y_1^2}$$

Example 1: Find the distance of the point $(5, -6)$ from the origin.

(a) $\sqrt{61}$ (b) $\sqrt{51}$ (c) $\sqrt{41}$ (d) $\sqrt{31}$

Solution: **(a)** Origin has co–ordinate (0,0)

$\therefore$ Required Distance $= \sqrt{(5-0)^2 + (-6-0)^2}$

$= \sqrt{25+36} = \sqrt{61}$ units

Example 2: Find the point on y–axis, each of which is at a distance of 13 units from the point $(-5,7)$.

(a) $(0, 5), (0, 19)$ (b) $(0, -5), (0, 19)$

(c) $(0, 5), (0, -19)$ (d) None of these

Solution: **(a)** Let $A(-5, 7)$ be the given point and $B(0, y)$ be the point on axis

$$\therefore \quad AB = 13$$

$$AB^2 = 13^2$$

$$\Rightarrow \quad (0+5)^2 + (y-7)^2 = 13^2$$

$$\Rightarrow \quad 25 + y^2 + 49 - 14y = 169$$

$$\Rightarrow \quad y^2 - 14y - 95 = 0$$

$$\Rightarrow \quad y^2 - 19y + 5y - 95 = 0$$

$$\Rightarrow \quad y(y-19) + 5y - 95 = 0$$

$$\Rightarrow y(y-19) + 5(y-19) = 0$$

$$\Rightarrow \quad y = -5, y = 19$$

The points are $(0, -5)$ and $(0, 19)$.

Section Formula

The co–ordinates of the point $P(x, y)$ which divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ is given by

$$x = \frac{mx_2 + nx_1}{m+n}, \; y = \frac{my_2 + ny_1}{m+n}$$

in case of external division

$$x = \frac{mx_2 - nx_1}{m-n}, y = \frac{my_2 - ny_1}{m-n}$$

Example 3: What is the co–ordinate of the points which divides the join of $A(-1,7)$ and $B(4,-3)$ in the ratio 2:3?

(a) 1, 3 (b) 3, 1 (c) 2, 3 (d) 3, 3

Solution: **(a)**

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{2(4)+3(-1)}{2+3} = \frac{5}{5} = 1$$

$$y = \frac{my_2 + ny_1}{m+n} = \frac{2(-3)+3(7)}{2+3} = \frac{15}{5} = 3$$

The point is (1, 3).

Mid-point Formula

If $A(x_1, y_1)$ and $B(x_2,y_2)$ are two points, then the mid–point of AB is given by $P(x,y)$ where

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

Example 4: Find the co-ordinate of the mid-point of the line segment $A(4,-6)$ and $B(2,4)$.

(a) (1, 3) (b) (–1, 3) (c) (3, –1) (d) (3, 1)

Solution: **(c)** Here $= \frac{x_1 + x_2}{2} = \frac{4+2}{2}$

$$= \frac{6}{2} = 3.$$

and $\frac{y_1 + y_2}{2} = \frac{-6+4}{2} = \frac{-2}{2} = -1.$

$\therefore$ Mid point is (3, –1).

Centroid of a Triangle

The point of intersection of the medians of a triangle is called centroid

Co-ordinate of Centroid of a Triangle

If $A(x_1,y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a ΔABC then co-ordinate of centroid (G) is given by

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$

Example 5: Find the centroid of ΔPQR whose vertices are $P(-2, 4)$, $Q(4, 2)$, $R(1,-3)$.

(a) 1, 1 (b) 1, 2 (c) 2, 2 (d) 2, 3

Solution: **(a)** Centroid $(G) = \left[\frac{-2+4+1}{3}, \frac{4+2-3}{3}\right]$

$$= \left(\frac{3}{3}, \frac{3}{3}\right) = (1, 1)$$

Area of a Triangle

The area of ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by area (ΔABC)

$$= \left|\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$$

If the area in (i) comes out to be negative number sometimes then you take its modules or absolute value because area cannot be negative.

If area is zero the points are collinear and vice-versa.

Example 6: Find the area of triangle whose vertices are $A(4,4)$, $B(3, -16)$ and $C(3,-2)$.

(a) 7 sq. units (b) 5 sq. units (c) 6 sq. units (d) 4 sq. units

Solution: **(a)**

Area (ΔABC)

$$= \left|\frac{1}{2}[4(-16+2) + 3(-2-4) + 3(4+16)]\right|$$

$$= \left|\frac{1}{2}[-56 - 18 + 60]\right| = \left|\frac{1}{2}[-14]\right|$$

$$= \frac{1}{2} \times 14 = 7 \text{ sq. units.}$$

Three points $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ will be collinear if area of $\Delta ABC = 0$.

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

Example 7: Show that the points $(-1,1)$ $(5,7)$ and $(8, 10)$ are collinear.

Solution: Let the points be $A(-1, 1)$, $B(5, 7)$ and $C(8, 10)$.

Area of ΔABC

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[(-1)(7-10) + 5(10-1) + 8(1-7)]$$

As area $(\Delta ABC) = 0$

So, the given points are collinear.

Multiple Choice Questions

1. Find the value of k for which the points $A(3, 2)$, $B(4, k)$ and $C(5, 3)$ are collinear.

   (a) $\frac{5}{2}$ (b) $\frac{2}{5}$

   (c) $\frac{3}{5}$ (d) $\frac{1}{5}$

2. Find the third vertex of a ΔABC if two of its vertices are $B(-3,1)$, $C(0, -2)$ and its centroid is at the origin.

   (a) (1, 3) (b) (3, 1)

   (c) (3, −1) (d) (0, 3)

3. Find ratio in which the line $2x + y - 4 = 0$ divides the line segment joining $A(2, -2)$ and $B(3, 7)$.

   (a) 9:7 (b) 7:2

   (c) 2:9 (d) 1:2

4. The circumcentre of a triangle is (3, 3). If its two vertices are (4, 6) and (0, 4) find the third vertex of the triangle.

(a) (2, 4) (b) (6, 2)
(c) (2, 6) (d) (4, 4)

5. The co-ordinates of ends of a diameter of a circle are (4, –1) and (–2, –5). Find the centre of the circle.
(a) (–3, 1) (b) (1, –3)
(c) (0, –3) (d) (–3, 0)

6. In which ratio does the point P (1, 2) divides the join of A (–2, 1) and B (7, 4)?
(a) 1:2 (b) 2:1
(c) 2:3 (d) 3:2

7. The area of a triangle with vertices $(a, b + c)$ and $(b, c + a)$ and (c, $b + a$) is
(a) 0 (b) abc
(c) $a + b + c$ (d) $(a + b + c)^2$

8. If (2, –2), (–2, 1) and (5, 2) are vertices of a right angled triangle, then the area of triangle is
(a) 24.559 units (b) 12.5 sq. units
(c) 12 sq. units (d) 24 sq. units.

9. Find the co-ordinates of the points which trisects the line joining (–3, 5) and (6, –7).
(a) (2, 0) (b) (2, 2) and (0, 1)
(c) (–1, –1) and (0, 3) (d) (0, 1) and (3, 3)

10. The centre of a circle is (4, 5) A(8, 10) is a point on the circumference. Find the other end of diameter of the circle through A.
(a) (0, 0) (b) (0, 1)
(c) (1, 0) (d) (2, 0)

11. The ends of a diagonal of a square have the co–ordinates $(a, 1)$ and $(-1, a)$, find a if the area of the square is 50 square units.
(a) ±7 (b) ±5
(c) ±6 (d) ±4

12. The ends of a diameter of a circle have the co–ordinates (4, 3) and (–4, –3), PQ is another diameter where P has co–ordinate $\left(\frac{5}{\sqrt{2}}, \frac{-5}{\sqrt{2}}\right)$ find the co-ordinates of Q.
(a) $\left(\frac{-5}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$ (b) $\left(\frac{5}{\sqrt{2}}, 0\right)$
(c) $\left(0, \frac{5}{\sqrt{2}}\right)$ (d) (0, 0)

13. Two vertices of a triangle are (2, –4) and (1, 3). If the origin is the centroid of the triangle then what is the third vertex?
(a) (1, 3) (b) (–3, 1)
(c) (1, –1) (d) (3, –3)

14. The vertices of a triangle are A(1, 1), B(–2, –5) and C(2, 2) find the length of medium through C.
(a) $\frac{5}{\sqrt{2}}$ (b) $\frac{25}{2}$
(c) $\frac{5}{2}$ (d) $\frac{\sqrt{61}}{2}$

15. What is the locus of a point equidistant from the point (2, 4) and y–axis?
(a) $y^2 - 8x - 4y + 20 = 0$
(b) $y^2 - 4x - 8y + 20 = 0$
(c) $x^2 - 4x + 4y + 20 = 0$
(d) $y^2 - 4x - 8y + 12 = 0$

16. What is the circumradius of the triangle whose vertices are (2, –2), (8, 6) and (8, –2)?
(a) 25 (b) 5
(c) $\sqrt{5}$ (d) None of these

17. If the co-ordinates of mid–point of the sides of a triangle are (1, 1) (2, –3) and (3, 4) what is the centroid?
(a) (6, 2) (b) $\left(2, \frac{2}{3}\right)$
(c) (2, 1) (d) (3, 2)

18. The sum of square of the distance of a moving point from two fixed points $(a, 0)$ and $(-a, 0)$ is equal to constant quantity $2c^2$. Find the equation of its locus.
(a) $x^2 + y^2 = c^2$
(b) $x^2 + y^2 = 2c^2$
(c) $x^2 + y^2 = c^2 - a^2$
(d) $x^2 + y^2 = a^2$

19. What is the value of k, so that the points A(8, 1), B(3, –4) and C(2, K) are collinear?
(a) 5 (b) –5 (c) 7 (d) –6

20. If the points $(a, 0)$, $(0, b)$ and (1, 1) are collinear then what is the value of $\frac{1}{a} + \frac{1}{b}$?
(a) 1 (b) 2 (c) –1 (d) –2

21. What is the distance of the point (4, 7) from the y–axis?

(a) 7 (b) 4 (c) 11 (d) 12

22. What is the distance between the points $(\cos\theta, \sin\theta)$ and $(\sin\theta, -\cos\theta)$ is?

(a) $\sqrt{2}$ (b) 2

(c) 1 (d) $\sqrt{3}$

23. If the distance between the points (3, 0) and $(0, y)$ is 5 units. y is positive then what is value of y?

(a) 3 (b) 2 (c) 4 (d) 1

24. If the centroid of the triangle formed by points $P(a, b)$, $Q(b, c)$ and $R(c, a)$ is at the origin. What is the value of $a + b + c$?

(a) 0 (b) 1 (c) 2 (d) 3

25. What is the value of 'a' except zero for which the area of the triangle formed by the points $A(a, 2a)$, $B(-2, 6)$ $C(3, 1)$ is 10 square units?

(a) $\frac{3}{8}$ (b) $\frac{8}{3}$

(c) -3 (d) None of these

26. If the point $P(-1, 2)$ divides externally the line segment joining $A(2, 5)$ and B in the ratio 3:4. What is the co–ordinate of the point B?

(a) (5, 2) (b) $(-5, -2)$

(c) $(5, -2)$ (d) None of these.

27. In what ratio does the y-axis divide the line segment joining the point $A(-4, 5)$ and $B(3, -7)$?

(a) 4:3 (b) 3:4

(c) 2:5 (d) 5:2

28. The points $A(3, 1)$, $B(0, 4)$, $C(-3, 1)$, $D(0, -2)$ are vertices of a

(a) rectangle (b) square

(c) parallelogram (d) rhombus

29. P is a point on x-axis at a distance of 3 units from y-axis to the right. What is the co-ordinate at P?

(a) (3, 0) (b) (0, 3)

(c) (3, 3) (d) $(-3, 3)$

30. If the point $(x, 4)$ lies on a circle whose centre is at the origin and radius is 5, What is the value of x?

(a) ± 4 (b) 0

(c) ± 3 (d) ± 5

31. If $(-2, 1)$ is the centroid of the triangle having its vertices $(x, 0)$, $(5, -2)$ and $(-8, y)$ then which of the following is correct?

(a) $3x - 8y = 0$ (b) $5x + 3y = 0$

(c) $8x + 3y = 0$ (d) $8x - 3y = 0$

32. If the centroid of the triangle formed by $(7, x)$, $(y, -6)$ and (9, 10) is at (6, 3) then $(x, y) = ?$

(a) (4, 5) (b) (5, 4)

(c) $(-5, -2)$ (d) (5, 2)

33. Find the area of the triangle formed by the line $5x - 3y + 15 = 0$ with coordinate axes.

(a) 15 cm^2 (b) 5 cm^2

(c) 8 cm^2 (d) $\frac{15}{2}$ cm^2

34. Equation of a line whose inclination is 45° and making an intrercept of 3 units on X-axis is

(a) $x + y - 3 = 0$ (b) $x - y - 3 = 0$

(c) $x - y + 3 = 0$ (d) $x + y + 3 = 0$

35. The centre of a circle is $C(2, k)$. If $A(2, 1)$ and $B(5, 2)$ are two points on its circumference, then the value of k is

(a) 6 (b) 2 (c) -6 (d) -2

36. The lines $x = -1$ and $y = 4$ are _____.

(a) perpendicular to each other

(b) parallel to each other

(c) neither parallel nor perpendicular to each other

(d) None of these

37. The distance between the points $(2k + 4, 5k)$ and $(2k, -3 + 5k)$ in units is

(a) 1 (b) 2 (c) 4 (d) 5

Answer Key

1. (a)	2. (b)	3. (c)	4. (b)	5. (b)	6. (a)	7. (a)	8. (b)	9. (d)	10 (a)
11. (a)	12. (a)	13. (b)	14. (d)	15. (b)	16. (b)	17. (b)	18. (b)	19. (a)	20. (a)
21. (b)	22. (a)	23. (c)	24. (a)	25. (d)	26. (b)	27. (a)	28. (b)	29. (a)	30. (c)
31. (b)	32. (d)	33. (d)	34. (b)	35. (a)	36. (a)	37. (d)			

Hints and Soltions

1. **(a)** The points A (3, 2), B (4, K) and C (5, 3) are collinear then area (ΔABC) = 0

$$\Rightarrow \frac{1}{2}[3(k-3)+4(3-2)+5(2-k)]=0$$

$$\Rightarrow 3k-9+4+10-5k=0$$

$$\Rightarrow -2k+5=0 \Rightarrow k=\frac{5}{2}$$

2. **(b)** B (–3, 1) C (0, –2)

Let the third vertex be A (x, y)

G = (0, 0)

$$\frac{x-3+0}{3}=0 \Rightarrow x=3 \text{ and } \frac{y+1-2}{3}=0$$

$$\Rightarrow y-1=0 \Rightarrow y=1$$

A (3, 1)

3. **(c)**

A (2, –2) — P (x, y) — B (3, 7)

Let the point P (x, y) divides the line AB in the ratio k : 1

$$x=\frac{3k+2}{k+1}; y=\frac{7k-2}{k+1}$$

This point $P(x,y)$ lies on the line

$2x+y-4=0$

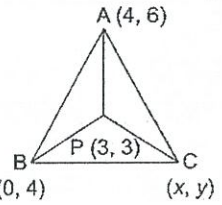
$$\Rightarrow 2\frac{(3k+2)}{k+1}+\frac{7k-2}{k+1}-4=0$$

$$\Rightarrow 6k+4+7k-2-4k-4=0$$

$$\Rightarrow 9k-2=0 \Rightarrow 9k=2$$

$$\Rightarrow k=\frac{2}{9}=2:9$$

4 **(b)** Let P (3, 3) be the circumcentre of ΔABC, $C(x, y)$ be third vertex.



$PA = PB = PC$

$PA^2 = PB^2 = PC^2$

$PA^2 = PC^2$

$$\Rightarrow (4-3)^2+(6-3)^2 = (x-3)^2+(y-3)^2$$

$$\Rightarrow 1+9=x^2+9-6x+y^2+9-6y$$

$$\Rightarrow x^2+y^2-6x-6y+8=0 \quad ...(1)$$

and $PB^2 = PC^2$

$$\Rightarrow (0-3)^2+(4,-3)^2=(x-3)^2+(y-3)^2$$

$$\Rightarrow 9+1=x^2+9-6x+y^2-6y+9$$

Equations (1) and (2) are identical

$(x-3)^2+(y-3)^2=10$

So, (6, 2) is the required point.

5. **(b)** Ends of diameter of circle are (4, –1) and (–2, –5)

centre = Mid–point of (4, –1) and (–2, –5)

$$=\left[\frac{4-2}{2},\frac{-1-5}{2}\right]=(1,-3)$$

6. **(a)**

A (–2, 1) — P (1, 2) — B (7, 4)

Let the ratio be k:1.

$$1=\frac{7k+1(-2)}{k+1} \Rightarrow k+1=7k-2$$

$$\Rightarrow 6k=3$$

$$\Rightarrow k=\frac{3}{6}=\frac{1}{2}$$

7. **(a)** Area of triangle

$$= \frac{1}{2}\Big[a(c+a-a-b)+b(a+b-b-c) + c(b+c-c-a)\Big]$$

$$= \frac{1}{2}[a(c-b)+b(a-c)+c(b-a)]$$

$$= \frac{1}{2}[ac-ab+ab-bc+bc-ac]$$

$$= \frac{1}{2}\times 0 = 0$$

8. **(b)** Area of triangle

$$= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

$$= \frac{1}{2}[2(1-2)+(-2)(2+2)+5(-2-1)]$$

$$= \frac{1}{2}[-2-8-15] = \frac{25}{2} = 12.5 \text{ sq. units}$$

9. **(d)** Let $P(x,y)$ be the point

$$x = \frac{2(6)+1(-3)}{2+1} = \frac{12-3}{3} = \frac{9}{3} = 3$$

$$y = \frac{2(-7)+1(5)}{2+1} = \frac{-14+5}{3} = \frac{-9}{3} = -3$$

or

$$x = \frac{1(6)+2(-3)}{1+2} = \frac{6-6}{3} = \frac{0}{3} = 0$$

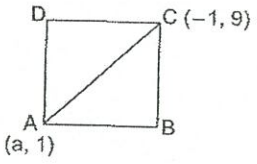
$$y = \frac{1(-7)+2(5)}{1+2} = \frac{3}{3} = 1$$

10. (a) Let the other end be (x, y)

$$\frac{8+x}{2} = 4 \Rightarrow x = 8-8 = 0$$

$$\frac{10+y}{2} = 5 \Rightarrow y = 10-10 = 0$$

11. **(a)**



Let side of the square be x.

$$x^2 + x^2 = (-1-a)^2 + (a-1)^2$$

$$\Rightarrow 2x^2 = 1 + a^2 + 2a + a^2 + 1 - 2a$$

$$\Rightarrow x^2 = a^2 + 1$$

$$\Rightarrow 50 = a^2 + 1 \Rightarrow a^2 = 49$$

$$\Rightarrow a = \pm 7$$

12. **(a)** Mid–point of (4, 3) and (–4, –3)

$$= \left[\frac{4-4}{2}, \frac{3-3}{2}\right] = (0, 0)$$

$$0 = \frac{\frac{5}{\sqrt{2}}+x}{2} \Rightarrow x = \frac{-5}{\sqrt{2}}$$

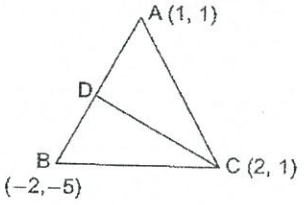
$$0 = \frac{\frac{-5}{\sqrt{2}}+y}{2} \Rightarrow y = \frac{5}{\sqrt{2}}$$

13. **(b)** $\frac{2+4+x}{3} = 0 \Rightarrow x = -3$

$$\frac{-4+3+y}{3} = 0 \Rightarrow y = 1$$

third vertex = (–3, 1)

14. **(d)**

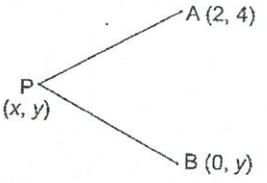


$$CD = \sqrt{\left(\frac{1}{2}-2\right)^2 + (-2-1)^2}\left(-\frac{1}{2}, -2\right)$$

$$CD = \sqrt{\frac{25}{4}+9}$$

$$= \sqrt{\frac{25+36}{4}} = \frac{\sqrt{61}}{2}$$

15. **(b)**



Let, the point be $P(x,y)$.

Given $AP = BP$

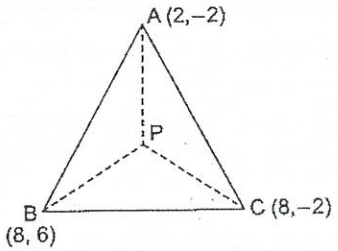
$\Rightarrow AP^2 = BP^2$

$\Rightarrow (x-2)^2 + (y-4)^2 = (x-0)^2 + (y-y)^2$

$\Rightarrow x^2 + 4 - 4x + y^2 + 16 - 8y = x^2$

$\Rightarrow y^2 - 4x - 8y + 20 = 0$

16. (b)



Let $P(x, y)$ be the circumcentre

$PA = PB = PC$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-2)^2 + (y+2)^2$

$= (x-8)^2 + (y-6)^2$

$\Rightarrow x^2 + 4 - 4x + y^2 + 4 - 4y$

$= x^2 + 64 - 16x + y^2 + 36 - 12y$

$\Rightarrow -4x + 16x + 4y + 12y = 100 - 8$

$\Rightarrow 12x + 16y = 92 \Rightarrow 3x + 4y = 23$ ...(1)

and $PB^2 = PC^2$

$\Rightarrow (x-8)^2 + (y-6)^2 = (x-8)^2 + (y+2)^2$

$\Rightarrow y^2 + 36 - 12y = y^2 + 4 + 4y$

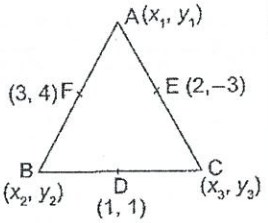
$\Rightarrow 16y = 32 \Rightarrow y = 2$

$3x = 23 - 4 \Rightarrow 2 = 15 \Rightarrow x = 5$

$P(5, 2) \therefore PA = \sqrt{(5-2)^2 + (2+2)^2}$

$= \sqrt{9+16} = 5$

17. (b) $\frac{x_1 + x_2}{2} = 3$



$\Rightarrow \quad x_1 + x_2 = 6$

$y_1 + y_2 = 8$

$x_2 + x_3 = 2$

$y_2 + y_3 = 2$

$x_1 + x_3 = 4$

$y_1 + y_3 = -6$

$x_1 + x_2 + x_3 = \frac{12}{2} = 6$

$x_1 = 6 - 2 = 4; x_2 = 2, x_3 = 0$

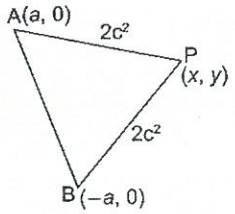
$y_1 + y_2 + y_3 = 2$

$y_1 = 0; y_2 = 8; y_3 = -6$

Centroid $= \left[\frac{4+2+0}{3}, \frac{0+8-6}{3}\right]$

$= \left(2, \frac{2}{3}\right)$

18. (b)



Here $(x-a)^2 + (y-0)^2 = 2c^2$

$x^2 + a^2 - 2ax + y^2 = 2c^2$

and $(x+a)^2 + (y-0)^2 = 2c^2$

$\Rightarrow x^2 + a^2 + 2ax + y^2 + 2c^2$

Now $x^2 + a^2 - 2ax + y^2 = x^2 + a^2 + 2ax + y^2$

$\Rightarrow 4ax = 0 \Rightarrow x = 0$ or $a = 0$

$(x-0)^2 + y^2 = 2c^2$

$\Rightarrow x^0 + y^2 = 2c^2$

20. (a) area of $\Delta ABC = 0$

$\frac{1}{2}[a(b-1) + 0(1-0) + 1(0-b)] = 0$

$\Rightarrow ab - a - b = 0$

$\Rightarrow ab = a + b$

$\Rightarrow \frac{ab}{ab} = \frac{a}{ab} + \frac{b}{ab} \Rightarrow 1 = \frac{1}{b} + \frac{1}{a}$

21. (b) Distance of the point (4, 7) from $(0, y)$

$= x$ co-ordinate

$= 4$

22. (a) Required Distance

$= \sqrt{(\sin\theta - \cos\theta)^2 + (-\cos\theta - \sin\theta)^2}$

$= \sqrt{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta}$

$= \sqrt{2(\sin^2\theta + \cos^2\theta)} = \sqrt{2 \cdot 1}$

$= \sqrt{2}$

23. **(c)** $(3-0)^2+(0-y)^2=5^2$

$9+y^2=25 \Rightarrow y^2=16 \Rightarrow y=\pm 4$

y is positive, $\therefore y=4$

24. **(a)** $\frac{a+b+c}{3}=0 \Rightarrow a+b+c=0$

25. **(d)** Area of triangle = 10

$\Rightarrow \frac{1}{2}[a(6-1)+(-2)(1-2a)+3(2a-6)]=10$

$\Rightarrow 6a-6-2+4a+6a-18=20$

$\Rightarrow 16a=20+26$

$\Rightarrow 16a=46 \Rightarrow a=\frac{46}{16}=\frac{23}{8}$

26. **(b)** $\frac{3x+4\times 2}{3+4}=-1$

$\Rightarrow 3x=-7-8 \Rightarrow 3x=-15$

$\Rightarrow x=-5$

$\frac{3y+4\times 5}{3+4}=2 \Rightarrow 3y=14-20$

$\Rightarrow 3y=-6 \Rightarrow y=-2$

$\therefore B=(-5,-2)$

27. **(a)** $\therefore$ Point is on y-axis

A (−4, 5) — P (0, y) — B (3, −7)

By section formula

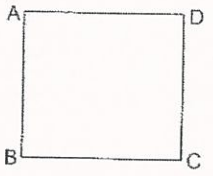
$0=\frac{k(3)+1(-4)}{k+1}$

$\Rightarrow 3k-4=0$

$\Rightarrow k=\frac{4}{3}$

Ratio = 4 : 3

28. **(b)**



By distance formula, we get

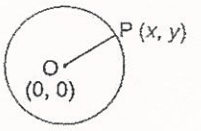
$AB=BC=CD=DA$

and $AC=BD$

so, the given points are vertices of a square.

29. **(a)** As point P on x-axis, then its co-ordinate $=(3, 0)$

30. **(c)** $OP=5$



$\Rightarrow OP^2=25$

$\Rightarrow (x-0)^2+(4-0)^2=25$

$\Rightarrow x^2+16=25$

$\Rightarrow x^2=9 \Rightarrow x=\pm 3$

31. **(b)** Here $\frac{x+5-8}{3}=-2 \Rightarrow x-3=-6 \Rightarrow x=-3$

and $\frac{0-2+y}{3}=1 \Rightarrow y=3+2=5$

Clearly $5x+3y$ is correct.

32. **(d)** $\frac{7+y+9}{3}=6 \Rightarrow y=18-16=2$

and $\frac{x-6+10}{3}=3 \Rightarrow x=9-4=5$

$\therefore (x, y)=(5, 2)$

33. **(d)** The given line is $5x-3y+15=0$

$\Rightarrow \frac{5x}{-15}-\frac{3y}{-15}=1 \Rightarrow \frac{x}{-3}+\frac{y}{5}=1$

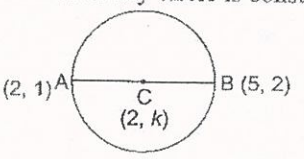
Here $a=-3, b=5$

$\therefore$ Required area $=\frac{1}{2}|ab|$

$=\frac{1}{2}|-3\times 5|=\frac{15}{2}$ cm^2

34. **(b)** Slope = $\tan\theta$, and intercept on x-axis is 3.

35. **(a)** Radius of any circle is constant.



36. **(a)** $x=-1$ is a vertical line and $y=4$ is a horizontal line. Hence lines are perpendicular to each other.

37. **(d)** Required distance

$=\sqrt{(2k+4-2k)^2+(5k+3-5k)^2}$

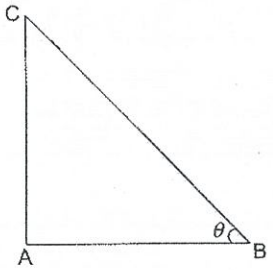
$=\sqrt{16+9}=\sqrt{25}=5$

8. Trigonometry

Trigonometry is the study of relation between sides and angles of a triangle. We can come across so many problems involving sides and angles e.g. Foot of ladder, height of a tower including of ladder etc.

Trigonometric Ratios

In a right triangle



$$\sin\theta = \frac{AC}{BC}, \operatorname{cosec}\theta = \frac{BC}{AC}$$

$$\cos\theta = \frac{AB}{BC}, \sec\theta = \frac{BC}{AB}$$

$$\tan\theta = \frac{AC}{AB}, \cot\theta = \frac{AB}{AC}$$

and $\tan\theta = \dfrac{\sin\theta}{\cos\theta}, \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$$\tan\theta = \frac{1}{\cot\theta}$$

Example 1: If $\sin A = \frac{3}{4}$, calculate tan A

(a) $\frac{3}{7}$ (b) $\frac{3}{\sqrt{7}}$

(c) $\frac{1}{7}$ (d) None of these

Solution: **(b)** $\because \sin A = \frac{3}{4} = \frac{\text{Perpendicular}}{\text{Hyopentuse}}$

Using Pythagoras theorem,

$\text{Base} = \sqrt{\text{Hyp}^2 - \text{Perp}^2}$

$= \sqrt{4^2 - 3^2} = \sqrt{16-9} = \sqrt{7}$

Hence $\tan A = \dfrac{\text{Perp.}}{\text{Base}} = \dfrac{3}{\sqrt{7}}$

Trigonometric Ratios of some specific Angles:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\operatorname{cosec}\theta$	$\sec\theta$	$\cot\theta$
0	0	1	0	∞	1	∞
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90	1	0	∞	1	∞	0

Example 2: Evaluate $\dfrac{\tan 30}{\sec 60 + \operatorname{cosec} 60}$

(a) $\dfrac{\sqrt{3}+1}{4}$ (b) $\dfrac{\sqrt{3}-1}{4}$

(c) $\dfrac{\sqrt{3}-1}{2}$ (d) None of these

Solution: **(b)** $\dfrac{\tan 30}{\sec 60 + \operatorname{cosec} 60}$

$$= \frac{\frac{1}{\sqrt{3}}}{2+\frac{2}{\sqrt{3}}} = \frac{\frac{1}{\sqrt{3}}}{\frac{2\sqrt{3}+2}{\sqrt{3}}} = \frac{1}{2(\sqrt{3}+1)}$$

$$= \frac{\sqrt{3}-1}{2(3-1)} = \frac{\sqrt{3}-1}{4}$$

Trigonometric Ratio of Complementary Angles

$\sin(90-\theta) = \cos\theta$

$\cos(90-\theta) = \sin\theta$

$\tan(90-\theta) = \cot\theta$

$\cot(90-\theta) = \tan\theta$

$\sec(90-\theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(90-\theta) = \sec\theta \qquad 0 < \theta < 90°$

Example 3: Find the value of $\dfrac{\sin 10}{\cos 80} + \dfrac{\tan 40}{\cot 50} - 2$.

(a) 0 (b) 1

(c) 2 (d) None of these

Solution: (a) We have $\frac{\sin 10}{\cos 80}+\frac{\tan 40}{\cot 50}-2$

$$=\frac{\sin(90-80)}{\cos 80}+\frac{\tan(90-80)}{\cot 50-2}$$

$$=\frac{\cos 80}{\cos 80}+\frac{\cot 50}{\cot 50}-2=1+1-2=0$$

Trigonometric Identities

$$\sin^2\theta+\cos^2\theta=1$$
$$\sec^2\theta+\tan^2\theta=1$$
$$\operatorname{cosec}^2\theta+\cot^2\theta=1$$

Example 4: Evaluate $\frac{1}{1+\tan^2\theta}+\frac{1}{1+\cot^2\theta}$

(a) 1 (b) 0

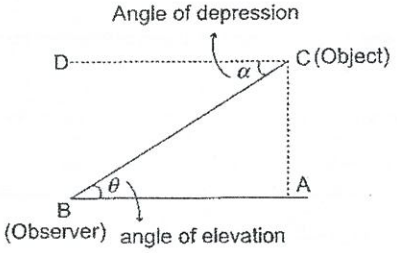
(c) 2 (d) None of these

Solution: (a) We have $\frac{1}{1+\tan^2\theta}+\frac{1}{1+\cot^2\theta}$

$$=\frac{1}{\sec^2\theta}+\frac{1}{\operatorname{cosec}^2\theta}=\cos^2\theta+\sin^2\theta=1$$

Some Applications of Trigonometry

- The height and distance of object can be determined with the help of T-ratios.
- The line of sight is the line drawn form the eye of an observer to a point on the object where the person is viewing.



Here BC is the line of sight.

- Angle of elevation of an object is the angle formed by the line of sight with the horizontal line when the object is above the horizontal level.
- Angle of depression of an object is the angle formed by the line of sight with the horizontal line when the object is below the horizontal level.

Example 5: The angle of elevation of the top of a tower from a point on the ground, which is 20 m away from the foot of the tower, is 30. Find the height of the tower.

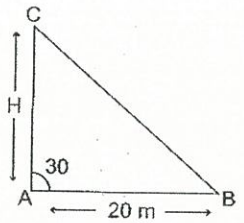
(a) $\frac{20\sqrt{3}}{7}$ m (b) $\frac{20\sqrt{3}}{3}$ m

(c) $\frac{50\sqrt{3}}{7}$ (d) None of these

Solution: **(b)** Here $AB = 30$ m, $\angle A = 30$

$\therefore$ In ΔABC $\tan A = \dfrac{BC}{20} \Rightarrow \tan 30 = \dfrac{H}{20}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{H}{20} \Rightarrow H = \dfrac{20}{\sqrt{3}} = \dfrac{20\sqrt{3}}{3} m$



Multiple Choice Questions

1. If in a triangle ABC, A and B are complementary, then tan C is
(a) ∞ (b) 0
(c) 1 (d) $\sqrt{3}$

2. If $\sin \sin \alpha = \dfrac{4}{5}$ and $\cos \beta = \dfrac{4}{5}$, then which of the following is true?
(a) $\alpha < \beta$ (b) $\alpha > \beta$
(c) $\alpha = \beta$ (d) None of these

3. $\sin^2 20 + \sin^2 70$ is equal to ——.
(a) 1 (b) –1
(c) 0 (d) 2

4. $\sin\theta \cos(90° - \theta) + \cos\theta \sin(90° - \theta)$ ——.
(a) –1 (b) 2
(c) 0 (d) 1

5. A wheel makes 20 revolutions per hour. The radians it turns through 25 minutes is
(a) $\dfrac{50\pi^c}{7}$ (b) $\dfrac{250\pi^c}{3}$
(c) $\dfrac{150\pi^c}{7}$ (d) $\dfrac{50\pi^c}{3}$

6. $\dfrac{\sin^4\theta - \cos^4\theta}{\sin^2\theta - \cos^2\theta} =$
(a) –1 (b) 2
(c) 0 (d) 1

7. Simplified expression of $(\sec\theta + \tan\theta)(1 - \sin\theta)$ is
(a) $\sin^2\theta$ (b) $\cos^2\theta$
(c) $\tan^2\theta$ (d) $\cos\theta$

8. If $a = \sec\theta - \tan\theta$ and $b = \sec\theta + \tan\theta$, then
(a) $a = b$ (b) $\dfrac{1}{a} = \dfrac{-1}{b}$
(c) $a = \dfrac{1}{b}$ (d) $a - b = 1$

9. If $\sec\alpha + \tan\alpha = m$, then $\sec^4\alpha - \tan^4\alpha - 2\sec\alpha\tan\alpha$ is
(a) m^2 (b) $-m^2$
(c) $\dfrac{1}{m^2}$ (d) $\dfrac{-1}{m^2}$

10. The value of $\tan 15° \tan 20° \tan 70° \tan 75°$ is
(a) –1 (b) 2
(c) 0 (d) 1

11. If $\tan(A - 30°) = 2 - \sqrt{3}$, then find A.
(a) $\dfrac{\pi^c}{2}$ (b) $\dfrac{\pi^c}{4}$
(c) $\dfrac{\pi^c}{6}$ (d) $\dfrac{\pi^c}{3}$

12. If $\sin^4\theta - \cos^4\theta = K^4$ then $\sin^2\theta - \cos^2\theta$ is
(a) K^4 (b) K^3
(c) K^2 (d) K

13. $\frac{\tan^3\theta - 1}{\tan\theta - 1} =$

(a) $\sec^2\theta + \tan\theta$ (b) $\sec^2\theta - \tan\theta$
(c) 0 (d) $\tan\theta - \sec^2\theta$

14. For all values of θ, $1 + \cos\theta$ can be ____.

(a) positive (b) negative
(c) non-positive (d) non-negative

15. $(\text{cosec}A - \sin A)(\sec A - \cos A)(\tan A + \cot A) =$

(a) -1 (b) 2
(c) 0 (d) 1

16. If $x = a(\text{cosec}\,\theta + \cot\theta)$ and $y = b(\cot\theta - \text{cosec}\,\theta)$, then

(a) $xy - ab = 0$ (b) $xy + ab = 0$
(c) $\frac{x}{a} + \frac{y}{b} = 1$ (d) $x^2y^2 = ab$

17. The value of $\frac{\cos^4 x + \cos^2 x \sin^2 x + \sin^2 x}{\cos^2 x + \sin^2 x \cos^2 x + \sin^4 x}$ is

(a) 2 (b) 1
(c) 3 (d) 0

18. $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$ is equal to

(a) $2\sec^2\theta$ (b) $2\cos^2\theta$
(c) 0 (d) 1

19. If $\tan(\alpha + \beta) = \frac{1}{2}$ and $\tan\alpha = \frac{1}{3}$, then $\tan = \beta$

(a) $\frac{1}{6}$ (b) $\frac{1}{7}$ (c) 1 (d) $\frac{7}{6}$

20. The value of $\log\sin 0° + \log\sin 1° + \log\sin 2° + \ldots\ldots + \log\sin 90°$ is

(a) 0 (b) 1
(c) -1 (d) Undefined

21. Which of the following is not possible?

(a) $\sin\theta = \frac{3}{5}$ (b) $\sec\theta = 100$
(c) $\text{cosec}\,\theta = 0.14$ (d) None of these

22. $\sin^2 20° + \cos^2 160° - \tan^2 45° =$

(a) 2 (b) 0 (c) 1 (d) -2

23. $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} + \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} =$

(a) $\frac{2}{1 - 2\cos^2\theta}$ (b) $\frac{2}{2\sin^2\theta - 1}$
(c) Both (1) and (2) (d) None of these

24. The length of the side (in cm) of an equilateral triangle inscribed in a circle of radius 8 cm is

(a) $16\sqrt{3}$ (b) $12\sqrt{3}$
(c) $8\sqrt{3}$ (d) $10\sqrt{3}$

25. Which among the following is true?

(a) $\sin 1° > \sin 1°$ (b) $\sin 1° < \sin 1°$
(c) $\sin 1° = \sin 1°$ (d) None of these

26. If $\frac{1+\sin\alpha}{1-\sin\alpha} = \frac{m^2}{n^2}$, then $\sin\alpha$ is

(a) $\frac{m^2+n^2}{m^2-n^2}$ (b) $\frac{m^2-n^2}{m^2+n^2}$
(c) $\frac{m^2+n^2}{n^2-m^2}$ (d) $\frac{n^2-m^2}{m^2+n^2}$

27. If $\sin\theta - \cos\theta = \frac{3}{5}$, then $\sin\theta\cos\theta =$

(a) $\frac{16}{25}$ (b) $\frac{9}{16}$
(c) $\frac{9}{25}$ (d) $\frac{8}{25}$

Answer Key

1. (a)	2. (b)	3. (a)	4. (d)	5. (d)	6. (d)	7. (d)	8. (c)	9. (c)	10 (d)
11. (b)	12. (a)	13. (a)	14. (d)	15. (d)	16. (b)	17. (b)	18. (a)	19. (b)	20. (d)
21. (a)	22. (b)	23. (c)	24. (c)	25. (b)	26. (a)	27. (b)			

Hints and Solutions

1. Given A and B are complementary angles then

$\angle A+\angle B=90$ (i)

and in $\Delta ABC\ \angle A+\angle B+\angle C=180$ (ii)

From (i) and (ii) $\angle C=90$

$\therefore \tan\angle C=\tan 90=\infty$

2. Here $\sin\beta=\sqrt{1-\cos^2\beta}=\sqrt{1-\left(\frac{4}{5}\right)^2}$

$=\sqrt{1-\frac{16}{25}}=\sqrt{\frac{9}{25}}=\frac{3}{5}$

Now $\sin\alpha>\sin\beta\Rightarrow\alpha>\beta$

3. Given

$\sin^2 20+\sin^2 70=\sin^2 20+[\cos(90-20)]^2$

$=\sin^2 20+\cos^2 20=1$

4. Here $\sin\theta\cos(90-\theta)+\cos\theta-\sin(90-\theta)$

$=\sin\theta-\sin\theta+\cos\theta-\cos\theta$

$=\sin^2\theta+\cos^2\theta=1$

6. $\frac{\sin^4\theta-\cos^4\theta}{\sin^2\theta-\cos^2\theta}=\frac{(\sin^2\theta)^2-(\cos^2\theta)^2}{\sin^2\theta-\cos^2\theta}$

$=\frac{(\sin^2\theta+\cos^2\theta)(\sin^2\theta-\cos^2\theta)}{(\sin^2\theta-\cos^2\theta)}$

$\sin^2\theta+\cos^2\theta=1$

7. We have $(\sec\theta+\tan\theta)(1-\sin\theta)$

$=\left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)(1-\sin\theta)$

$=\frac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta}=\frac{1-\sin^2\theta}{\cos\theta}=\frac{\cos^2\theta}{\cos\theta}$

$=\cos\theta$

8. Given $a=\sec\theta-\tan\theta$ (i)

and $b=\sec\theta+\tan\theta$ (ii)

Multiplying (i) and (ii) we get

$\sec^2\theta-\tan^2\theta=a-b$

$\Rightarrow 1=ab$

$\Rightarrow a=\frac{1}{b}$

9. $\sec\alpha+\tan\alpha=m$ then $\sec^4\alpha-\tan^4\alpha-2\sec\alpha\tan\alpha$

$=(\sec^2\alpha-\tan^2\alpha+\sec^2\alpha+\tan^2\alpha)-2\sec\alpha\tan\alpha$

$=\sec^2\alpha+\tan^2\alpha-2\sec\alpha\tan\alpha$

$=(\sec\alpha+\tan\alpha)=m^2$

10. We have $\tan 15\tan 20\tan 70\tan 75$

$=\tan 15\tan 20\tan(90-20)\tan(90-15)$

$=\tan 15\tan 20-\cot 20\cot 15$

$=\tan 15\times\frac{1}{\tan 15}-\tan 20\times\frac{1}{\tan 20}$

$=1$

11. Given $\tan(A-30)=2-\sqrt{3}$

$\text{LHS}=\frac{\tan A-\tan 30}{1+\tan A-\tan 30}$

$\Rightarrow\frac{\tan A-\frac{1}{\sqrt{3}}}{1+\tan A-\frac{1}{\sqrt{3}}}$

Now let $A=\frac{\pi}{4}$

then $\frac{\tan 45-\frac{1}{\sqrt{3}}}{1+\tan 45-\frac{1}{\sqrt{3}}}$

$\Rightarrow\frac{1-\frac{1}{\sqrt{3}}}{1+1-\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$=\frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$=\frac{(\sqrt{3}-1)^2}{3-1}=\frac{3+1-2\sqrt{3}}{2}$

$=\frac{4-2\sqrt{3}}{2}=2-\sqrt{3}$

Hence $A=\frac{\pi}{4}$

13. $\dfrac{\tan^3\theta-1}{\tan\theta-1}=\dfrac{(\tan\theta-1)}{(\tan\theta-1)}(\tan^2\theta+1+\tan\theta)$

$=\sec^2\theta+\tan\theta$

15. Given

$(\operatorname{cosec}A-\sin A)(\sec A-\cos A)(\tan A+\cot A)$

$=\left(\dfrac{1}{\sin A}-\sin A\right)\left(\dfrac{1}{\cos A}-\cos A\right)(\tan A+\cot A)$

$=\left(\dfrac{1-\sin^2 A}{\sin A}\right)\left(\dfrac{1-\cos^2 A}{\cos A}\right)(\tan A+\cot A)$

$=\left(\dfrac{\cos^2 A}{\sin A}\cdot\dfrac{\sin^2 A}{cpsA}\cdot\right)(\tan A+\cot A)$

$=(\cos A\sin A)\left(\dfrac{\sin A}{\cos A}+\dfrac{\cos A}{\sin A}\right)$

$=\dfrac{\sin^2 A+\cos^2 A}{(\sin A-\cos A)}(\sin A-\cos A)$

$=\sin^2 A+\cos^2 A=1$

18. **(a)** We have

$\dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta}=\dfrac{1-\sin\theta+1+\sin\theta}{(1-\sin^2\theta)}$

$=\dfrac{2}{\cos^2\theta}=2\sec^2\theta$

20. **(a)** Here

$\log\sin 0+\log\sin 1+\log\sin 2+\ldots+\log\sin 90$

$=\log(\sin 0\times\sin 1\times\sin 2\ldots\sin 90)$

$=\log(0)=0$

22. **(b)** $\sin^2 20+\cos^2 160-\tan^2 45$

$=\sin^2 20+\cos^2 160-\tan^2 45$

$=\sin^2(180-160)+\cos^2 160-\tan^2 45$

$=\sin^2 160-\cos^2 160-\tan^2 45$

$=1-1=0$

23. **(c)** We have $\dfrac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\dfrac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

$=\dfrac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta}{+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}$

$=\dfrac{2}{\sin^2\theta+\sin^2\theta-1}=\dfrac{2}{2\sin^2\theta-1}$

$=\dfrac{2}{1-\cos^2\theta-\cos^2\theta}=\dfrac{2}{1-2\cos^2\theta}$

$\therefore$ Both 1 and 2 are correct.

24. **(c)** Length of the side (cm) of an equilateral triangle

$=28\sin 60=2\times 8\times\dfrac{\sqrt{3}}{2}=8\sqrt{3}$ cm.

26. **(b)** We have $\dfrac{1+\sin\alpha}{1-\sin\alpha}=\dfrac{m^2}{n^2}$

$\Rightarrow\dfrac{1+\sin\alpha+1-\sin\alpha}{1+\sin\alpha-1+\sin\alpha}=\dfrac{m^2+n^2}{m^2-n^2}$

$\Rightarrow\dfrac{\cancel{2}}{\cancel{2}\sin\alpha}=\dfrac{m^2+n^2}{m^2-n^2}$

$\Rightarrow\sin\alpha=\dfrac{m^2-n^2}{m^2+n^2}$

Here $\sin\theta-\cos\theta=\dfrac{3}{5}$

27. **(d)** Squaring both circles, we get

$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=\dfrac{9}{25}$

$\Rightarrow 1+2\sin\theta\cos\theta=\dfrac{9}{25}$

$\Rightarrow\quad 2\sin\theta\cos\theta=1-\dfrac{9}{25}=\dfrac{16}{25}$

$\Rightarrow\quad\sin\theta\cos\theta=\dfrac{8}{25}$

9. Circles

Definitions

Radius: It is a line segment joining the centre of a circle and a point on the circle.

Chord: A line segment joining any two points on a circle is called a chord of the circle.

Diameter: A chord of a circle passing through the centre is called a diameter of the circle.

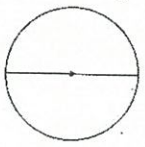
Diameter = 2 × radius

Circumference: The perimeter of a circle is called its circumference.

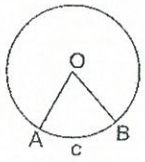
Circumference of a circle = $2\pi r = \pi d$

Arc: A continuous piece of a circle is called an arc.

Semi-circle: A diameter of a circle divides it into two equal arcs. Each of these two arcs are semicircle.



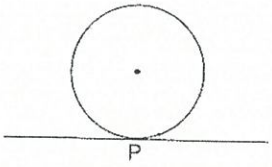
Sector of a circle: The region enclosed by an arc of a circle and its two bounding radii is called a sector of the circle.



Quadrant: One fourth of a circular disc is called a quadrant. Its central angle is 90°.

Tangent

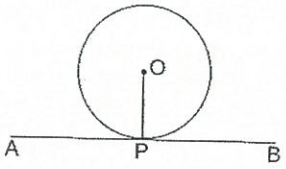
A line meeting a circle only in one point is called a tangent to the circle at that point. The point at which the tangent line meets the, circle is called the point of contact.



There is one and only one tangent passing through a point lying on a circle.

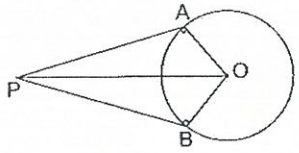
There are exactly two tangents through a point lying outside the circle.

The tangent at any point of a circle is perpendicular to the radius through the point of contact.



$OP \perp AB$

The lengths of tangents drawn from an external point to a circle are equal.



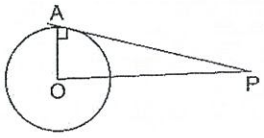
$PA = PB$

If two tangents are drawn from an external point then they subtend equal angle at the centre.

Example 1: From a point 10 cm away from the centre of a circle, a tangent is drawn whose length is 8 cm. What is the radius of the circle?

(a) 5 cm (b) 4 cm

(c) 6 cm (d) 3 cm



Solution: **(a)** In right angle $\Delta\, OPA$,

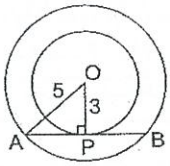
$$OA = \sqrt{OP^2 - PA^2} = \sqrt{10^2 - 8^2}$$

$$= \sqrt{100 - 64} = \sqrt{36} = 6 \text{ cm.}$$

Example 2: In the given fig., O is the centre of the circle. The circles are concentric with radii 3 cm and 5 cm. AB is a chord of outer circle which touches the inner circle? What is the length of chord AB?

(a) 8 cm (b) 4 cm

(c) 5 cm (d) 6 cm



Solution: **(a)** In the given Fig.

In right angled triangle ΔOAP,

$$AP = \sqrt{OA^2 - OP^2}$$

$$= \sqrt{5^2 - 3^2}$$

$$= \sqrt{25 - 9} = \sqrt{16}$$

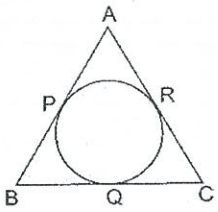
$$= 4$$

$\therefore$ Length of $AB = 2 \times AP = 2 \times 4 = 8$ cm.

Example 3: In the given figure, $AP = 4$ cm, $BP = 6$ cm, $AC = 12$ cm. What is the length of BC?

(a) 15 cm (b) 14 cm

(c) 12 cm (d) 10 cm



Solution: **(b)**

$$AP = AR = 4 \text{ cm}$$

$$CR = AC - AR$$

$$= 12 - 4 = 8 \text{ cm}$$

$$BP = BQ = 6 \text{ cm}$$

$$BC = BQ + QC$$

$$= BQ + CR = 6 + 8 = 14 \text{ cm.}$$

Areas related to Circles

For a circle of radius r,

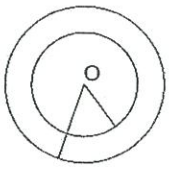
Circumference $= 2\pi r$

Area of a circle $= \pi r^2$

Area of semicircle $= \frac{1}{2}\pi r^2$

Perimeter of semi-circle $= \pi r + 2r$

Area of a ring $= \varpi(R^2 - r^2)$



Area of the sector $= \frac{\pi r^2 \theta}{360}$

Length of an arc $= \frac{2\pi r\theta}{360}$

Area of a segment = Area of sector – Area of triangle

Cost = Length × Rate

and Cost = Area × Rate

The value of $\pi = \frac{22}{7} = 3.14$

Angle described by the minute hand of a clock in 60 minutes = 360°

Angle described by the hour hand of a clock in 12 hours = 360°

Distance moved by a wheel in one rotation = circumference of the wheel.

No. of rotations in 1 minute $= \frac{\text{distance covered in 1 minute}}{\text{circumference}}$

Example 4: What is the circumference of the circle whose radius is 21 cm?

(a) 130 cm (b) 132 cm (c) 140 cm (d) 125 cm

Solution: **(b)** Circumference $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 21$$

$$= 6 \times 22 = 132 \text{ cm}$$

Example 5: What is the area of circle whose circumference is 88 cm.

(a) 616 cm^2 (b) 600 cm^2 (c) 500 cm^2 (d) 516 cm^2

Solution: **(a)** Let r be the radius of the circle.

$$2\pi r = 88$$

$$2 \times \frac{22}{7} \times r = 88$$

$$r = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm}$$

Area of circle $= \pi r^2$

$$= \frac{22}{7} \times 14^2 = \frac{22}{7} \times 14 \times 14$$

$$= 616 \text{ cm}^2.$$

Example 6: Find the area of the ring whose outer and inner radii are 23 cm and 12 cm respectively.

(a) 1200 cm^2 (b) 1210 cm^2 (c) 1000 cm^2 (d) 500 cm^2

Solution: **(b)**

Area of the ring $= \pi(R^2 - r^2)$

$$= \pi (23^2 - 12^2)$$

$$= \frac{22}{7} \times (23+12)(23-12)$$

$$= \frac{22}{7} \times 35 \times 11$$

$$= 22 \times 55 = 1210 \text{ cm}^2.$$

Example 7: Find the area of sector of a circle whose radius is 14 cm and central angle 45°.

(a) 60 cm^2 (b) 80 cm^2 (c) 50 cm^2 (d) 77 cm^2

Solution: **(d)** Area $= \frac{\pi r^2 \theta}{360°}$

$$= \frac{22}{7} \times \frac{14^2 \times 45°}{360°}$$

$$= \frac{22}{7} \times \frac{14 \times 14}{8} = 77 \text{ cm}^2.$$

Example 8: The area of sector of a circle of radius 10.5 cm is 69.3 cm^2. What is the central angle of the sector.

(a) 50° (b) 60° (c) 72° (d) 80°

Solution: **(c)** Area of the sector $= \frac{\pi r^2 \theta}{360°}$

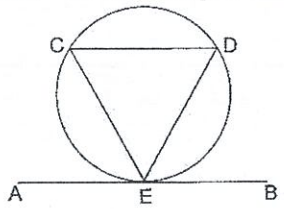
$$69.3 = \frac{\frac{22}{7} \times (10.5)^2 \times \theta}{360°}$$

$$\theta = \frac{69.3 \times 360 \times 7}{22 \times 10.5 \times 10.5}$$

$$\theta = 72°$$

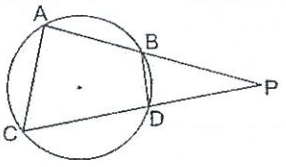
Multiple Choice Questions

1. AB is a tangent to the circle at E. If $EC = ED$ and $\angle CDE = 62°$. Find angle $\angle AED$.



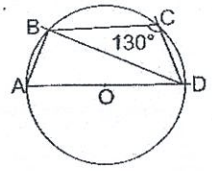
(a) 118° (b) 100°
(c) 108° (d) 98°

2. Two chords AB and CD of a circle intersect at an external point P as shown in figure. If $AB = 8$ cm, $BP = 10$ cm, $PD = 12$ cm. What is length of CP?



(a) 15 cm (b) 12 cm
(c) 18 cm (d) 10 cm

3. In the given figure AD is diameter of the circle. If $\angle BCD = 130°$. What is the value of $\angle DAB$?



(a) 50° (b) 65°
(c) 40° (d) 70°

4. What is the length of tangent segment PT to a circle whose centre is at O. $OP = 17$ cm and $OT = 8$ cm?
(a) 15 cm (b) 16 cm
(c) 9 cm (d) 10 cm

5. What is the area of quadrant of a circle whose circumference is 22 cm?
(a) 9.625 cm^2 (b) 10.625 cm^2
(c) 6.75 cm^2 (d) 8.75 cm^2

6. The side of a square is 10 cm. What is the area of circumscribed circle?
(a) 78.5 cm^2 (b) 157 cm^2
(c) 135 cm^2 (d) 314 cm^2

7. In a circle of radius 10.5 cm, the minor arc is one-fifth of the major arc. What is the area of major arc?
(a) 288.75 cm^2 (b) 281.75 cm^2
(c) 271.25 cm^2 (d) 262.75 cm^2

8. What is the area of circle in which the difference between the radius and circumference of the circle be 37 cm?
(a) 144 cm^2 (b) 154 cm^2
(c) 124 cm^2 (d) 224 cm^2

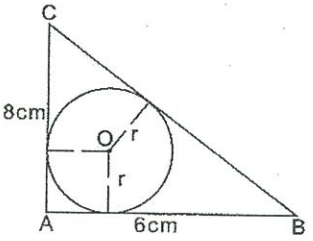
9. A Pendulum swings through an angle of 30° and describes an arc 8.8 cm in length, then what is the length of the Pendulum?
(a) 16.8 cm (b) 12.8 cm
(c) 14.2 cm (d) 15.6 cm

10. How the tangent at any point of a circle and radius through the point is related?
(a) Perpendicular to each other
(b) Parallel to each other
(c) having same length
(d) None of these

11. The minute hand of a clock is 12 cm long. Find the area of the face of the clock described by the minute hand in 35 minutes.
(a) 252 cm^2 (b) 264 cm^2
(c) 184 cm^2 (d) 1284 cm^2

12. The diameter of the front and rear wheels of a tractor are 80 cm and 200 cm respectively. What are the number of revolution that a rear wheel makes to cover the distance which the front wheel covers in 800 revolutions?
(a) 640 (b) 320
(c) 240 (d) 300

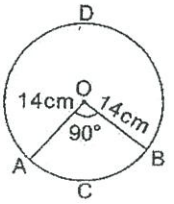
13. In the given figure, ΔABC is right angled at A with $AB = 6$ cm and $AC = 8$ cm. A circle with centre O has been inscribed inside the triangle. Find the value of the radius of the inscribed circle.



(a) 2 cm (b) 3 cm
(c) 4 cm (d) 5 cm

14. A copper wire when bent in the form of a square encloses an area of 484 cm^2. The same wire is now bent in the form of a circle. Find the area enclosed by the circle.
(a) 616 cm^2 (b) 456 cm^2
(c) 216 cm^2 (d) None of these

15. A chord of a circle of radius 14 cm makes a right angle at the centre. Find the area of minor segment of the circle.



(a) 98 cm^2
(b) 56 cm^2
(c) 16 cm^2
(d) 64 cm^2

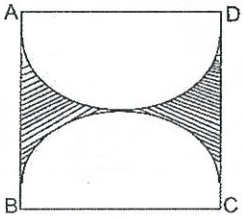
16. A race track is in the form of a ring whose inner and outer circumference are 437 m and 50 m respectively. Find the area of the track.
(a) 4935 m^2 (b) 4065 m^2
(c) 4135 m^2 (d) None of these.

17. A field is in the form of a circle. The cost of ploughing the field at ₹ 1.50 per m^2 is ₹ 5775. Find the cost of fencing the field at ₹ 8.50 per meter.
(a) ₹ 1870 (b) ₹ 1670
(c) ₹ 1980 (d) ₹ 1780

18. A square park has each side of 100 m. At each corner of the park, there is a flower bed in the form of a quadrant inside the park of radius 14 m. What is the area of remaining part?

(a) 9384 m^2 (b) 9684 m^2
(c) 9224 m^2 (d) 9386 m^2

19. What is the area of shaded region if *ABCD* is a square of side 14 cm and *APD* and *BPC* are semi circles?



(a) 52 cm^2
(b) 32 cm^2
(c) 42 cm^2
(d) 62 cm^2

20. If the area of a sector of a circle is $\frac{7}{20}$ of the area of the circle, then what is the sector angle of the circle?

(a) 126° (b) 130°
(c) 110° (d) 120°

21. If the perimeter of a circle is equal to that of square, then what is the ratio of their area?

(a) 11 : 14 (b) 22 : 13
(c) 14 : 11 (d) 13 : 22

22. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm, then what is area of sector?

(a) 14.6 cm^2 (b) 15.6 cm^2
(c) 19.6 cm^2 (d) None of these

23. A car has wheels which are 80 cm in diameter. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 kmph?

(a) 4375 (b) 4125
(c) 4275 (d) 4325

24. A bicycle wheel makes 5000 revolutions in moving 11 km. What is the radius of the wheel?

(a) 45 cm (b) 25 cm
(c) 35 cm (d) None of these

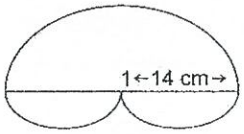
25. If an isosceles triangle *PQR* in which $PQ = PR = 6$ cm is inscribed in a circle of radius 9 cm. What is the area of triangle

(a) $56\sqrt{2}\ cm^2$ (b) $8\sqrt{2}\ cm^2$
(c) $7\sqrt{2}\ cm^2$ (d) $9\sqrt{2}\ cm^2$

26. What is the length of tangent drawn from a point whose distance from the centre of a circle is 25 cm, if radius of the circle is 7 cm?

(a) 24 cm (b) 25 cm
(c) 26 cm (d) 30 cm

27. What is the area of the fig if the radius of bigger semi-circle be 14 cm?

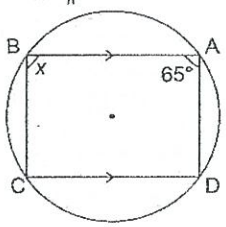


(a) 264 cm^2 (b) 462 cm^2
(c) 468 cm^2 (d) None of these

28. A bucket is raised from a well by means of a rope which is wound round a wheel of diameter 77 cm. Given that the bucket ascends in 1 minute 28 seconds with a uniform speed of 1.1 m/s. What is the number of complete revolutions the wheel makes in raising the bucket?

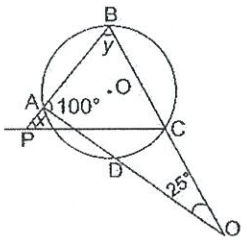
(a) 60 (b) 50
(c) 30 (d) 40

29. What is the value of x in the given figure? It is given that $AB \parallel CD$.



(a) 65° (b) 115°
(c) 130° (d) None of these

30. What is the value of y and x respectively in the given figure?



(a) $x = 55°, y = 45°$
(b) $x = 45°, y = 55°$
(c) $x = 65°, y = 55°$
(d) $x = 55°, y = 65°$

Answer Key

1. (a)	2. (a)	3. (a)	4. (a)	5. (a)	6. (b)	7. (a)	8. (b)	9. (a)	10 (a)
11. (b)	12. (b)	13. (a)	14. (a)	15. (b)	16. (a)	17. (a)	18. (a)	19. (c)	20. (a)
21. (c)	22. (b)	23. (a)	24. (c)	25. (d)	26. (d)	27. (a)	28. (d)	26. (a)	30. (b)

Hints and Solutions

1. (a)

$\because EC = ED$

$\therefore \angle ECD = \angle EDC = 62°$

$\therefore \angle DEC = 180° - (2 \times 62°) = 56°$

$\angle AEC = \angle DEB = \dfrac{180° - 56°}{2} = 62°$

$\therefore \angle AED = \angle CED + \angle AEC$

$= 56° + 62° = 118°$

2. (a) For any external point P,

$AP \times BP = PD \times PC$

$\Rightarrow (AB + BP) \times BP = 12 \times PC$

$\Rightarrow (8 + 10) \times 10 = 12 \times PC$

$\Rightarrow PC = \dfrac{180}{12} = 15$ cm.

3. (a)

$\because A, B, C$ and D are on the circumference of the circle.

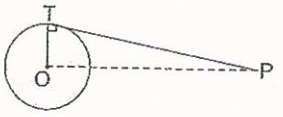
$\therefore ABCD$ is a cyclic quadrilateral, i.e.,

$\angle A + \angle C = 180°$

$\Rightarrow \angle A = 180° - \angle C = 180° - 130° = 50°$

$\therefore \angle DAB = 50°$.

4. (a) In ΔOPT,



$OT^2 + PT^2 = OP^2$

$\Rightarrow PT = \sqrt{OP^2 - OT^2}$

$= \sqrt{(17)^2 - (8)^2}$

$= \sqrt{289 - 64} = \sqrt{225} = 15$ cm.

5. (a) Let r be the radius

$2\pi r = 22$

$\Rightarrow r = \dfrac{22}{2\pi} = \dfrac{22 \times 7}{2 \times 22} = \dfrac{7}{2}$ cm.

Area of quadrant $= \dfrac{1}{4} \times \pi r^2$

$= \dfrac{1}{4} \times \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2}$

$= \dfrac{77}{8} = 9.625$ cm^2.

6. (b) Diameter of the circumscribed circle

= diagonal of the square

$= 10\sqrt{2}$ cm

Radius $= \dfrac{10\sqrt{2}}{2} = 5\sqrt{2}$ cm

Area of circumscribed circle $= \pi r^2$

$= \dfrac{22}{7} \times (5\sqrt{2})^2$

$= \dfrac{22}{7} \times 25 \times 2$

$= 157$ cm^2.

7. (a) Let the major arc be x cm.

Length of minor arc $= \dfrac{x}{5}$

Circumference $= x + \dfrac{x}{5} = \dfrac{6x}{5}$

$\Rightarrow \dfrac{6x}{5} = 2\pi r$

$\Rightarrow \dfrac{6x}{5} = 2 \times \dfrac{22}{7} \times 10.5$

$\Rightarrow x = 55$ cm

Required area $= \dfrac{1}{2} \times 55 \times 10.5$

$= 288.75$ cm^2.

8. **(b)** Let r be the radius of the circle

$2\pi r - r = 37$

$\Rightarrow r(2\pi - 1) = 37$

$\Rightarrow r\left(\frac{44}{7} - 1\right) = 37$

$\Rightarrow r\left(\frac{37}{7}\right) = 37$

$\Rightarrow r = 7$ cm

Area $= \pi r^2 = \frac{22}{7} \times 7^2 = 22 \times 7 = 154$ cm^2.

9. **(a)** Length of pendulum

= radius of the sector $= x$ cm.

Arc length = 8.8 cm

$\Rightarrow 2 \times \frac{22}{7} \times x \times \frac{30}{360} = 8.8$

$x = \frac{8.8 \times 360 \times 7}{2 \times 22 \times 30}$

$= \frac{88 \times 36 \times 7}{44 \times 30} = 16.8$ cm

11. **(b)** Angle described by the minute hand in

35 minutes $= \frac{360}{60} \times 35 = 210°$

$\theta = 210°$, $r = 12$ cm

Area of the sector $= \frac{\pi r^2 \theta}{360}$

$= \frac{22}{7} \times 12 \times 12 \times \frac{210}{360}$

$= 264$ cm^2.

12. **(b)** Radius of front of wheel = 40 cm

$= \frac{40}{100} = \frac{2}{5}$ m.

Circumference $= 2\pi \times \frac{2}{5} = \frac{4\pi}{5}$ m.

Distance moved by it in 800 revolutions

$= \frac{4\pi}{5} \times 800 = 640$ pm

Circumference of real wheel $= 2\pi \times 1 = 2\pi$ m.

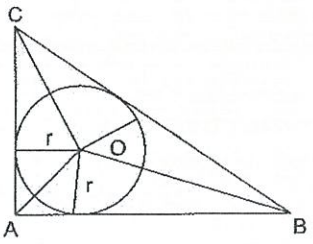
No. of revolutions $= \frac{640\pi}{2\pi} = 320$

13. **(a)** $BC = \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$

$= \sqrt{100} = 10$ cm

(area ΔOAC) + (area ΔOAB) + (area ΔOBC)

= area (ΔABC)



$\Rightarrow \frac{1}{2} \times 8 \times r + \frac{1}{2} \times 6 \times r + \frac{1}{2} \times 10 \times r$

$= \frac{1}{2} \times 6 \times 8$

$\Rightarrow 8r + 6r + 10r = 48$

$\Rightarrow 24r = 48 \Rightarrow r = 2$

14. **(a)** Area of square = 484

Side of square $= \sqrt{484} = 22$ cm

$\Rightarrow 2\pi r = 22 \times 4$

$\Rightarrow r = \frac{22 \times 4 \times 7}{2 \times 22} = 14$ cm

Area of circle $= \pi r^2$

$= \frac{22}{7} \times 14 \times 14$

$= 44 \times 14$

$= 616$ cm^2.

15. **(b)** Area of the sector $OACBO$,

$= -\frac{\pi r^2 \theta}{360}$ cm^2

$= \frac{22}{7} \times \frac{14 \times 14 \times 90}{360}$

$= 154$ cm^2.

Area of $\Delta OAB = \frac{1}{2} r^2 \sin\theta$

$= \frac{1}{2} \times 14 \times 14 \times \sin 90°$

$= 98$ cm^2.

$\therefore$ Area of minor segment of circle

$= 154 - 98 = 56$ cm^2.

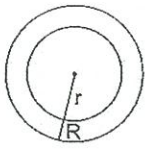
16. **(a)** Let R be the radius of outer circle and r be the radius of inner circle.

$2\pi R = 503$

$\Rightarrow R = \frac{503\times 7}{2\times 22} = \frac{3521}{44}$

and $2\pi r = 437$

$\Rightarrow r = \frac{437\times 7}{2\times 22} = \frac{3059}{44}$



Width of the track $= R - r = \frac{3521-3059}{44}$

$= \frac{462}{44} = \frac{21}{2}$ cm

Area of the track $= \pi(R^2 - r^2)$

$= \pi(R+r)(R-r)$

$= \frac{22}{7}\times\frac{6580}{44}\times\frac{21}{2}$

$= 4935\ \text{m}^2$

17. **(a)**

Area of the circular filed $= \frac{5775}{1.5} = 3580\ \text{m}^2.$

$\pi r^2 = 3850$

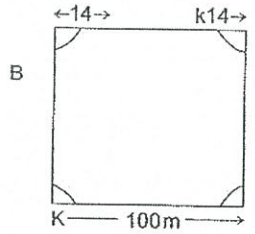
$r^2 = \frac{3850\times 7}{22} = 1225$

$r = 35$ m

Circumference $= 2\pi r = 2\times\frac{22}{7}\times 35 = 220$ m

Cost of fencing $= 220 \times 8.50 = ₹1870$

18. **(a)** Let A be the area of each quadrant of the circle.



Radius f circel $= 14$ cm

$A = \frac{1}{4}\times\pi r^2 = \frac{1}{4}\times\frac{22}{7}\times 14\times 14 = 154\ \text{m}^2.$

Area of 4 quadrants $= 4\times 154 = 616\ \text{m}^2$

Area of square park $= (100)^2 = 10000\ \text{m}^2$

Area of remaining part $= 10000 - 616$

$= 9384\ \text{m}^2$

19. **(c)** Area of the shaded region

= Area of square $ABCD$ – Area of two semicircles

$= 14\times 14 - 2\left(\frac{1}{2}\times\frac{22}{7}\times 7^2\right)$

$= 196 - 154 = 42\ \text{cm}^2$

20. **(a)** Area of the sector $= \frac{7}{20}\times\pi r^2$

$\Rightarrow \frac{\pi r^2\theta}{360} = \frac{7}{20}\times\pi r^2$

$\Rightarrow \theta = \frac{7\times 360}{20} = 126°$

21. **(c)** $2\pi r = 4\times a$

$\pi r = 2a$

$r = \frac{2a}{\pi}$

Area of circle $= \pi r^2$

$= \pi\frac{4a^2}{\pi^2} = \frac{4a^2}{\pi}$

Area of square $= a^2$

Ratio of their area $= \frac{\frac{4a^2}{\pi}}{a^2}$

$= \frac{4a^2}{\pi}\times\frac{1}{a^2} = \frac{4}{\pi} = \frac{4}{\frac{22}{7}}$

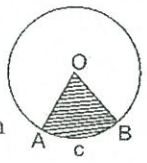
$= \frac{4\times 7}{22} = \frac{14}{11} = 14:11$

22. **(b)** Let O be the centre of a circle of radius 5.2 cm. Let $OABO$ be the sector with perimeter 16.4 cm.

$OA + OB + \text{arc } AB = 27.2$

$5.2 + 5.2 + \text{arc } AB = 16.4$

$\Rightarrow$ arc $AB = 16.4 - 16.4 = 6$ cm



Area of sector $OACBO$

$= \frac{1}{2}\times\text{radius}\times\text{arc} = \frac{1}{2}\times 5.2\times 6 = 15.6\ \text{cm}^2.$

23. **(a)** Speed $= 66$ km/h $= 66\times\frac{5}{18}$ m/s

$\therefore$ Distance moved in 10 min

$= 66\times\frac{5}{18}\times 60\times 10$ m

In one complete revolution, distance moved

$= \pi \times d$

$= \pi\left(\frac{80}{100}\right)$m

Let it makes h complete revolutions in 10 min.

$\therefore \quad \pi \times \frac{80}{100} \times h = 66 \times \frac{5}{18} \times 60 \times 10$

$\Rightarrow \quad h \times \frac{22}{7} \times \frac{80}{100} = 66 \times \frac{5}{18} \times 600$

$\Rightarrow \quad h = \frac{5 \times 100 \times 7 \times 100}{80} = 4375$

24. (c) Distance moved by wheel

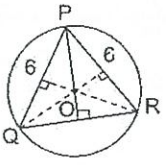
$= 2 \times \pi \times r \times 5000 = 11000$ m

$r = \frac{11 \times 7}{5 \times 2 \times 22} = \frac{7}{20}$ m

$= \frac{7}{20} \times 100$ cm

$= 35$ cm

25. (d)

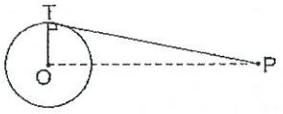


Area $(DPOR) = \frac{1}{2} \times 6 \times \sqrt{9^2 - 3^2}$

$= \frac{1}{2} \times 6 \times 6\sqrt{2} = 18\sqrt{2}$ cm^2

$\therefore$ Area of triangle $= \frac{18\sqrt{2}}{2} = 9\sqrt{2}$ cm^2

26. (a)



$PT^2 + OT^2 = OP^2$

$\Rightarrow \quad PT^2 = (25)^2 - (7)^2$

$\Rightarrow \quad PT = \sqrt{576} = 24$ cm.

27. (b) Area of bigger semicircle

$= \frac{1}{2}(\pi r^2)$

$= \frac{1}{2} \times \frac{22}{7} \times 14 \times 14$

$= 22 \times 98$ cm$^2 = 308$ cm^2

Area of 2 small semicircles

$= 2 \times \left(\frac{1}{2}\pi\left(\frac{r}{2}\right)^2\right)$

$= 2 \times \frac{1}{2} \times \frac{22}{7} \times \left(\frac{14}{2}\right)^2 = 154$

$\therefore$ Required area $= (308 + 154)$ cm^2

$= 462$ cm^2.

28. (d) Distance moved by rope

$= (1.1) \times (60 + 28)$

$= 1.1 \times 88 = 96.8$ cm

$\therefore$ No. of revolutions $= \frac{96.8 \times 100 \times 7}{22 \times 77} = 40$

29. (a) $ABCD$ is a cyclic quadilateral

$(\angle B + \angle D = 180°)$

$\because \quad AB \parallel CD$

$\angle A + \angle D = 180°$ (Alternate interior angles)

$\Rightarrow \quad CD = 180° - 65° = 115°$

$\angle D + \angle B = 180° \Rightarrow x = 180° - 115° = 65°$

30. (b) In ΔABO,

$\angle y + 25° + 100° = 180°$

$\Rightarrow \quad y = 55°$

$\angle C + \angle A = 180°$ [$\because ABCD$ is cycle]

$\Rightarrow \quad \angle = 80°$

In ΔPBC,

$\angle x + 55° + 80° = 180°$

$\Rightarrow \quad \angle x = 45°$

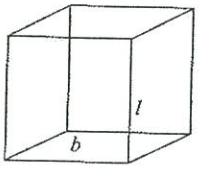
10. Surface Area and Volume

Surface area and volume help us to measure the size of 3D objects. In this chapter we will calculate the surface area and volume of 3D shapes such as Cubes, Cuboids, Cylinders, Spheres and Cones.

Solids

The objects which have definite shapes and sizes are called solids.

Cube

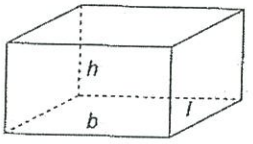


Volume of cube whose each edge be a units = a^3 cubic units

Total surface area of cube = $6a^2$ square units

Lateral surface area of cube = $4a^2$ square units

Cuboid



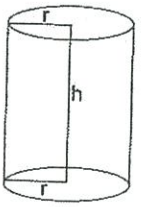
If length = l, breadth = b, height = h.

Then volume of cuboid = lbh cubic units.

Total surface area of cuboid = $2(lb + bh + lh)$ sq. units

Lateral surface area of cuboid = $2(l + b) \times h$ sq. units

Cylinder



If r be the radius of cylinder and h be the height, then

Volume of cylinder = $\pi r^2 h$ cubic units

Curved surface area of cylinder = $2\pi rh$ sq. units

Total surface area of cylinder = $(2\pi rh + 2\pi r^2)$ sq. units

Hollow Cylinder

Let R be the external radius and r be the internal radius of hollow cylinder and h be the height. Then

I. Volume of the material = $\pi(R^2 - r^2)\, h$ cubic units

II. Curved surface area of hollow cylinder = $2\pi(R - r)h$ square units.

Total Surface area of Hollow Cylinder = $[(2\pi Rh + 2\pi rh) + 2\pi(R^2 - r^2)]$ square units

Cone

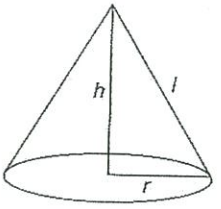
In this figure, there is a cone in which

base radius = r,

height = h

Slant height = l

$= \sqrt{h^2 + r^2}$



Volume of the cone = $\frac{1}{3}\pi r^2 h$ cubic units

Curved Surface area of cone = πrl square units

Total Surface area of cone = $\pi rl + \pi r^2$ square units

Example 1: Find the total surface area of a closed right circular cylinder whose radius is 5 m and height 7 m.

(a) 188 m^2 (b) 188.57 m^2

(c) 189 m^2 (d) 190 m^2

Solution: **(b)** Total Surface area $= 2\pi r\,(r+h)$

$$= 2\times\frac{22}{7}\times 3(3+7)$$

$$= \frac{44}{7}\times 3\times 10$$

$$= 188.57\ m^2$$

Example 2: Find the volume of the cylinder if the circumference of the base of a cylinder is 198 cm and its height is 30 cm.

(a) 93560 cm^3 (b) 93555 cm^3

(c) 9555 cm^3 (d) 93550 cm^3

Solution: **(b)** Let r be the radius of cylinder.

$\Rightarrow 2\pi r = 198$

$$\Rightarrow r = \frac{198}{2\pi} = \frac{198}{2\times\frac{22}{7}} = \frac{198\times 7}{2\times 22} = \frac{63}{2}\ \text{cm}$$

$h = 30$ cm

Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7}\times\left(\frac{63}{2}\right)^2\times 30$$

$$= \frac{22}{7}\times\frac{63}{2}\times\frac{63}{2}\times 30$$

$$= 93555\ cm^3.$$

Example 3: The volume of a cylindrical pipe is 1408 cm^3. If length is 14 cm and its external radius is 9 cm. What is the thickness of pipe.

(a) 3 cm (b) 1 cm (c) 5 cm (d) 2 cm

Solution: **(d)** Here $R = 9$ cm

$h = 14$ cm

$V = 1408\ cm^3$

Let r be its internal radius

$$V = \pi(R^2 - r^2)h$$

$$1408 = \frac{22}{7}(9^2 - r^2)14$$

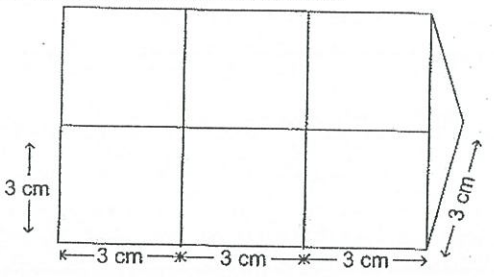
$$\Rightarrow 9^2 - r^2 = \frac{1408}{44} = 32$$

$$\Rightarrow r^2 = 81 - 32 = 49$$

$$\Rightarrow r = 7\ \text{cm};$$

Thickness $= R - r = 9 - 7 = 2$ cm

Example 4: Three cubes each of edge 3 cm long are placed together as shown in the figure given below. Find the surface area of the cuboid so formed.



(a) 120 cm^2 (b) 126 cm^2

(c) 130 cm^2 (d) None of these

Solution: **(b)** The dimension of the cuboid,

$l = 9$ cm, $b = 3$ cm, $h = 3$ cm

The surface area of the cuboid $= 2(lb + bh + lh)$

$= 2[9 \times 3 + 3 \times 3 + 9 \times 3]$

$= 2[27 + 9 + 27] = 2 \times 63 = 126\ cm^2$.

Example 5: Find the surface area of the cube whose volume is 3375 m^3.

Solution: Let x be the edge of the cube, then

$x^3 = 3375 = 15^3$

$\Rightarrow \quad x = 15$ m

Surface area of the cube $= 6x^2$

$= 6 \times 15^2$

$= 6 \times 225$

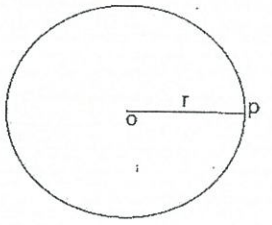
$= 1350\ m^2$

Sphere

If r be the radius of the sphere, then

Volume of sphere $= \frac{4}{3}\pi r^3$ cubic units.

Surface area of sphere $= 4\pi r^2$ sq. units.



Hemisphere

If r be the radius of the sphere.

Volume of the hemisphere $= \frac{2}{3}\pi r^3$ cubic units

Curved surface area of the hemisphere $= 2\pi r^2$ square units

Total surface area of the hemisphere $= 3\pi r^2$ square units

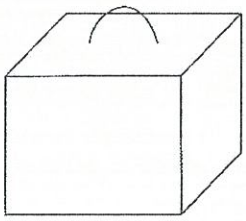
Volume of frustum of a cone $= \frac{1}{3}\pi h\sqrt{r_1^2 + r_2^2 + r_1 r_2}$

Curved surface area of frustum of a cone $= \pi l(r_1 + r_2) = \pi(r_1 + r_2)\sqrt{h^2 + (r_1 - r_2)^2}$

Total surface area of frustum of a cone $= \pi l(r_1 + r_2) = \pi(r_1^2 + r_2^2)$.

When one solid is converted into another, the volume of the material of both the solids will be same or equal.

Example 6: A solid is made up of a cube and a hemisphere attached on the top as shown in the figure. Each edge of the cube measures 5 cm and the hemisphere has a diameter of 4.2 cm.



(a) 163 cm^2 (b) 164 cm^2

(c) 163.8 cm^2 (d) none of them

Solution: **(c)** Total surface area of the cube $= 6 \times 5^2 = 150\ cm^2$

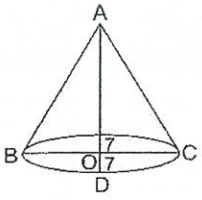
Area to be painted = Total surface area of the cube – base area of hemisphere + curved surface area of hemisphere

$$= 150 - \pi r^2 + 2\pi r^2 = 150 + \pi r^2$$

$$= 150 + \frac{22}{7} \times 21 \times 21 = 150 + 13.86$$

$$= 163.86\ cm^2.$$

Example 7: A toy is in the form of a cone mounted on a hemisphere of common base radius 7 cm. The total height of the toy is 31 cm. Find the total surface area of the toy.



(a) 858 cm^2 (b) 750 cm^2

(c) 860 cm^2 (d) none of them

Solution: **(a)** $OD = 7$, $OC = 7$, $AD = 31$ cm

$$AO = AD - OD = 31 - 7 = 24 \text{ cm}$$

$$l = AC = \sqrt{24^2 + 7^2}$$

$$AC = 25 \text{ cm}$$

Surface area of cone $= \pi rl$

$$= \frac{22}{7} \times 7 \times 25 = 550\ cm^2$$

Surface area of hemisphere $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 7 \times 7$$

$$= 308\ cm^2$$

$\therefore$ Total surface area $= 550 + 308 = 858\ cm^2$.

Multiple Choice Question

1. Marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm containing some water. What is the number of marbles that should be dropped into the beaker so that the water level rises by 5.6 cm?
   (a) 120 (b) 130
   (c) 150 (d) 180

2. A spherical shell of lead whose external and internal diameters are 24 cm and 18 cm respectively is melted and recast into a right circular cylinder 37 cm high. Find the diameter of the base of the cylinder.
   (a) 6 cm (b) 12 cm
   (c) 18 cm (d) 24 cm

3. A hollow sphere of internal and external diameter 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone.
   (a) 12 cm (b) 16 cm
   (c) 14 cm (d) 7 cm

4. A cylindrical container is filled with ice–cream whose diameter is 12 cm and height 15 cm. The whole ice-cream cone?
   (a) 6 cm (b) 4 cm
   (c) 8 cm (d) None of these

5. A solid cylinder has height 12 cm and diameter 10 cm. A conical cavity of same height and same diameter is hollowed out. What is the total surface area of the remaining solid?
   (a) 660 cm^2 (b) 600 cm^2
   (c) 560 cm^2 (d) 760 cm^2

6. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into 12 toys in the shape of a right circular cone mounted on a hemisphere. Find the radius of the hemisphere, if height of the cone is 3 times the radius.
   (a) 3 cm (b) 6 cm
   (c) 5 cm (d) 9 cm

7. The surface area of the cylinder is 2992 cm^2 and its height is 20 cm, what is the diameter of the cylinder?
   (a) 14 cm (b) 28 cm
   (c) 7 cm (d) 56 cm

8. The volume of the cube is 1728 cm^3. What is the total surface area of the cube?
   (a) 824 cm^2 (b) 864 cm^2
   (c) 924 cm^2 (d) None of these

9. The curved surface area of a cylindrical pillar is 264 m^2 and its volume is 924 m^3. What is the height of pillar?
   (a) 4 m (b) 5 m
   (c) 6 m (d) 7 m

10. A circus tent is cylindrical to a height of 4 cm and cone above it. If its diameter is 105 m and its lant height is 40 m. What is the total area of canvas required?
    (a) 1760 m^2 (b) 3960 m^2
    (c) 7920 m^2 (d) 2640 m^2

11. A solid iron hemisphere is melted and a number of solid iron balls of equal size are made. The radius of each ball is one-fourth the radius of the hemisphere. What are the number of balls that can be made?
    (a) 32 (b) 16 (c) 64 (d) 30

12. A right cylinder, a right cone and a hemisphere have the same height and same base area. What is ratio of their volumes?
    (a) 1 : 2: 3 (b) 3 : 1 : 2
    (c) 2 : 3 : 1 (d) None of these

13. A solid sphere of copper of radius 10.5 cm is melted and right cones of radius 3.5 cm and height 3 cm are made from the material. What is the number of cones made?
    (a) 136 (b) 126
    (c) 156 (d) 146

14. The height of a right cylinder is 165 cm and the diameter of its base is 24 cm. Find the radius of the sphere whose volume is equal to the volume of the given cylinder.
    (a) 12 cm (b) 6 cm
    (c) 16 cm (d) 24 cm

15. The weight of a metallic spherical shell is 11.176 kg. If the inner radius of the cell is

6 cm. 1 cm^3 of the metal weights 21 g then what is the thickness of the shell?

(a) 1 cm (b) 2 cm
(c) 3 cm (d) 4 cm

16. If the radius of a sphere is doubled then how many times will its surface area?

(a) 8 (b) 4 (c) 2 (d) 0.5

17. The ratio of the volumes of two spheres is 27 : 8. Find the ratio of their surface areas.

(a) 4 : 9 (b) 9 : 4
(c) 2 : 3 (d) 1 : 3

18. The inner and outer surface area of a spherical shell are 324 πcm^2 and 576 πcm^2. What is the thickness of the shell?

(a) 6 cm (b) 3 cm
(c) 2 cm (d) 4 cm

19. The surface area of a sphere is 616 cm^2, what is the diameter of the sphere?

(a) 7 cm (b) 14 cm
(c) 16 cm (d) 24 cm

20. Five persons will live in a tent. If each person requires 16 m^2 of floor area and 100 m^3 space for air then find the required height of the cone of the smallest size to accommodate those persons.

(a) 18.75 m (b) 20 m
(c) 21.75 m (d) None of these

21. The ratio of curved surface area of two right cones of equal base is 5 : 4. What is the ratio of their slant heights?

(a) 5 : 4 (b) 4 : 5
(c) 1 : 4 (d) 1 : 5

22. The ratio of the heights of two right cones is 3 : 2. Ratio of their radii of the base is 2 : 3. What is the ratio of their volumes?

(a) 2 : 3 (b) 3 : 2
(c) 3 : 1 (d) 1 : 3

23. The radius of the base of right cone is 7 cm and its height is 24 cm. What is the total surface area?

(a) 704 cm^2 (b) 550 cm^2
(c) 599 cm^2 (d) None of these

24. The total surface area of a right cone is 1760 cm^2 and radius of its base is 14 cm. What is the lateral surface area of the cone?

(a) 1148 cm^2 (b) 1144 cm^2
(c) 1198 cm^2 (d) None of these

25. The height of a conical tent is 14 m and its floor area is 346.5 m^2. How much canvas 1.1 m wide will be required for it?

(a) 525 m (b) 860 m
(c) 665 m (d) 425 cm

26. On increasing each of the radius of the base and the height of a cone by 20%. By what percent its volume will be increased?

(a) 72.8% (b) 60%
(c) 40% (d) 30%

27. The volume of a hemisphere is 19404 cm^3. What is the total surface area of the hemisphere?

(a) 4158 cm^2 (b) 3696 cm^2
(c) 8316 cm^2 (d) 4996 cm^2

28. The ratio between the volumes of two spheres is 8 : 27. What is the ratio between their surface area?

(a) 2 : 3 (b) 4 : 5
(c) 5 : 6 (d) 4 : 9

29. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. What is the ratio of their volumes?

(a) 20 : 27 (b) 27 : 20
(c) 9 : 4 (d) 4 : 9

30. The areas of three adjacent faces of a cuboid are x, y, z respectively then the volume of the cuboid is

(a) xyz (b) $\sqrt{xyz}$
(c) $2xyz$ (d) $3\sqrt{xyz}$

31. A metallic sphere of radius 10.5 cm is melted and then recast into smaller cones, each of radius 3.5 cm and height 3 cm. How many cones are obtained?

(a) 126 (b) 124
(c) 128 (d) 130

Answer Key

1. (c)	2. (b)	3. (b)	4. (a)	5. (a)	6. (a)	7. (b)	8. (b)
9. (c)	10. (c)	11. (a)	12. (b)	13. (b)	14. (a)	15. (a)	16. (b)
17. (b)	18. (b)	19. (b)	20. (a)	21. (a)	22. (a)	23. (a)	24. (d)
25. (a)	26. (a)	27. (a)	28. (d)	29. (a)	30. (b)	31. (a)	

Hints and Solutions

1. **(c)** Let the no. of marbles $= x$

Radius of marble $= \frac{1.4}{2} = 0.7$ cm

Radius of cylinder $= \frac{7}{2} = 3.5$ cm

$$x \times \frac{4}{3}\pi \times (0.7)^3 = \pi(3.5)^2 \times 5.6$$

$$\Rightarrow x = \frac{3.5 \times 3.5 \times 5.6 \times 3}{4 \times 0.7 \times 0.7 \times 0.7}$$

$$\Rightarrow x = 150$$

2. **(b)** Volume of lead in the shell = volume of cylinder

$$\Rightarrow \frac{4}{3}\pi(12^3 - 9^3) = \pi r^2 h$$

$$\Rightarrow \frac{4}{3}\pi(1728 - 729) = \pi \times r^2 \times 37$$

$$\Rightarrow r^2 = 36 \Rightarrow r = 6$$

$\therefore$ diameter $= 2r = 2 \times 6 = 12$ cm

3. **(b)** Volume of cone = Volume of metal in the sphere

$$\frac{1}{3}\pi r_1^2 h = \frac{4}{3}\pi(R^3 - r^3)$$

$$\Rightarrow \frac{1}{3} \times \pi \times 4^2 \times h = \frac{4}{3}\pi(4^3 - 2^3)$$

$$\Rightarrow 4^2 h = 4 \times 56$$

$$\Rightarrow h = \frac{4 \times 56}{4 \times 4} = 14 \text{ cm}$$

4. **(a)** Let r be the radius of base of the conical part.

Height of the conical part $= 4r$

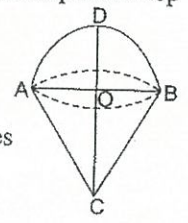
Volume of cone with hemispherical top = volume of cone + volume of hemispherical top

$$= \left[\frac{1}{3}\pi r^2 \times 4r + \frac{2}{3}\pi r^3\right] \text{cm}^3$$

$$= 2\pi r^3 \text{ cm}^3$$

Volume of 10 such cones with hemispherical top

$$= 10 \times 2\pi r^3 = 20\pi r^3 \text{ cm}^3$$



Volume of cylindrical container $= \pi \times \left(\frac{12}{2}\right)^2 \times 15$

$$= 540\ \pi \text{cm}^3$$

$$20\ \pi r^3 = 540\ \pi$$

$$r^3 = 27 \Rightarrow r = 3 \text{ cm.}$$

diameter $= 3 \times 2 = 6$ cm

5. **(a)** Total surface area of the remaining solid = curved surface area of the cylinder + curved surface area of the cone + area of upper base of the cylinder

$$= 2\pi rh + \pi rl + \pi r^2$$

$$= 2 \times \frac{22}{7} \times 5 \times 12 + \frac{22}{7} \times 5 \times 13 + \frac{22}{7} \times 5 \times 5$$

$$= \frac{2640}{7} + \frac{1430}{7} + \frac{550}{7} = \frac{4620}{7} = 660 \text{ cm}^2.$$

6. **(a)** Radius of cylinder $= \frac{12}{2} = 6$ cm

height $= 15$ cm

Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 6^2 \times 15$$

$$= 540\pi \text{ cm}^3$$

Volume of 12 toys = 540π cm^3

Volume of 1 toy = $\frac{540\pi}{12} = 45\pi$ cm^3

Let the radius of the hemisphere be r cm.

Height of the cone = $3r$ cm

Volume of one toy = Volume of hemisphere + volume of cone

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 \times 3r$$

$$= \frac{5}{3}\pi r^3$$

Now $\frac{5\pi r^3}{3} = 45\pi$

$\Rightarrow$ $r^3 = \frac{45\times 3}{5} = 27 = 3^3$.

$\Rightarrow$ $r = 3$ cm

7. (b) Let r be the radius of cylinder

Total surface area = $2\pi r(h + r)$

$\Rightarrow 2\times\frac{22}{7}\times r(20 + r) = 2992$

$\Rightarrow 20r + r^2 = \frac{2992\times 7}{2\times 22} = 476$

$\Rightarrow r^2 + 20r - 476 = 0$

$\Rightarrow r^2 + 34r - 14r - 476 = 0$

$\Rightarrow r(r + 34) - 14(r + 34) = 0$

$\Rightarrow (r - 14)(r + 34) = 0$

$\Rightarrow r = 14; r = -34$ (Not possible)

diameter = $2 \times 14 = 28$ cm

8. (b) Volume of the cube = 1728

$a^3 = 1728 \Rightarrow A = 12$ cm

Total surface area = $6a^2 = 6 \times (12)^2$

$= 6 \times 144 = 864$ cm^2.

9. (c) Curved surface area of cylindrical pillar = 264

$\Rightarrow 2\pi rh = 264 \Rightarrow h = \frac{264}{2\pi r}$ ...(1)

$\therefore$ Volume = 924

$\pi r^2 h = 924 \Rightarrow h = \frac{924}{\pi r^2}$ ...(2)

From (1) and (2)

$\frac{264}{2\pi r} = \frac{924}{\pi r^2}$

$264r = 924 \times 2$

$h = \frac{924\times 2}{2\times 22\times 7} = 6$ cm

10. (c) Area of canvas = $2\pi rh + \pi rl$

$= 2\times\frac{22}{7}\times\frac{105}{2}\times 4 + \frac{22}{7}\times\frac{105}{2}\times 40$

$= 1320 + 6600 = 7920$ cm^2.

11. (a) Radius of hemisphere = R

$\therefore$ Volume of hemisphere = $\frac{2}{3}\pi R^3$

radius of spherical balls = $\frac{R}{4}$

Let the no. of spherical balls casted be x.

$\therefore$ Volume of resulting spheres = $x\left(\frac{4}{3}\pi\left(\frac{R}{4}\right)^3\right)$

Now,

Initial volume = Final volume,

$\therefore \frac{2}{3}\pi R^3 = x\times\frac{4}{3}\times\pi\times\left(\frac{R}{4}\right)^3$

$\Rightarrow x = \frac{2}{4}\times 4\times 4\times 4$

$\Rightarrow x = 32$

12. (b) Let the base areas and heights be A and h respectively.

For cylinder, volume = $V_C = \pi r^2 h = Ah$

For cone, volume = $V_{CO} = \frac{1}{3}\pi r^2 h = \frac{1}{3}Ah$

For hemisphere, volume = $V_h = \frac{2}{3}\pi r^3$

$\therefore$ Ratio of their volumes = $\frac{2}{3}(\pi r^2)r$

$= \frac{2}{3}Ah \quad [\because h = r]$

$\therefore \frac{2}{3}Ah \quad [\because h = r] = 1 : \frac{1}{3} : \frac{2}{3} = 3 : 1 : 2$

13. (b) Let the number of cones made be n.

Volume of solid sphere = volume of resulting cones

$\Rightarrow \frac{4}{3}\pi(10.5)^3 = h\times\frac{1}{3}\times\pi\times(3.5)^2\times 3$

$$\Rightarrow h = \frac{4\times(10.5)^2\times10.5}{(3.5)^2\times3}$$
$$= 12 \times 10.5 = 126.$$

14. **(a)** Volume of given cylinder $= \pi r^2 h$
$$= \pi(12)^2 \times 16$$
Now,
$$\frac{4}{3}\pi R^3 = \pi(12)^2\times16$$
$$\Rightarrow R^3 = \frac{12\times12\times16\times3}{4} \Rightarrow R = 12 \text{ cm}$$

15. **(a)** Let the thickness of the shell be t.
Inner radius $= r$, outer radius $= R$
$$\therefore \text{ Volume of spherical shell} = \frac{4}{3}\pi(R^3 - r^3)$$
$$= \frac{4}{3}\times\pi\times(R^3 - 216)\text{cm}^3$$
$$\therefore \text{ Weight of spherical shell}$$
$$= \frac{4}{3}\times\pi\times(R^3-216)\times\frac{21}{1000}\text{kg}$$
$$= 11.1786 \text{ kg}$$
$$\Rightarrow R^3 = 343 \Rightarrow R = 7 \text{ cm}$$
$$\therefore \text{ Thickness} = t = (R - n) = (7 - 6) = 1 \text{ cm}$$

16. **(b)** Surface area of sphere $= 4\pi R^2$
If the radius of sphere is doubled, i.e., R becomes $2R$, then
New surface area $= 4\pi(2R)^2 = 16\pi R^2$
$$= 4\ (4\pi R^2)$$
$$= 4 \text{ (surface area)}$$
$\therefore$ Surface area will become 4 times.

17. **(b)** Ratio of volumes of spheres of radii r_1 and r_2 respectively
$$= \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3.$$
Similarly,
$$\text{ratio of surface areas} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$
According to condition
$$\left(\frac{r_1}{r_2}\right)^3 = \frac{27}{8} \Rightarrow \frac{r_1}{r_2} = \frac{3}{2}$$
$$\therefore \text{ Ratio of surface areas} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{2}\right)^2$$
$$= \frac{9}{4} = 9:4$$

18. **(b)** Inner surface Area $= 4\pi r^2 = 324\ \pi \text{ cm}^2$
$$\Rightarrow r^2 = \frac{324\pi}{4\pi}\text{cm}^2$$
$$\Rightarrow r = 9 \text{ cm}$$
$$\therefore \text{ Outer radius} = \sqrt{\frac{576\pi}{4\pi}\text{cm}^2} = 12 \text{ cm}$$
$$\therefore \text{ thickness of the shell} = (12 - 9) = 3 \text{ cm}$$

19. **(b)** Surface area of sphere $= 4\pi r^2 = 616$
$$\Rightarrow r^2 = \frac{616}{4\pi} = \frac{616\times7}{22\times4} = 7\times7$$
$$\Rightarrow r = 7 \text{ cm}$$
$$\therefore \text{ diameter} = 2r = 2\times7 = 14 \text{ cm}$$

20. **(a)** Let the height of the required cone be h cm
$$\therefore \text{ Required base area} = (16)\times5$$
$$= 80 \text{ cm}^2 = \pi r^2$$
Height $= h$ cm volume $= \frac{1}{3}(\pi r^2)h$
According to given condition
Total volume required $= 5\times100 \text{ cm}^3 = 500 \text{ cm}^3$
$$\Rightarrow \frac{1}{3}(\pi r^2)h = 500 \text{ cm}^3$$
$$\Rightarrow h = \frac{500\times3}{16\times5} = 18.75 \text{ cm}$$

21. **(a)** Ratio of curved surface areas
$$= \frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{5}{4}$$
$r_1 = r_2$
$\therefore$ Ratio of curved surface area
$$= \frac{l_1}{l_2} = \frac{5}{4} = 5:4 \text{ (Given)}$$

22. **(a)** Ratio of volumes $= \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \frac{h_1}{h_2}$
$$= \left(\frac{2}{3}\right)^2 \times \frac{3}{2} = \frac{2}{3} = 2:3$$

23. **(a)** We know that,

$$l = \sqrt{h^2 + r^2}$$
$$= \sqrt{(24)^2 + (7)^2}$$
$$= 25 \text{ cm}$$

Total surface area of cone

$$= \pi r (l + r)$$
$$= \frac{22}{7} \times 7(l + 7) \quad \ldots(1)$$
$$= \frac{22}{7} \times 7 \times (25 + 7) \text{cm}^2$$
$$= 22 \times 32 \text{ cm}^2 = 704 \text{ cm}^2$$

24. **(d)** Total surface area of cylinder

$$= 2\pi r (r + h)$$
$$= 2 \times \frac{22}{7} \times 14(14 + h)$$
$$= 88 (14 + h)$$
$$= 1760 \text{ cm}^2.$$

$$\Rightarrow h = \frac{1760}{88} - 14 = 6 \text{ cm}$$

$\therefore$ Lateral Surface Area $= 2 \times \pi \times r \times h$

$$= 2 \times \frac{22}{7} \times 14 \times 6$$
$$= 498 \text{ cm}^2.$$

25. **(a)** Lateral surface area of canvas $= \pi rl$

Given, $\pi r^2 = 346.5 \text{ m}^2$

$$\Rightarrow r^2 = \frac{346.5 \times 7}{22} \text{m}^2 \times r = 10.5 \text{ cm}$$

$$\therefore \quad l = \sqrt{h^2 + r^2}$$
$$= \sqrt{(14)^2 + (10.5)^2} = 17.5 \text{ m}$$

$\therefore$ Length of canvas needed

$$= \frac{\pi \times (10.5) \times (17.5)}{1.1}$$
$$= \frac{22 \times 10.5 \times 17.5}{7 \times 1.1}$$
$$= 30 \times 17.5 = 525 \text{ m}$$

26. **(a)** Let the original radius be r and height be h.

Original volume $= V = \frac{1}{3}\pi r^2 h$

New radius $= 120\%$ of $r = \frac{120r}{100} = \frac{6r}{5}$

New height $= 120\%$ of $h = \frac{120h}{100} = \frac{6h}{5}$

$$\text{New volume} = \frac{1}{3}\pi\left(\frac{6r}{5}\right)^2 \times \frac{6h}{5}$$
$$= \frac{216}{125}\left(\frac{1}{3}\pi r^2 h\right) = \frac{216}{125}V$$

$$\text{Increase in volume} = \frac{216V}{125} - V = \frac{91V}{125}$$

$$\text{Increase \%} = \frac{91V}{125} \times \frac{1}{V} \times 100$$
$$= \frac{91V}{125} \times \frac{1}{V} \times 100$$
$$= \frac{91 \times 4}{5}$$
$$= 72.8\%$$

27. **(a)** Given $\frac{2}{3}\pi r^3 = 19404$

where r is radius

$$\Rightarrow r^3 = \frac{19404 \times 3 \times 7}{2 \times 22} = (21)^3$$

$$\Rightarrow \quad r = 21 \text{ cm}$$

Total surface area $= 3\pi r^2$

$$= 3 \times \frac{22}{7} \times 21 \times 21$$
$$= 4158 \text{ cm}^2.$$

28. **(d)** Let the radius of two spheres be R and r.

$$\therefore \quad \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{8}{27}$$

$$\Rightarrow \frac{R^3}{r^3} = \frac{2^3}{3^3} \Rightarrow \frac{R}{r} = \frac{2}{3}.$$

$$\text{Ratio of their surface area} = \frac{4\pi R^2}{4\pi r^2} = \left(\frac{R}{r}\right)^2$$
$$= \left(\frac{2}{3}\right)^2 = \frac{4}{9} = 4:9.$$

29. **(a)** Let the radii of two cylinders are $2r$, $3r$ and heights be $5h$ and $3h$.

$$\text{Ratio of their volumes} = \frac{\pi(2r)^2 5h}{\pi(3r)^3 3h}$$

$$= \frac{4\times5}{9\times3} = \frac{20}{27} = 20:27.$$

30. **(b)** Let l, b, h be the length, breadth and height

$x = lb$, $y = bh$, $z = lh$

$\therefore\ lb \times bh \times hl = xyz$

$$(lbh)^2 = xyz$$

$$lbh = \sqrt{xyz}$$

= Volume of the cuboid

31. **(a)** $$\text{No. of cones} = \frac{\text{Volumes of sphere}}{\text{Volume of each cone}}$$

$$= \frac{\frac{4}{3}\pi\times\left(\frac{21}{2}\right)^3}{\frac{1}{3}\pi\times\left(\frac{7}{2}\right)^2\times3} = 126.$$

11. Statistics

A certain value representative of the whole data and signifying its characteristics is called an average of the data.

There are three types of average for analysing data

(1) Mean

(2) Median

(3) Mode

Mean

If a variable x takes values $x_1, x_2, \ldots x_n$ with corresponding frequencies $f_1, f_2, \ldots f_n$ respectively, then arithmetic mean is given by

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \ldots f_nx_n}{f_1 + f_2 + \ldots + f_n} = \frac{\sum x_i f_i}{\sum x_i}$$

when $N = f_1 + f_2 + \ldots + f_n$ = total frequency.

Example 1: Find the mean of the following data:

x	19	21	23	25	27	29	31
f	13	15	16	18	16	15	13

(a) 25 (b) 20 (c) 26 (d) 30

Solution: **(a)** Mean $= \dfrac{\sum f_i x_i}{\sum f_i}$

$$\text{Mean} = \frac{\begin{array}{r}19\times13+21\times15+23\times16+25\times18\\+27\times16+29\times15+31\times13\end{array}}{13+15+16+18+16+15+13}$$

$$= \frac{\begin{array}{r}247+315+368+450+432\\+435+403\end{array}}{106}$$

$$= \frac{2650}{106} = 25$$

Example 2: If the mean of the following distribution is 6, then what is the value of P?

x	2	4	6	10	$P+5$
f	3	2	3	1	2

(a) 6 (b) 7 (c) 8 (d) 9

Solution: **(b)** Mean $= \dfrac{\sum f_i x_i}{\sum f_i}$

We know Mean $= \dfrac{2\times3+4\times2+6\times3+10\times1+(P+5)2}{3+2+3+1+2}$

$\Rightarrow \quad 6 = \dfrac{52+2P}{11} \Rightarrow 52+2P = 66$

$\Rightarrow \quad P = \dfrac{66-52}{2} = \dfrac{14}{2} = 7$

Example 3: If the mean of the given data is 15. Find f_1.

x	5	10	15	20	25
f	6	f_1	6	10	5

(a) 6 (b) 7 (c) 8 (d) 9

Solution: **(c)** Here Mean $= \dfrac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow \quad 15 = \dfrac{5\times6+10f_1+15\times6+20\times10+25\times5}{6+f_1+6+10+5}$

$\Rightarrow \quad 15 = \dfrac{445+10f_1}{27+f_1} \Rightarrow 445+10f_1 = 405+15f_1$

$\Rightarrow \quad f_1 = \dfrac{40}{5} = 8$

Example 4: Find the mean of the given frequency distribution.

Class Interval	0–8	8–16	16–24	24–32	32–40
Frequency	5	6	4	3	2

(a) 16.4 (b) 15 (c) 16 (d) 18.4

Solution: **(a)** Here

Class interval	**Class Mark** (x_i)	**Frequency** (f_i)	$f_i x_i$
0–8	4	5	20
8–16	12	6	72
16–24	20	4	80
24–32	28	3	84
32–40	36	2	72
		$\Sigma f_i = 20$	$\Sigma f_i x_i = 328$

$\therefore$ Mean $= \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{328}{20} = 16.4$

Median

It is the middle value of a distribution.

Steps for finding Median

(I) Arrange the observations in ascending or descending order.

(II) Determine the number of observations (n).

(III) If n is odd then median is the value of $\left(\frac{n+1}{2}\right)$ th term.

(IV) If n is even, then median is the arithmetic mean of $\left(\frac{n}{2}\right)$ th and $(n/2)$th term.

Example 5: Following are the weight of 15 students in a class. Find Median.

60, 35, 48, 37, 52, 41, 38, 54 ,43, 36, 32, 62, 53, 44, 45

(a) 20 (b) 44 (c) 43 (d) 45

Solution: **(b)** The given data in ascending order is

32, 35, 36, 37, 38, 41, 43, 44, 45, 48, 52, 53, 54, 60, 62.

No. of terms = 15 (odd)

so, Median = $\left(\frac{15+1}{2}\right)$ th term = 8^{th} term

$\therefore$ Median = 44

Median of a Grouped Frequency Distribution

1. Obtain the frequency distribution.
2. Prepare the cumulative frequency column and obtain $N = \sum f$
3. Find $N/2$
4. See the cumulative frequency just greater than $\frac{N}{2}$ and determine the corresponding class. It is called Median class.
5. Use the formula

$$\text{Median} = l + \left[\frac{\frac{N}{2} - F}{f}\right] \times h$$

where

l = lower limit of Median class

f = frequency of Median class

h = size of Median class

f = cumulative frequency of the class preceding the Median class

Example 6: Calculate the Median of the given data.

Marks	0–10	10–20	20–30	30–40	40–50
No. of candidates	5	25	25	18	7

(a) 20 (b) 24 (c) 18 (d) 10

Solution: **(b)**

Class interval	**Frequency**	**Cumulative frequency**
0–10	5	5
10–20	25	30
20–30	25	55
30–40	18	73
40–50	7	80

$N = 80$

$\frac{N}{2} = \frac{80}{2} = 40$

$20 - 30$ is the median class.

$$\text{Median} = l + \left(\frac{\frac{N}{2} - F}{f}\right) h$$

$$= 20 + \left(\frac{40-30}{25}\right) \times 10$$

$$= 20 + \frac{10 \times 10}{25}$$

$$= 20 + 4 = 24$$

Mode

The variable having maximum frequency is the mode.

Computation of Mode of a series of individual observation:

For finding Mode, we convert the given data in discrete frequency distribution by preparing a frequency table. From the frequency table, identify the value having maximum frequency.

Example 7: Find the mode of the following data

25, 16, 19, 48, 19, 20, 34, 15, 19, 20, 21, 24, 19, 16, 22, 16, 18, 20, 16, 19.

Solution: **(c)** The frequency table is as given below

Value	15	16	18	19	20	21	22	24	25	34	48
Frequency	1	4	1	5	3	1	1	1	1	1	1

(a) 18 (b) 15 (c) 19 (d) 10

Clearly maximum frequency = 5

Then mode = 19

Computation of Mode for a Continuous Frequency Distribution

1. Obtain the continuous frequency distribution.
2. Determine the class having maximum frequency. It is called Modal class.
3. Use the formula

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

where

l = lower limit of Modal class

f = frequency of Modal class

f_1 = frequency of the class preceding the Modal class

f_2 = frequency of the class following the Modal class

h = width of Modal class.

Example 8: Find the mode of the following frequency distribution.

Class	0–20	20–40	40–60	60–80	80–100	100–120
Frequency	10	35	52	61	38	29

(a) 65.625 (b) 65.425 (c) 60.425 (d) 61.425

Solution: **(a)**

Here 60 – 80 has maximum frequency 61.

So, 60–80 is the modal class

$$\therefore \quad \text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 60 + \frac{61 - 52}{2 \times 61 - 52 - 38} \times 20$$

$$= 60 + \frac{9}{122 - 90} \times 20$$

$$= 60 + 5.625 = 65.625$$

Multiple Choice Questions

1. If the mean of following distribution is 25, then what is the value of x?

Class	0–10	10–20	20–30	30–40	40–50
Frequency	2	3	5	3	x

(a) 2 (b) 4 (c) 6 (d) 8

2. What is the mode of the given data?

7, 8, 5, 6, 8, 3, 12 ,11, 8, 6, 8, 7, 8, 5, 3, 8

(a) 8 (b) 6 (c) 5 (d) 3

3. The median of first 10 Prime numbers is

(a) 11 (b) 12
(c) 13 (d) 14

4. If the difference of mode and median of a data is 24, then what is the difference of median and mean?

(a) 8 (b) 24 (c) 12 (d) 36

5. The arithmetic mean of a data is 48 and mode is 24 then the median is

(a) 40 (b) 20
(c) 42 (d) 50

6. If the mean of 6, 7, x, 8, y, 14 is 9, then which of the following is correct?

(a) $x+y=21$ (b) $x-y=19$
(c) $x+y=19$ (d) $x-y=21$

7. If the arithmetic mean of x, $x+3$, $x+6$, $x+9$, $x+12$ is 10 then what is the value of x.

(a) 58 (b) 5 (c) 6 (d) 4

8. In the following distribution.

Class	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45
Frequency	5	6	15	10	5	4	2	2

What is the cumulative frequency of 25–30?

(a) 26 (b) 36
(c) 41 (d) 45

9. In the given distribution, which is the modal class?

Class	0–10	10–20	20–30	30–40	40–50
Frequency	6	10	12	32	20

(a) 0–10 (b) 40–50
(c) 30–40 (d) 20–30

10. Find the median of the following data:

20, 25, 17, 18, 8, 15, 9, 11, 14, 22

(a) 16 (b) 12
(c) 18 (d) 17

11. What is the value of P, if the mean of the following distribution is 7.5?

x	3	5	7	9	11	13
f	6	8	15	P	8	4

(a) 3 (b) 6 (c) 5 (d) 7

12. Find the median of the following data

Class	0–10	10–30	30–60	60–80	80–90
Frequency	5	15	30	8	2

(a) 30 (b) 40
(c) 35 (d) 45

13. The median of the given data

41, 43, 127, 99, 61, 71, 92, 58, 57, is 61.

If 58 is replaced by 85, then what is difference of the new median and the given median?

(a) 9 (b) 8
(c) 10 (d) 12

14. The number of problems worked out by a student on seven days of a week are 5, 9, 15, 11, 13, 17, 7, then what is the sum of lower quartile and upper quartile?

(a) 15 (b) 18
(c) 22 (d) 19

15. What is the interquartile range for the data 2, 1, 0, 3, 1, 2, 3, 4, 3, 5?

(a) 1 (b) 2 (c) 3 (d) 4

16. What is difference of mean and median of the following data?

Variate	2	4	6	8	10	12	14	16
Frequency	5	8	10	13	4	12	6	2

(a) 1.23 (b) 0.43
(c) 2.43 (d) 2

17. If the mean of first n natural number is 15, what is the value of n?

(a) 29 (b) 30
(c) 15 (d) 14

18. The arithmetic mean and mode of a data are 24 and 12 respectively. What is the median of that data?

(a) 18 (b) 20
(c) 23 (d) 22

19. If the mean of a frequency distribution is 8.1 and $\sum f_i x_i = 132 + 5k$, $\sum f_i = 20$.
What is the value of k?

(a) 5 (b) 6
(c) 7 (d) 8

20. If the numbers 25, 31, x –3, 37, x + 4, 42, 43, 45, 46 are in ascending order and their median is 39. What is the number x?

(a) 40 (b) 35
(c) 32 (d) 42

21. What is the median of the possible values of x such that $1 \leq x \leq 7$, where x is an integer?

(a) 4 (b) 3
(c) 5 (d) 4.5

22. The median of first five natural numbers be 3. If 6 is included. then what is the median, Also find the difference of two medians respectively?

(a) 3.5, 0.5 (b) 4, 1
(c) 4.5, 1.5 (d) None of these

23. What is the mode of the given data?

Variate	12	13	14	15	16	17	18
Frequency	21	8	9	51	38	12	25

(a) 12 (b) 16
(c) 15 (d) 18

24. What is the mean, median, mode of the following data respectively?

0, 2, 2, 3, 3, 3, 4, 5, 5, 5, 5, 6, 6, 7, 8, 8

(a) 4.5, 5, 5 (b) 5, 4, 5
(c) 5, 5, 6 (d) 4.5, 6, 6

25. What is the difference between mean and mode of the data

3, 1, 5, 6, 3, 4, 5, 3, 7, 2?

(a) 0.5 (b) 0.9
(c) 0.8 (d) 0.6

26. What is the mean of the following data?

Class Interval	0–50	50–100	100–150	150–200	200–250	250–300
Frequency	4	8	16	13	6	3

(a) 145 (b) 144
(c) 143 (d) 148

27. If the mean of the given data is 7.5 then what is the value of f?

Variable	5	6	7	8	9	10	11	12
Frequency	20	17	f	10	8	6	7	6

(a) 12 (b) 14
(c) 15 (d) 16

28. What is the median of the following distribution?

Variate	1	2	3	4	5	6	7	8	9
Frequency	8	10	11	16	20	25	15	9	6

(a) 7 (b) 6
(c) 5 (d) 4

29. What is the median of the given data?

Marks	0–10	10–20	20–30	30–40	40–50	50–60
No. of students	5	8	20	15	7	5

(a) 28.5 (b) 27.5
(c) 26.5 (d) 29.5

30. What is the mode of the following distribution?

Class	3–6	6–9	9–12	12–15	15–18	18–21	21–24
Frequency	2	5	10	23	21	12	3

(a) 14.6 (b) 15.6
(c) 12.6 (d) 13.6

31. The median of the data 5, 6, 7, 8, 9, 10 is _____.

(a) 7 (b) 8
(c) 7.5 (d) 8.5

32. If a mode exceeds a mean by 12 then the mode exceeds the median by _____.

(a) 4 (b) 8
(c) 6 (d) 10

33. If the less than cumulative frequency of a class is 50 and that of the previous class is 30, then the frequency of that class is _____.
(a) 10 (b) 20
(c) 40 (d) 30

34. If the median of the data $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$ is a, then find the median of the data x_3, x_4, x_5, x_6.
(where $x_1 < x_2 < x_3 < x_4 < x_5 < x_6 < x_7 < x_8$)
(a) a (b) $\frac{a}{2}$
(c) $\frac{a}{4}$ (d) Cannot say

35. The mode of the data 6, 4, 3, 6, 4, 3, 4, 6, 5 and x can be
(a) Only 5 (b) Both 4 and 6
(c) Both 3 and 6 (d) 3, 4 or 6

36. If the greater than cumulative frequency of a class is 60 and that of the next class is 40, then find the frequency of that class.
(a) 10 (b) 20
(c) 50 (d) Cannot say

37. If the difference between the mode and median is 2, then the difference between the median and mean is _____ (in the given order).
(a) 2 (b) 4 (c) 1 (d) 0

Answer Key

1. (a)	2. (a)	3. (b)	4. (c)	5. (a)	6. (c)	7. (d)	8. (c)	9. (c)	10 (a)
11. (a)	12. (b)	13. (c)	14. (c)	15. (b)	16. (b)	17. (a)	18. (b)	19. (b)	20. (b)
21. (a)	22. (a)	23. (c)	24. (a)	25. (b)	26. (c)	27. (d)	28. (c)	29. (a)	30. (a)
31. (c)	32. (b)	33. (b)	34. (a)	35. (d)	36. (b)	37. (c)			

Hints and Solutions

1. (a)

Class	0–10	10–20	20–30	30–40	40–50
Class marks	5	15	25	35	45
Frequency	2	3	5	3	x
xf	10	45	125	105	$45x$

$$\therefore \text{Mean} = \frac{\sum x_i f_i}{\sum f_i} = \frac{10+45+125+105+45x}{2+3+5+3+x}$$

$$\Rightarrow 25 = \frac{285+45x}{13+x}$$

$$\Rightarrow 325 + 25x = 285 + 45x$$

$$\Rightarrow 20x = 40 \Rightarrow x = \frac{40}{20} = 2$$

2. (a) Here the frequency of 8 is maximum. (6 times)
$\therefore$ Mode = 8

3. (b) The first 10 Prime numbers are
2, 3, 5, 7, 11, 13, 17, 19, 23, 29
$\because$ $n = 10$, which is even

$$\therefore \text{Median} = \frac{1}{2}\left[\frac{n}{2}th + \left(\frac{n}{2}+1\right)th\right] \text{ terms}$$

$$= \frac{1}{2}\left(\left[\frac{10}{2}th + \left(\frac{10}{2}+1\right)th\right]\right) \text{ terms}$$

$$= \frac{1}{2}\,(5^{\text{th}} + 6^{\text{th}} \text{ term})$$

$$= \frac{1}{2}\,(11 + 13) = \frac{1}{2} \times 24 = 12$$

4. (c) Given, Mode – Median = 24
and we know, Mode = 3 Median – 2 Mean
$\therefore$ 24 + Median = 3 Median – 2 Mean
$\Rightarrow$ 24 = 2 (Median – Mean)
$\Rightarrow$ median – mode = 12

5. (a) Here Mean = 48; Mode = 24

$\because$ Mode = 3 Median − 2 Mean

$\therefore$ $24 = 3 \text{ Median} - 2 \times 48$

$\Rightarrow$ $3 \text{ Median} = 24 + 96$

$\Rightarrow$ $\text{Median} = \frac{120}{3} = 40$

6. (c) Given $\frac{6+7+x+8+y+14}{6} = 9$

$\Rightarrow$ $x + y + 35 = 54$

$\Rightarrow$ $x + y = 19$

7. (d) Given $\frac{x+x+3+x+6+x+9+x+12}{5} = 10$

$\Rightarrow$ $5x + 30 = 50$

$\Rightarrow$ $5x = 20 \Rightarrow x = 4$

8. (c) Here

Class	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45
Frequency	5	6	15	10	5	4	2	2
c.f	5	11	26	36	41	45	47	49

$\therefore$ *c.f.* of 25 − 30 = 41

9. (c) The class having maximum frequency is called modal class i.e. 30–40.

10. (a) The given data is ascending order is

8, 9, 11, 14, 15, 17, 18, 20, 22, 25

$n = 10$ (even)

$$\text{Median} = \frac{1}{2}\left[\frac{n}{2}th + \left(\frac{n}{2}+1\right)th\right] \text{ term}$$

$$= \frac{1}{2}\,[5^{\text{th}} + 6^{\text{th}} \text{ term}]$$

$$= \frac{1}{2}\,[15 + 17]$$

$$= \frac{1}{2} \times 32 = 16$$

11. (a) $\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$

$\Rightarrow$ $7.5 = \frac{3\times6+5\times8+7\times15+9P+11\times8+13\times4}{6+8+15+P+8+4}$

$\Rightarrow$ $7.5 = \frac{18+40+105+88+52+9P}{41+P}$

$\Rightarrow$ $307.5 + 7.5P = 303 + 9P$

$\Rightarrow$ $1.5P = 4.5 \Rightarrow P = \frac{4.5}{1.5} = 3$

12. (b)

Class	0–10	10–30	30–60	60–80	80–90
f	5	15	30	8	2
cf	5	20	50	58	60

$N = 60$

$\frac{N}{2} = 30$

c.f just greater than 30 is 50

Median class is 30 – 60

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 30 + \frac{30-20}{30} \times 30$$

$$= 30 + 10 = 40$$

13. (c) The given data in ascending order is

41, 43, 57, 61, 71, 85, 92, 99, 127

$n = 9$ (odd)

$\text{Median} = \left(\frac{9+1}{2}\right)^{th} = 5^{\text{th}}$ term $= 71$

Required Difference $= 71 - 61 = 10$

14. (c) The given data in ascending order is

5, 7, 9, 11, 13, 15, 17

$n = 7$ (odd)

$\text{Lower quartile} = \frac{n}{4}th = \frac{7}{4}th = 2^{\text{nd}}$ term $= 7$

$\text{Upper quartile} = \frac{3n}{4}th = \frac{3\times7}{4}th = \frac{21}{4}th$

$= 6^{\text{th}}$ term $= 15$

$\therefore$ Sum $= 7 + 15 = 22$

15. (b) The given data in ascending order is

0, 1, 1, 2, 2, 3, 3, 3, 4, 5

$n = 10$ (even)

$\text{Lower quartile} = \frac{n}{4}th = \frac{10}{4}th = 2.5^{\text{th}}$

$= 3^{\text{rd}}$ term

$= 1$

Upper quartile $(Q_3) = \frac{3n}{4}th$ term

$= \frac{3\times10}{4} = 7.5^{th}$ term

$= 8^{th}$ term

$= 3$

Interquartile range $= Q_3 - Q_1 = 3 - 1 = 2$

16. **(b)** Here Mean $= \frac{\sum x_i f_i}{\sum f_i}$

$= \frac{10+32+60+104+40+144+84+32}{5+8+10+13+4+12+6+2}$

$= \frac{506}{60} = 8.43$

$N = 60$

$\frac{N}{2} = \frac{60}{2} = 30$ lies between $24 - 36$

Variate $= 8 \Rightarrow$ Median $= 8$

Required difference $= 8.43 - 8 = 0.43$

17. **(a)**

Given $\frac{1+2+3+.....+n}{n} = 15$

$\Rightarrow \frac{\frac{n}{2}[1+n]}{n} = 15$

$\Rightarrow \frac{1+n}{2} = 15 \Rightarrow n = 30 - 1 = 29.$

18. **(b)** $\because$ Mode $= 3$ Median $- 2$ Mean

$\therefore \ 12 = 3$ Median $- 2 \times 24$

$\Rightarrow$ Median $= \frac{60}{3} = 20$

19. **(b)** We have Mean $= \frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow 8.1 = \frac{132+5k}{20}$

$\Rightarrow 162 = 132 + 5k \Rightarrow 5k = 30$

$\Rightarrow \quad k = 6$

20. **(b)** The given data in ascending order is

$25, 31, x-3, 37, x+4, 42, 43, 45, 46$

$n = 9$ (odd)

Median $= \frac{n+1}{2}th$ term $= \frac{9+1}{2}th$ term

$= 5^{th}$ term

$\Rightarrow \quad x + 4 = 39$

$\Rightarrow \quad x = 35$

21. **(a)** Values of x are 1, 2, 3, 4, 5, 6, 7

$n = 7$ (odd)

Median $= \frac{n+1}{2}th$ term

$= \frac{7+1}{2}th = 4^{th}$ term $= 4$

22. **(a)** First 5 natural numbers are 1, 2, 3, 4, 5

$n = 5$ (odd)

$\therefore$ Median $= \frac{n+1}{2}th = \frac{5+1}{2}th = 3^{rd}$ term

$\Rightarrow$ Median $= 3$

1, 2, 3, 4, 5, 6

Median $= \frac{1}{2}(3+4) = \frac{7}{2} = 3.5$

Difference $= 3.5 - 3 = 0.5$

23. (c) The variate having highest frequency is the mode.

24. **(a)**

Here Mean $= \frac{0+2+2+3+3+3+4+5+5+5+5+6+6+7+8+8}{16}$

$= \frac{72}{16} = 4.5$

and $n = 16$ (even)

$\therefore$ Median $= \frac{1}{2}\left[\frac{16}{2}th + \left(\frac{16}{2}+1\right)th\right]$

$= \frac{1}{2}[5+5] = \frac{10}{2} = 5$

also Mode $= 5$

25. **(b)** The given data in ascending order is

1, 2, 3, 3, 3, 4, 5, 5, 6, 7

$\therefore$ Mean $= \frac{1+2+3+3+3+4+5+5+6+7}{10}$

$= \frac{39}{10} = 3.9$

Mode $= 3$

Difference $= 3.9 - 3 = 0.9.$

26. (c)

Class Interval	0–50	50–100	100–150	150–200	200–250	250–300
f	4	8	16	13	6	3
$c.f$	4	12	28	41	47	50
x	25	75	125	175	225	275

$$\therefore \text{ Mean} = \frac{\sum x_i f_i}{\sum f_i} = \frac{7150}{50} = 143$$

27. (d)

$$\text{Mean} = \frac{100+102+7f+80+72+60+77+72}{20+17+f+10+8+6+7+6}$$

$$\Rightarrow 7.5 = \frac{563+7f}{74+f}$$

$$\Rightarrow 555 + 7.5f = 563 + 7f$$

$$\Rightarrow 0.5f = 8$$

$$\Rightarrow f = \frac{8}{0.5} = \frac{8\times10}{5} = 16$$

28. (c)

$N = 8 + 10 + 11 + 16 + 20 + 25 + 15 + 9 + 6$

$= 120$

$$\therefore \frac{N}{2} = \frac{120}{2} = 60$$

60 lies in between 45 – 65

$\therefore$ Variate = 5

29. (a)

Class	0–10	10–20	20–30	30–40	40–50	50–60
f	5	8	20	15	7	5
$c.f$	5	13	33	48	55	60

$N = 60$; $\frac{N}{2} = \frac{60}{2} = 30$

Median class = 20 – 30

$l = 20$; $F = 13$; $h = 10$; $f = 20$

$$\text{Median} = 20 + \frac{30-13}{20}\times10$$

$$= 20 + \frac{17}{2} = 20 + 8.5 = 28.5$$

30. (a)

Class	3–6	6–9	9–12	12–15	15–18	18–21	21–24
f	2	5	10	23	21	12	3

Class 12 – 15 has maximum frequency.

12 – 15 is the modal class

$$\text{Mode} = l + \frac{f-f_1}{2f-f_1-f_2}\times h$$

$$= 12 + \frac{23-10}{2\times23-10-21}\times3$$

$$= 12 + \frac{13}{46-31}\times3$$

$$= 12 + \frac{13}{15}\times3$$

$$= 12 + \frac{13}{5} = \frac{73}{5} = 14.6.$$

31. If the no. of observations is even then the median of the data is the average of $\left(\frac{n}{2}\right)$ and $\left(\frac{n}{2}+1\right)$ observations.

Hence median of the data = $\frac{\left(\frac{6}{2}\right)^{th} + \left(\frac{6}{2}+1\right)^{th}}{2}$ observation.

$$= \frac{7+8}{2} = \frac{15}{2} = 7.5$$

33. (b) Frequency of a particular class = cumulative frequency of that class – cumulative frequency of the previous class

$= 50 - 30 = 20$

35. (d) An observation which has more frequency in the data is called the mode of the data.

12. Probability

Probability: It is a concept which measures numerically the degree of certainty of the occurrence of events.

Experiment: An operation which can produce some well defined outcomes is called an experiment.

Random Experiment: An experiment in which all possible outcomes are known and the exact outcome cannot be predicted is called a random experiment.

Example: Tossing a fair coin, Rolling an unbiased die.

Die: A die is a solid cube having 6 faces, marked 1, 2, 3, 4, 5, 6.

In throwing a die, the outcome is the number of dots appearing on the uppermost face.

A well shuffled deck of playing cards has 52 cards. It has 13 cards each of four suits namely Spades, Clubs, Hearts and Diamonds. Cards of spades and clubs are black cards. Cards of hearts and diamonds are red cards. Kings, queens, Jacks are known as face cards.

When we toss a coin, we get either a head or a tail.

Event: The collection of all or some of the possible outcomes is called an event.

Probability of occurrence of an event denoted by P(E) is given by

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes}}$$

The probability of an impossible event is zero.

If E be an event.

(not E) be an event which occurs only when E does not occur.

$$P(E) = 1 - P(\text{not E}).$$

For some event E, then $0 \le P(E) \le 1$.

P (E) = 1, when E is a sure event.

Example 1: A die is thrown once. What is the probability of getting a prime number?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) 1

Solution: **(a)**

In a single throw of a die, all possible outcomes are 1, 2, 3, 4, 5, 6.

Total no. of possible outcomes = 6.

The prime numbers are 2, 3, 5.

No. of favourable outcomes = 3.

$$P(E) = \frac{3}{6} = \frac{1}{2}$$

Example 2: One card is drawn at random from a well shuffled deck of 52 cards. What is the probability of drawing a king?

(a) $\frac{1}{12}$ (b) $\frac{1}{13}$ (c) $\frac{1}{15}$ (d) $\frac{1}{4}$

Solution: **(b)**

Total no. of all possible outcomes = 52.

Total number of kings = 4.

$$P(E) = \frac{4}{52} = \frac{1}{13}$$

Example 3: Sohan draws a card from a well shuffled deck of 52 cards. What is the probability that he draws a black card?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{5}$ (d) $\frac{1}{6}$

Solution: **(b)**

Total no. of possible outcomes = 52.

No. of black cards = 26.

$$P \text{ (getting a black card)} = \frac{26}{52} = \frac{1}{2}$$

Example 4: In a lottery, there are 10 prizes and 25 blanks. What is the probability of getting a prize?

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{4}$ (d) $\frac{4}{7}$

Solution: **(b)**

Total no. of tickets = 10 + 25 = 35.

No. of prizes = 10

$$P \text{ (getting a prize)} = \frac{10}{35} = \frac{2}{7}$$

Example 5: A box contains 20 balls bearing numbers 1, 2, 3, ... 20 respectively. A ball is drawn at random from the box. What is the probability that the number on the ball is an odd number?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

Solution: **(a)**

Total no. of possible outcomes = 20

Odd numbers are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

No. of favourable outcomes = 10

$$P \text{ (getting an odd number)} = \frac{10}{20} = \frac{1}{2}$$

Example 6: Find the probability of getting 53 Tuesday in a leap year.

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{4}$ (d) 1

Solution: **(b)**

In a leap year, there are 366 days. There are 52 weeks and 2 days. For 52 weeks, there are 52 Tuesday. The 2 days may be (Sunday, Monday), (M, T), (T, W), (W, Th), (Th, F), (F, S), (S, Sunday)

$$P(E) = \frac{\text{No. of favourable outcomes}}{\text{No. of Total outcomes}} = \frac{2}{7}$$

Mulitple Choice Questions

1. The probability that it will rain today is 0.76. What is the probability that it will not rain today?
   (a) 0.24 (b) 0.2 (c) 0.5 (d) 1

2. A card is drawn at random from a well shuffled deck of 52 cards. What is the probability that the card drawn is neither a king nor a queen?
   (a) $\frac{12}{13}$ (b) $\frac{11}{13}$
   (c) $\frac{2}{13}$ (d) $\frac{1}{2}$

3. A bag contains 19 balls having numbers 1, 2, 3, ... 19 respectively. A ball is drawn at random from the bag, what is the probability that the number on the ball is a prime number?
   (a) $\frac{8}{19}$ (b) $\frac{3}{19}$ (c) $\frac{7}{19}$ (d) $\frac{6}{19}$

4. What is the probability of getting 53 Tuesday in a leap year?
   (a) $\frac{1}{7}$ (b) $\frac{2}{7}$
   (c) $\frac{4}{7}$ (d) None of these

5. There are 35 tickets numbered 1, 2, 3, . . . 35. One ticket is drawn at random. What is the probability that the number on the ticket is a multiple of 3 or 5.
   (a) $\frac{16}{35}$ (b) $\frac{18}{35}$
   (c) $\frac{12}{35}$ (d) $\frac{17}{35}$

6. There are 25 cards numbered from 1 to 25. One card is drawn at random. What is the probability that the number on this card is not divisible by 4?
   (a) $\frac{19}{25}$ (b) $\frac{6}{25}$
   (c) $\frac{21}{25}$ (d) $\frac{4}{25}$

7. Two dice are thrown simultaneously. Who is the probability of getting 9 as the sum of two numbers that turn up?
   (a) $\frac{5}{36}$ (b) $\frac{1}{9}$
   (c) $\frac{1}{12}$ (d) $\frac{1}{6}$

8. A bag contains tickets marked with numbers 179, 180, 172, 127, 115, 115, 122, 143, 175, 222, 232, 162, 112, 132, 192, 182, 174, 132, 32, 131. A ticket is drawn at random. Find the probability that the ticket drawn has an even digit at 10's place.
   (a) $\frac{7}{19}$ (b) $\frac{3}{20}$
   (c) $\frac{7}{20}$ (d) $\frac{6}{19}$

9. A man calculates that the probability of his winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has he bought?
   (a) 480 (b) 720
   (c) 240 (d) 140

10. A lot of 24 bulbs contains 25% defective. A bulb is drawn at random from the lot. It is found to be not defective and it is not put back. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?
    (a) $\frac{15}{24}$ (b) $\frac{17}{23}$
    (c) $\frac{20}{23}$ (d) $\frac{18}{23}$

11. A die is thrown 350 times and the score of 6 obtained 28 times. Find the probability of getting a score under 6.
    (a) $\frac{2}{25}$ (b) $\frac{23}{25}$
    (c) $\frac{21}{25}$ (d) $\frac{1}{25}$

12. A box contains 5 red marbles, 7 black marbles and 3 white marbles. One marble is taken out from the box at random what is the probability that the marble taken out will be black or white?

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{4}{5}$ (d) None of these

13. What is the probability that a number selected at random from the numbers 1, 2, ... 35 is not a multiple of 7?

(a) $\frac{6}{7}$ (b) $\frac{5}{7}$ (c) $\frac{4}{7}$ (d) $\frac{3}{7}$

14. A bag contains 6 red balls, 8 white balls, 5 green balls and 3 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is not white.

(a) $\frac{7}{11}$ (b) $\frac{8}{11}$

(c) $\frac{4}{11}$ (d) $\frac{9}{22}$

15. In a class, there are 35 boys and 15 girls. What is the probability of a randomly selected student of the class to be a girl?

(a) $\frac{3}{10}$ (b) $\frac{1}{5}$

(c) $\frac{2}{5}$ (d) None of these

16. Find the probability of getting 52 sunday in a leap year.

(a) $\frac{1}{7}$ (b) $\frac{5}{7}$ (c) $\frac{2}{7}$ (d) $\frac{3}{7}$

17. A box contains 6 green balls, 4 red balls and some white balls. If the probability of not drawing a white ball in one draw is $\frac{2}{3}$, what is the number of white balls?

(a) 5 (b) 6 (c) 7 (d) 4

18. A bag contains 5 red balls and some black balls. If the probability of drawing a black ball from the bag is twice that of a red ball. Find the number of black balls in the bag.

(a) 7 (b) 12 (c) 10 (d) 15

19. In a pack of 52 playing cards, the king, the queen, the Jack and 10 are lost, All these cards are of spade. A card is drawn from the remaining well shuffled back. Find the probability of getting a king.

(a) $\frac{3}{16}$ (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $\frac{1}{12}$

20. A card is drawn from a well shuffled pack of 52 cards. Find the probability that the card drawn is a non ace.

(a) $\frac{12}{13}$ (b) $\frac{4}{13}$

(c) $\frac{2}{13}$ (d) $\frac{1}{13}$

21. A Jar contains 54 marbles each of which is blue, green or white. The probability of getting a blue marble at random from the Jar is $\frac{1}{3}$ and the probability of getting a green marble at random is $\frac{4}{9}$. What is the number of white marbles?

(a) 12 (b) 10 (c) 21 (d) 15

22. A number n is selected from the numbers 1, 2, 3 then a second number r is randomly selected from the numbers. 2, 5, 7. What is probability that a product of nr of the two numbers will be less than 15?

(a) $\frac{7}{9}$ (b) $\frac{5}{9}$ (c) $\frac{1}{3}$ (d) $\frac{4}{9}$

23. In a bag, there are 70, ₹1 coin, 30, ₹2 coin and 50, ₹5 coin. One coin is drawn at random. What is the probability that it will not be a ₹5 coin?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{7}{15}$ (d) $\frac{1}{5}$

24. If a number x is chosen at random from the numbers $-3, -2, -1, 1, 2, 3, 4$. What is the probability that $x^2 < 12$?

(a) $\frac{6}{7}$ (b) $\frac{4}{7}$ (c) $\frac{3}{7}$ (d) $\frac{2}{7}$

25. A letter is chosen at random from the letter of word communication. What is the probability that the chosen letter is a vowel?

(a) $\frac{5}{13}$ (b) $\frac{7}{13}$

(c) $\frac{6}{13}$ (d) $\frac{4}{13}$

26. A bag contains 7 red balls, 8 green balls, 4 white balls and 5 black balls. If one ball is drawn at random. What is the probability that the ball drawn is not green?

(a) $\frac{9}{24}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) None of these

27. A letter is chosen at random from the letter of word concentration. What is the probability that the chosen letter is a consonant?

(a) $\frac{8}{13}$ (b) $\frac{6}{13}$ (c) $\frac{5}{13}$ (d) $\frac{7}{13}$

28. Pravin and Navin are friends. What is the probability that both will have the same birthday, if the year was not a leap year?

(a) $\frac{1}{365}$ (b) $\frac{2}{365}$

(c) $\frac{364}{365}$ (d) None of these

29. 500 tickets of a lottery were sold. There are 15 prizes on these tickets. If Mohan has purchased one ticket, what is the probability that he will win a price?

(a) 0.03 (b) 0.01

(c) 0.05 (d) 0.04

30. There are 600 shirts in a carton in which 12 shirts are defective. One shirt is drawn at random. What is the probability that it is non-defective shirt?

(a) 0.98 (b) 0.96

(c) 0.02 (d) 0.04

31. One card is drawn from a pack of 52 cards. What is the probability that the card drawn is a face card?

(a) $\frac{3}{13}$ (b) $\frac{4}{13}$ (c) $\frac{1}{13}$ (d) $\frac{2}{13}$

32. A bag contains 5 red balls and some black balls. If the probability of drawing a black ball is double that of a red ball. What is the number of black balls in the bag?

(a) 10 (b) 12 (c) 12 (d) 8

33. Two dice are thrown simultaneously. Find the probability of getting the sum as a prime number.

(a) $\frac{5}{12}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{7}{12}$

Answer Key

1. (a)	2. (b)	3. (a)	4. (b)	5. (a)	6. (a)	7. (b)	8. (c)	9. (a)
10. (b)	11. (b)	12. (a)	13. (a)	14. (a)	15. (a)	16. (b)	17. (a)	18. (c)
19. (b)	20. (a)	21. (a)	22. (a)	23. (b)	24. (a)	25. (c)	26. (c)	27. (a)
28. (a)	29. (a)	30. (a)	31. (a)	32. (a)	33. (a)			

Hints and Solutions

1. (a)

Let E be the event that it will rain today.

$P(E) = 0.76$

P (That it will not rain today) $= 1 - 0.76$

$= 0.24$

2. (b)

P (getting neither a king nor a queen)

$= 1 -$ P (getting a king or a queen)

$$= 1 - \left(\frac{4}{52} + \frac{4}{52}\right) = 1 - \frac{8}{52} = \frac{52-8}{52} = \frac{44}{52}$$

$$= \frac{11}{13}$$

3. (a)

Total number of possible outcomes = 19

Out of the given numbers, prime numbers are –2, 3, 5, 7, 11, 13, 17, 19

P (the number on the ball is a prime) $= \frac{8}{19}$

4. (b)

In a leap year, there are 52 weeks and 2 days. These two days may be (Monday, Tuesday) (Tuesday, Wednesday), (Wednesday, Thursday) (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday), (Sunday, Monday).

For 53 Tuesday, Probability $= \frac{2}{7}$

5. (a)

1, 2, 3, ... 35

The multiples of 3 or 5 = 3, 5, 6, 9, 10, 12, 15, 18, 20, 21, 24, 25, 27, 30, 33, 35

P (ticket is a multiple of 3 or 5) $= \frac{16}{35}$

6. (a)

1, 2, 3, ... 35

The numbers which are divisible by 4. 4, 8, 12, 16, 20, 24

P (Number divisible by 4) $= \frac{6}{25}$

P (Number not divisible by 4) $= 1 - \frac{6}{25} = \frac{19}{25}$

7. (b)

Total number of possible outcomes = 36

E = The sum of two numbers be 9 is as (3, 6), (6, 3), (4, 5), (5, 4)

$$P(E) = \frac{4}{36} = \frac{1}{9}$$

8. (c)

Total no. of tickets = 20

Let E = The ticket drawn has an even digit at 10's place

E = 180, 127, 122, 143, 222, 162, 182,

$$P(E) = \frac{7}{20}$$

9. (a)

Required probability = $6000 \times 0.08 = 480$

10. (b)

Total no. of bulbs = 24

No. of defective bulbs = 25% of 24

$$= \frac{25}{100} \times 24 = 6$$

Now a bulb is drawn at random from the 10%. It is found to be not defective and it is not put back. Now one bulb is drawn at random from the rest. Hence the probability that this bulb is not defective $= \frac{17}{23}$

11. (b)

Total number of outcomes = 350

No. of score of 6 obtained = 28

No. of score under 6 obtained = 350 – 28

= 322

Probability of getting the score of under 6

$$= \frac{322}{350} = \frac{23}{25}$$

12. (a)

No. of possible outcomes = 5 + 7 + 3 = 15

Probability of black marble $= \frac{7}{15}$

Probability of white marble $= \frac{3}{15}$

Probability that the marble taken out will be black or white $= \frac{7}{15}+\frac{3}{15}=\frac{10}{15}=\frac{2}{3}$

13. (a)

Numbers are 1, 2, 3, ... 35

Total no. of possible outcomes = 35

E = The number which is multiple of 7

= 7, 14, 21, 28, 35

$P(E)=\frac{5}{35}=\frac{1}{7}$

P (Not multiple of 7) $= 1-\frac{1}{7}=\frac{6}{7}$

14. (a)

Total no. of balls = 6 + 8 + 5 + 3 = 22

Probability that ball drawn is not white

$=\frac{6+5+3}{22}=\frac{14}{22}=\frac{7}{11}$

15. (a)

Total no. of students = 35 + 15 = 50

Probability that selected student is a girl

$=\frac{15}{50}=\frac{3}{10}$

16. (b)

In a leap year, there are 366 days. There are 52 weeks and 2 days. There are 52 Sundays. There may be five possibilities of being not a Sunday, which are (Monday, Tuesday) (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday).

Probability for 52 Sunday $=\frac{5}{7}$

17. (a)

Let no. of white balls $= x$

Total no. of balls $= 6 + 4 + x = 10 + x$

Probability of drawing not a white ball

$=\frac{4+6}{10+x}=\frac{10}{10+x}$

$\frac{10}{10+x}=\frac{2}{3}$ (given)

$\Rightarrow 20 + 2x = 30$

$2x = 30 - 20 \Rightarrow 2x = 10$

$\Rightarrow x = 5$

18. (c)

Let the no. of black balls $= x$

Total no. of balls $= x + 5$

Probability of drawing a black ball $= \frac{x}{5+x}$

Probability of drawing a red ball $= \frac{5}{5+x}$

According to question,

$\frac{x}{5+x}=\frac{2\times5}{5+x}\Rightarrow x=10$

19. (b)

Remaining cards = 52 – 4 = 48

There are 3 kings

$\therefore$ P (getting a king) $= \frac{3}{48}=\frac{1}{16}$

20. (a)

Total no. of outcomes = 52

No. of ace cards = 4

No. of Non ace cards = 52 – 4 = 48

P (getting a non-ace card) $= \frac{48}{52}=\frac{12}{13}$

21. (a)

Let there are b blue, g green and w white marbles.

$b + g + w = 54$

P (getting a blue marble) $= \frac{b}{54}$

$\Rightarrow \frac{1}{3}=\frac{b}{54}\Rightarrow b=18$

P (getting a green marble)

$=\frac{4}{9}\Rightarrow\frac{4}{9}=\frac{g}{54}\Rightarrow g=24$

No. of while marbles

$= 54 - (18 + 24) = 54 - 42 = 12$

22. (a)

Two numbers can be selected in 9 ways (1, 2), (1, 5), (1, 7) (2, 2) (2, 5), (2, 7), (3, 2) (3, 5), (3, 7)

Total no. of possible outcomes = 9

Favourable no. of outcomes = 7

P (getting a product less than 15) $= \frac{7}{9}$

23. (b)

Total no. of possible outcomes

$= 70 + 30 + 50$

$= 150$

No. of ₹5 coin = 50

No. of coins of ₹1 & ₹2 coin = 70 + 30 = 100

P (will not be ₹5 coin) $= \frac{100}{150} = \frac{2}{3}$

24. (a)

Total no. of possible outcomes = 7

$x^2 < 12$

P (getting that $x^2 < 12$) $= \frac{6}{7}$

25. (c)

Total no. of letters = 13

No. of vowel letters = 6

Requited probability $= \frac{6}{13}$

26. (c)

Total no. of balls = 7 + 8 + 4 + 5 = 24

The no. of balls which are not green

$= 7 + 4 + 5 = 16$

P (getting not a green ball) $= \frac{16}{24} = \frac{2}{3}$

27. (a)

Total no. of letters = 13

No. of consonant letters = 8

Required probability $= \frac{8}{13}$

28. (a)

Total no. of ways Pravin & Navin may have their birthday = 365 × 365

No. of ways in which they have the same birthday = 365

Required probability $= \frac{365}{365 \times 365} = \frac{1}{365}$

29. (a)

Total no. of tickets = 500

Total no. of prizes = 15

P (win a prize) $= \frac{15}{500} = \frac{3}{100} = 0.03$

30. (a)

Total no. of possible outcomes = 600

No. of defective shirts = 12

No. of non-defective shirts = 600 − 12 = 588

Required Probability $= \frac{588}{600} = \frac{49}{50} = 0.98$

31. (a)

Total no. of possible outcomes = 52

Total no. of face cards = 12

Required Probability $= \frac{12}{52} = \frac{3}{13}$

32. (a)

Let No. of black balls = x

Total no. of balls = $x + 5$

$$\frac{x}{x+5} = 2 \times \frac{5}{x+5}$$

$\Rightarrow x = 10$

33. (a)

Total no. of possible outcomes = 6 × 6 = 36

Favourable outcomes are

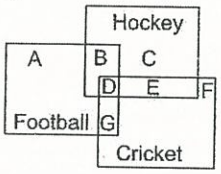
(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (6, 1), (6, 5), (5, 6)

Required probability $= \frac{15}{36} = \frac{5}{12}$

High Order Thinking Skills (Hots)

Multiple Choice Questions

1. The adjoining figure represents those students who play cricket, football and hockey. Study the figure and identify the student who play all the games.



(a) D (b) $G + E$
(c) $D + E + G$ (d) $A + B + C$

2. Naresh starts walking straight towards east. After walking 75 m he turns to the left and walks 25 m straight. Again he turns to the left walks a distance of 40 m straight, again he turns to the left and walks a distance of 25 m. How far is he from the starting point?
(a) 25 m (b) 50 m
(c) 75 m (d) 35 m

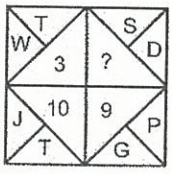
3. Pointing to a man, a woman said, "His mother is the only daughter of my mother." How is the woman related to the man?
(a) Mother (b) Sister
(c) Daughter (d) Grandmother

4. In a certain code language, '617' means 'sweet and hot'. '735' means 'coffee is sweet' and '263' means tea is hot. Which of the following would mean 'coffee is hot.'
(a) 731 (b) 536
(c) 367 (d) 753

5. Choose the pair in which the words are differently related.
(a) Bees : apiculture
(b) Silkworm : Sericulture
(c) Bird : Horiculture
(d) Fish : Pisciculture

6. Choose the pair in which the word bear the same relationship to each other as the words of the given pair bear.
Genuine : Authentic
(a) Ocean : water
(b) Breeze : Cyclone
(c) Mirage : Illusion
(d) Freeze : Threat

7. Find the missing number in the adjoining figure



(a) 15 (b) 11 (c) 13 (d) 12

8. If $525 \times 91 = 19525$; $137 \times 82 = 28731$
$84 \times 47 = 7448$; then $67 \times 45 = ?$
(a) 4576 (b) 5476
(c) 5467 (d) 5647

9. What is the product of all the numbers in the dial of a telephone?
(a) 1,59450 (b) 1,59,480
(c) 1,58,480 (d) None of these

10. If a student writes down all the numbers from 1 to 100, then how many times does he write 3?
(a) 20 (b) 18 (c) 21 (d) 12

11. Choose the correct alternative
c_bba_cab_ac_ab_ac
(a) *abcbc* (b) *acbcb*
(c) *babcc* (d) *bcacb*

12. Choose the missing term from the following
2, 7, 27, 107, ___, 1707.
(a) 427 (b) 417
(c) 407 (d) 437

13. If 'cinto baoli tsi nzro' means 'her village is sarurpur', 'mhi cinto keepi tsi onid', means 'her first love is literature, and 'oind geit tsi cinto pki' means 'literature collection is her hobby, which word would mean literature?
(a) geit (b) oind
(c) baoli (d) cinto

14. *A, B, C, D* are to be seated in a row. But *C* and *D* cannot be together. *B* cannot be at 3rd place. If *A* is not at 3rd place then what is the position of *C*?
(a) 1st place only
(b) the 3rd place only
(c) Any of the places
(d) the 1st and 3rd place only

15. Choose the word out of the given alternatives which cannot be formed from the letters of word CONSULTATION.
(a) SALUTE (b) NATION
(c) STATION (d) CONSTANT

16. Arrange the given words in a meaningful sequence and then choose the most appropriate sequence from the given alternatives.
1. Honey 2. Flower
3. Bee 4. Wax
(a) 2, 3, 1, 4 (b) 4, 3, 2, 1
(c) 2, 1, 4, 3 (d) 1, 3, 4, 2

17. In a family, the father took $\frac{1}{4}$ of the cake and he had 3 times as much as each of the other members had. The total number of family member is
(a) 3 (b) 7 (c) 10 (d) 12

18. If 100 cats kill 100 mice in 100 days then in how many days 4 cats would kill 4 rats?
(a) 1 day (b) 4 days
(c) 40 days (d) 100 days

19. From the word 'BEHIND' how many independent words can be made without changing the order of the letters and using each letter only one?
(a) 1 (b) 2 (c) 3 (d) 4

20. It being given that ∠ denotes 'equal to', □ denotes 'not equal to', + denotes 'greater than', − denotes 'less than', ÷ denotes 'not greater than', × denotes ' not less than. Choose the correct statement $a + b + c$ does not imply.
(a) $c + b - a$ (b) $b - a + c$
(c) $b \div a - c$ (d) None of these

21. If *L* stands for +, *M* stands for −, *N* stands for × and *P* stands for ÷, then 14N10L42P2M8 = ?
(a) 216 (b) 153
(c) 248 (d) 251

22. In a row of boys, *A* is 13th from the left and *D* is 17th from the right. If in this row *A* is 11th from the right then what is the position of *A* from the left?
(a) 7th (b) 10th
(c) 6th (d) 12th

23. If each of the odd digits in the number 54638 is decreased by 1 and each of the even digits is increased by 1 and each of the even digits is increased by 1, then which of the following will be the sum of digits of the new numbers?
(a) 27 (b) 29 (c) 28 (d) 26

24. In the series 6 4 1 2 2 8 7 4 2 1 5 3 8 6 2 1 7 1 4 1 3 2 8 6 how many pairs of successive numbers have a difference of 2 each?
(a) 4 (b) 6 (c) 5 (d) 7

25. How many meaningful English words can be formed by using any two letters of the word NOT?
(a) Two (b) Three
(c) One (d) None of these

26. Find out how many such pairs of letter are there in the given word each of which has many letters between them in the word as in English alphabet?
DECORATE
(a) One (b) Three
(c) Two (d) Four

27. In the given letter series, some of the letters are missing which are given in that order as one of the alternative below it. Choose the correct alternative.
_bc_ca_aba_c_ca
(a) *abcbb* (b) *bbbcc*
(c) *bacba* (d) *abbcc*

28. What is the missing number in the given series
6, 11, 21, 36, 56 _?
(a) 42 (b) 51
(c) 81 (d) 91

29. Find the term which does not fit into the given series
G4T, J10R, M20P, P43N, S90L
(a) J10R (b) G4T
(c) P43N (d) M209

30. If the English alphabet is arranged in reverse order, which letter will be twelfth to the left of the 16th letter from your left?
(a) *W* (b) *V* (c) *X* (d) *D*

31. In English alphabet which letter is 7th to the right of the 18th letter from the right end of the alphabet?
(a) *K* (b) *O* (c) *P* (d) *R*

32. How many 5s are there in the following number sequence which are immediately preceded by 7 and immediately followed by 6?
7 5 5 9 4 5 7 6 4 5 9 8 7 5 6 7 6 4 3 2 5 6 7 8
(a) One (b) Two
(c) Three (d) Four

33. Rakesh ranks 18th in a class of 49 students. What is his rank from the last?
(a) 32 (b) 18
(c) 19 (d) 31

34. One of the four alternatives under it specifies the interchange of signs in the equation which when made will make the equation correct. Find the correct alternative.
$2 \times 3 + 6 - 12 \div 4 = 17$
(a) × and + (b) + and –
(c) + and ÷ (d) – and ÷

35. In this question, jumbled letters of a meaningful word are given, you have to rearrage these letters and select from the given alternative, the word which is almost similar in meaningful to the rearranged word?
H N A G S R I
(a) Decorate (b) Compose
(c) Impress (d) Impose

36. If it was Saturday on 17th December 2002. What was the day on 22nd December 2004?
(a) Monday (b) Sunday
(c) Tuesday (d) Wednesday

37. In a certain code language '234' means 'spark and fire', '456' means 'spark is cause', '258' means 'fire is effect'. Which of the following numerals is used for cause?
(a) 3 (b) 4 (c) 5 (d) 6

38. Choose the group of letters which is different from other.
(a) CFIL (b) PSVX
(c) JMPS (d) ORUX

39. Choose the number which is different from the others.
(a) 3781 (b) 1593
(c) 9317 (d) 9175

40. Choose the odd pair of words
(a) Iodine : Goitre
(b) Iron : Anamia
(c) Sodium : Rickets
(d) Proteins : Marasmus

41. Choose the common characteristics among the give words
Viper : Krait : Mamba
(a) These are boot polishes
(b) These are insects living in bushes
(c) These are snakes
(d) These are haunting spirits

42. Bald is related to Bland in the same way as Barren is related to____
(a) Fertile (b) Inhabited
(c) Vegetarian (d) Farm

43. 1, 4, 10, 22, 46, ...
(a) 64 (b) 94
(c) 86 (d) 122

44. If STRONG is written as ROTNSG then how would NAGPUR be written in the same code?
(a) GPUANR (b) PGAURN
(c) PGUARN (d) GPAUNR

45. In a certain code, a number 13479 is written as AQFJL and 2568 is written as DMPN. How is 396824 written in that code?
(a) QLPNDF (b) QLPNMJ
(c) QLPMNF (d) QLPNMF

46. Choose the odd one out from the given group of letters.
(a) OUT (b) RAT
(c) LOT (d) BED

47. At the end of a business conference, the ten people present all shake hands with each other. How many handshakes will be there be altogether?
(a) 20 (b) 55
(c) 45 (d) 90

48. If '–' stands for division, '+' for multiplication, ÷ for subtraction and '×' for addition then which of the following is correct?
(a) $6 + 20 - 12 \div 7 - 1 = 38$
(b) $6 + 20 - 12 \div 7 \times 1 = 62$
(c) $6 \div 20 \times 12 + 7 - 1 = 70$
(d) $6 - 20 \div 12 \times 7 + 1 = 57$

49. If each of the odd digits in the number 54638 is decreased by 1 and each of the even digits is increased by 1 then which of the following is the sum of digits of the new number?
(a) 25 (b) 27
(c) 26 (d) 28

50. Aman is older than Mona. Gopi is older than Mona but younger than Aman. Kajal is younger than Rohan and Mona. Mona is older than Rohan? Who is the oldest?
(a) Rohan (b) Kajal
(c) Aman (d) Gopi

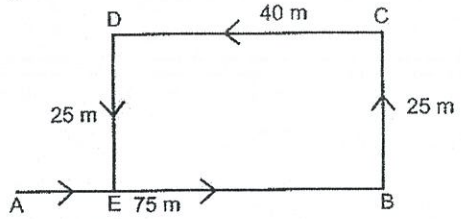
Answer Key

1. (a)	2. (d)	3. (b)	4. (b)	5. (c)	6. (c)	7. (a)	8. (b)	9. (d)	10 (a)
11. (d)	12. (a)	13. (b)	14. (d)	15. (a)	16. (a)	17. (a)	18. (d)	19. (c)	20. (d)
21. (b)	22. (a)	23. (a)	24. (b)	25. (b)	26. (b)	27. (a)	28. (c)	29. (a)	30. (a)
31. (c)	32. (a)	33. (a)	34. (a)	35. (a)	36. (b)	37. (d)	38. (b)	39. (a)	40. (c)
41. (c)	42. (a)	43. (b)	44. (d)	45. (a)	46. (a)	47. (c)	48. (c)	49. (b)	50. (a)

Hints and Solutions

1. (a) Only D plays all the games

2. (d)



Required Distance $= AE = AB - EB$
$= AB - CD$
$= 75 - 40 = 35$ m

4. (b) Given 617 = sweet and hot
735 = coffee is sweet
263 = tea is hot
$\therefore$ 7 = sweet, 6 = hot, 3 = is
Hence, coffee is hot = 536

7. (a) Here $W - T = 23 - 20 = 3$
$T - J = 20 - 10 = 10$
$P - G = 16 - 7 = 9$
$S - D = 19 - 4 = 15$

8. (b) $525 \times 91 = 19525$
$137 \times 82 = 28731$
$84 \times 47 = 7448$
$67 \times 45 = 5476$

9. **(d)** Product of all digits in the dial of a telephone = 0

10. **(a)** The numbers are 3, 13, 23, 33, 43, 53, 63, 73, 83, 93

$\therefore$ Total numbers = 10 + 10 = 20

11. **(d)** The given series is

cabbac cabbac cabbac

12. **(a)**

$2 \xrightarrow{+(5\times1^2)} 7 \xrightarrow{+(5\times2^2)} 27 \xrightarrow{+(5\times4^2)} 107 \xrightarrow{+(5\times8^2)} 437 \xrightarrow{+(5\times16^2)} 1707$

13. **(b)** cinto baoli tsi nzro →
her village is sarurpur
mhi cinto keepi tsi oind →
her first love is literature
and oind geit tsi cinto pki → literature
collection is her hobby.
clearly oind → literature

14. **(d)** CADB
DACB

15. **(a)** Because E is not in the word 'CONSULTATION'

16. **(a)** Flower → Bee → Honey → Wax

17. **(c)** Let there be $(x + 1)$ members

$$\text{Father's share} = \frac{1}{4}$$

$$\text{Other Member's share} = \frac{3}{4x}$$

$$3\left(\frac{3}{4x}\right) = \frac{1}{4} \Rightarrow 4x = 36 \Rightarrow x = 9$$

Total no. of members = 9 + 1 = 10

18. **(d)** Less cats, more days
Less mice, less days

$$\left.\begin{matrix}\text{cat } 4:100\\ \text{Mice } 100:4\end{matrix}\right\} :: x:100$$

$$x = \frac{4\times100\times100}{4\times100} = 100$$

21. **(b)** 14N10L42P2M8
= 14 × 10 + 42 × 2 − 8
= 14 × 10 + 21 − 8
= 140 + 21 − 8
= 161 − 8 = 153

22. **(a)** A is 13^{th} from the left and A is 11^{th} from the right end of the row.
Number of boys in the row = 12 + 1 + 10 = 23
D is 17^{th} from the right
Number of boys to the left of D = 23 − 17 = 6
D is 7^{th} from the left end of the row.

23. **(a)** 54638
45729
= 4 + 5 + 7 + 2 + 9 = 27

25. **(b)** There are three words – ON, NO, TO

26. **(b)** DE
ECORA
EAT

27. **(a)** a b c / b c a / c a b / a b c / b c a

28. **(c)** 6 + 5 = 11
11 + 10 = 21
21 + 15 = 36
36 + 20 = 56
56 + 25 = 81

29. **(a)**

$G \xrightarrow{+3} J \xrightarrow{+3} M \xrightarrow{+3} P \xrightarrow{+3} S$

$T \xrightarrow{-2} R \xrightarrow{-2} P \xrightarrow{-2} N \xrightarrow{-2} L$

4, 10, 20, 43, 90
4 × 2 + 1 = 9
9 × 2 + 2 = 20
20 × 2 + 3 = 43
43 × 2 + 4 = 90
$\therefore$ J10R is different.

30. **(a)**
ZYXWVU
TSRQPONMLKJIHGFEDCBA

31. **(c)**
A B C D E F G H I J K L M N O P Q R S T
U V W X Y Z ↑ 7^{th} →

32. **(a)** 7559457645981756764325678

33. **(a)** Number of students behind Rakesh in Rank
= 49 − 18 = 31
Rakesh is 32^{nd} from the last

34. **(a)** $2 \times 3 + 6 - 12 \div 4 = 17$

$2 + 3 \times 6 - 12 \div 4 = 2 + 3 \times 6 - 3$

$= 2 + 18 - 3 = 17$

35. **(a)** HNAGSRI → GARNISH

DECORATE

37. **(d)** Here 2 3 4 → spark and fire

4 5 6 → spark is cause

2 5 8 → fire is effect

Then 4 → spark

2 → Fire

5 → is

6 → Cause

38. **(b)**

$C \xrightarrow{+3} F \xrightarrow{+3} I \xrightarrow{+3} L$

$P \xrightarrow{+3} S \xrightarrow{+3} V \xrightarrow{+3} Y$

39. **(a)** 3 + 7 + 8 + 1 = 19 (odd)

1 + 5 + 9 + 3 = 18 (even)

9 + 7 + 1 + 7 = 20 (even)

9 + 1 + 7 + 5 = 22 (even)

43. **(b)**

$1 \xrightarrow{+3} 4 \xrightarrow{+6} 10 \xrightarrow{+12} 22 \xrightarrow{+24} 46 \xrightarrow{+48} 94$

44. **(d)**

S	T	R	O	N	G	→	R	O	T	N	S	G
1	2	3	4	5	6		3	4	2	5	1	6

N	A	G	P	U	R	→	G	P	A	U	N	R
1	2	3	4	5	6		3	4	2	5	1	6

45. **(a)** Here

1	3	4	7	9		2	5	6	8
↓	↓	↓	↓	↓		↓	↓	↓	↓
A	Q	F	J	L		D	M	P	N

3	9	6	8	2	4
↓	↓	↓	↓	↓	↓
Q	L	P	N	D	F

46. **(a)** There is only one vowel in each except OUT.

47. **(c)** Total number of handshakes

$= 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$

48. **(c)** Considering $6 \div 20 \times 12 + 7 - 1$

$= 6 - 20 + 12 \times 7 \div 1$

$= 6 - 20 + 12 \times 7$

$= 6 - 20 + 84$

$= 90 - 20 = 70$

49. **(b)** The new number is 4 5 7 2 9

Sum of digits = 4 + 5 + 7 + 2 + 9 = 27

50. **(a)** Given Aman > Mona

Aman > Gopi > Mona

Mona > Rohan > Kajal

Aman > Gopi > Mona > Rohan > Kajal

Model Test Paper - 1

1. 6, 13, 25, 51, 101 ?
   (a) 201 (b) 202
   (c) 203 (d) 205

2. Aryabhatta : Mathematician:: Varahamihira:?
   (a) Physician (b) Astronomer
   (c) Architect (d) Scientist

3. How many 7*S* are there in the given series which are preceded by 6 which is not preceded by 8?
   (a) One (b) Two
   (c) Three (d) Four

4. Which two months in a year have the same calendar?
   (a) April, July (b) April, November
   (c) June, October (d) April, November

5. One morning after sunrise Goyal was standing facing a pole. The shadow of the pole fell exactly to his right. Which direction was he facing?
   (a) North (b) South
   (c) East (d) West

6. The value of $\sqrt{6+\sqrt{6+\sqrt{6}}}+\ldots\ldots = ?$
   (a) 4 (b) 3 (c) –2 (d) 5

7. If the 1st, 2nd and last term of an A.P. are a, b and $2a$ respectively. 1st sum is given by
   (a) $\frac{3ab}{2(b-a)}$ (b) $\frac{ab}{b-a}$
   (c) $\frac{ab}{2(b-a)}$ (d) $\frac{ab}{a+b}$

8. The angle of elevation of a cloud from a point h metre above a lake is Q. The angle of depression of its reflection in the lake is 45°. The height of the cloud is

9. From the letters of the word 'MOBILE' a letter is selected. The probability that the letter is a vowel is
   (a) $\frac{1}{2}$ (b) $\frac{1}{6}$
   (c) $\frac{3}{7}$ (d) $\frac{1}{3}$

10. If points $(a, 0)$, $(0, b)$, and $(1, 1)$ are collinear. then $\frac{1}{a}+\frac{1}{b}=?$
    (a) 2 (b) 0 (c) 1 (d) –1

11. If mode of a series exceeds it mean by 12 then mode exceeds the median by
    (a) 4 (b) 6 (c) 8 (d) 10

12. Mode is
    (a) Middle most value
    (b) Most frequent value
    (c) Least frequent
    (d) None of these

13. If $\sin\theta = \cos\theta$. What is the value of Q?
    (a) 45° (b) 0°
    (c) 30° (d) 60°

14. If $\sin\alpha$ and $\cos\alpha$ are the roots of the eqn $ax^2 + bx + c = 0$ then what is the value of b^2?
    (a) $a^2 + 2ac$ (b) $a^2 - 2ac$
    (c) $a^2 + ac$ (d) $a^2 - ac$

15. If the altitude of the sum is at 60°, then the height of the vertical tower that will cast a shadow of length of 30m is
    (a) $30\sqrt{3}$ m (b) 15m
    (c) $10\sqrt{3}$ m (d) $15\sqrt{2}$ m

34. If the difference between the radius and circumference of a circle is 37 cm, then its area is

(a) 154 cm^2 (b) 200 cm^2
(c) 150 cm^2 (d) 160 cm^2

35. What is the distance between the points $(a\cos\theta + b\sin\theta, 0)$ and $(0, a\sin\theta - b\cos\theta)$?

(a) $a^2 + b^2$ (b) $a + b$
(c) $a^2 - b^2$ (d) $\sqrt{a^2 + b^2}$

36. The product of the zero of $x^3 + 4x^2 + x - 6$ is

(a) -4 (b) 4
(c) 6 (d) -6

37. The smallest number by which $\sqrt{27}$ should be multiplied so as to get a rational number is

(a) $3\sqrt{3}$ (b) $\sqrt{3}$
(c) $\sqrt{27}$ (d) 3

38. What is the exponent of 2 in the prime factorization of 144?

(a) 4 (b) 5
(c) 6 (d) 3

39. In an equilateral triangle ABC, if $AD \perp BC$, then what is the value of AD^2?

(a) CD^2 (b) $2CD^2$
(c) $3CD^2$ (d) $4CD^2$

40. If $\tan A = \sqrt{2} - 1$ then what is the value of $\sin A \cos A$?

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{1}{2}$ (d) 1

41. What is the mode of the following data

3, 5, 7, 4, 5, 3, 5, 6, 8, 9, 5, 3, 5, 3, 6, 9, 7, 4

(a) 5 (b) 3
(c) 7 (d) 6

42. What is the value of k for which $x^2 + 4x + k$ is a perfect square?

(a) 4 (b) 6
(c) 6 (d) 2

43. What is the sum of n terms of series

$\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots\ldots$

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n+1)}{\sqrt{2}}$
(c) 1 (d) $2n(n+1)$

44. If TP and TQ are two tangents to a circle with centre O so that $\angle POQ = 110°$, then what is the value of $\angle PTQ$?

(a) 80° (b) 90°
(c) 70° (d) 60°

45. A card is accidently dropped from a pack of 52 cards. The probability that it is an ace is

(a) $\frac{1}{4}$ (b) $\frac{1}{52}$
(c) $\frac{12}{13}$ (d) $\frac{1}{13}$

46. If four times the sum of areas of two circular faces of a cylinder of height 8 cm is equal to twice the curved surface area then diameter of the cylinder is

(a) 4 cm (b) 6 cm
(c) 8 cm (d) 2 cm

47. If two zeros of $x^3 + x^2 - 5x - 5$ are $\sqrt{5}$ and $-\sqrt{5}$, then the third root is

(a) 1 (b) 2 (c) -1 (d) 2

48. What is the value of k for which the system of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has no solution?

(a) -6 (b) 6 (c) 5 (d) $\frac{3}{2}$

Answers

1.(c)	2.(a)	3.(c)	4.(a)	5.(b)	6.(b)
7.(a)	8.(b)	9.(a)	10.(c)	11.(c)	12.(b)
13.(a)	14.(b)	15.(a)	16.(a)	17.(d)	18.(c)
19.(b)	20.(a)	21.(c)	22.(a)	23.(a)	24.(b)
25.(b)	26.(c)	27.(d)	28.(c)	29.(c)	30.(b)

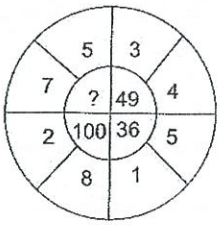
Mock Test Paper-2

1. Which number will be in place of question mark?
   1, 5, 7, 14, 18, 20, 40, 44, 46,?
   (a) 48 (b) 52
   (c) 92 (d) 50

2. Which one word cannot be made from the letter of the given word?
   (a) COIN (b) NOSE
   (c) SUN (d) SON

3. It was Saturday, on January 12, 1980. The day of the week on January 12, 1979 is
   (a) Sunday (b) Friday
   (c) Saturday (d) Thursday

4. What is the number in the question mark?



   (a) 75 (b) 144
   (c) 72 (d) None of these

5. Choose the number which has same relationship with the 3rd number as first two numbers are related?
   583 : 293 : : 488 : ?
   (a) 378 (b) 487
   (c) 291 (d) 581

6. For what value of k will the equation $2x + 2y + 7 = 0$, $2x + ky + 14 = 0$ represents coincident lines?
   (a) 3 (b) 4 (c) 5 (d) -4

7. If E is a point on side AC of equilateral triangle ABC such that $BE \perp AC$ then what is the value of $AB^2 + BC^2 + AC^2$?
   (a) $2BE^2$ (b) $3BE^2$
   (c) $6BE^2$ (d) $4BE^2$

8. If $\tan\theta = \frac{1}{\sqrt{3}}$ then what is the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$?

9. (a) $-\frac{1}{2}$ (b) $\frac{1}{2}$
   (c) -1 (d) -1

10. An arc of a circle is of length 5π cm and the sector it bounds has an area of 20π cm^2. What is the radius of the circle?
    (a) 8 cm (b) 4 cm
    (c) 12 cm (d) 16 cm

    Which of the following rational numbers have terminating decimal?
    (a) $\frac{7}{250}$ (b) $\frac{2}{21}$
    (c) $\frac{16}{225}$ (d) $\frac{5}{16}$

11. If the centroid of the triangle formed by the point (a, b), (b, c) and (c, a) is at the origin. then $a^3 + b^3 + c^3 = ?$
    (a) abc (b) $a + b + c$
    (c) $3abc$ (d) 0

12. The area of a circle whose area and circumference are numerically equal is
    (a) 2πsq. units (b) 8πsq. units
    (c) 4πsq. units (d) 5πsq. units

13. Find the quadratic equation whose one root is 2 and the sum of whose roots is zero.
    (a) $x^2 - 4 = 0$ (b) $x^2 + 4 = 0$
    (c) $x^2 - 2 = 0$ (d) $4x^2 - 1 = 0$

14. If the mean of first n natural number is $\frac{5n}{9}$, then what is the value of n?
    (a) 8 (b) 9 (c) 4 (d) 10

15. What is the value of $9\sec^2\theta - 9\tan^2\theta$?

(a) 8 (b) 9 (c) 0 (d) 1

16. One card is drawn from a well shuffled back of 52 cards. What is the probability of getting a black face card?

(a) $\frac{3}{26}$ (b) $\frac{3}{14}$

(c) $\frac{3}{13}$ (d) $\frac{1}{26}$

17. The ratio between the radius of the base and the height of the cylinder is 2:3. If its volume is 1617 cm^3, what is the total surface area of the cylinder?

(a) 462 cm^2 (b) 770 cm^2

(c) 540 cm^2 (d) 308 cm^2

18. A sector of 56°, cut out from a circle, contain 17.6 cm^2. What is radius of that circle?

(a) 4 cm (b) 5 cm

(c) 6 cm (d) 8 cm

19. On decreasing the radius of the circle by 30%, its area is decreased by what percent?

(a) 45% (b) 51%

(c) 60% (d) 30%

20. The points P (0, −2), Q (3, 1), R (0, 4) and S (−3, 1) are vertices of a

(a) Square (b) Rhombus

(c) Parallelogram (d) Rectangle

21. If the $\sin 3Q = \cos(Q - 6°)$ where $3Q$ and $(Q - 6)$ are acute angles, then what is the value of Q?

22. For what values of k, the system of equations $2x + 3y = 7$, $(k-1)x + (k+2)y = 3k$ has an infinite number of solutions?

(a) $k = 3$ (b) $k = 7$

(c) $k = 4$ (d) $k = 5$

23. Which term of A.P 24, 21, 18,15 . . . . is the first negative term?

(a) 10^{th} (b) 12^{th}

(c) 9^{th} (d) 8^{th}

24. If α, β, γ be the zeros of the polynomial $f(x)$ such that $\alpha + \beta + \gamma = 3$, $\alpha\beta + \beta\gamma + r\alpha = -10$ and $\alpha\beta\gamma = -24$ then what is the polynomial $f(x)$

(a) $x^3 + 3x^2 - 10x + 24$

(b) $x^3 + 6x^2 + 10x - 24$

(c) $x^3 - 3x^2 - 10x - 24$

(d) $x^3 + 3x^2 + 10x - 24$

25. $1.23\ \overline{48}$ is

(a) a rational number

(b) an integer

(c) an irrational number

(d) None of these

26. If one root of the equation $x^2 + ax + 3 = 1$ then the other root is

(a) −3 (b) 2 (c) −2 (d) 3

27. If the angles of elevation of the top of a tower from two points distant a and b from the base and in the same straight line with it are complementary. What is the height of the tower?

(a) ab (b) $\frac{a}{b}$ (c) $\sqrt{\frac{a}{b}}$ (d) $\sqrt{ab}$

28. If S_1, S_2, S_3 be the sum of n, $2n$, $3n$ forms of an A. P respectively then which of the following alternative is correct?

(a) $S_3 = 3S_2 - 3S_1$

(b) $S_2 = 2S_3 - 2S_1$

(c) $S_3 = 3S_1 - 3S_2$

(d) None of these

29. If $x = 1$ is a common root of $ax^2 + ax + 2 = 0$ and $x^2 + x + b = 0$ then what is the value of ab?

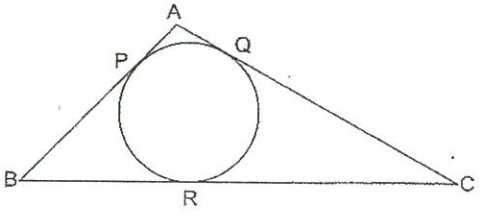
(a) 1 (b) 3 (c) 2 (d) 4

30. The radii of the circular ends of frustum are 6 cm and 14 cm. If its slant height is 10 cm then what is its vertical height?

(a) 7 cm (b) 4 cm

(c) 8 cm (d) 6 cm

31. It is given that $AP = PB$ then which of the following is correct?

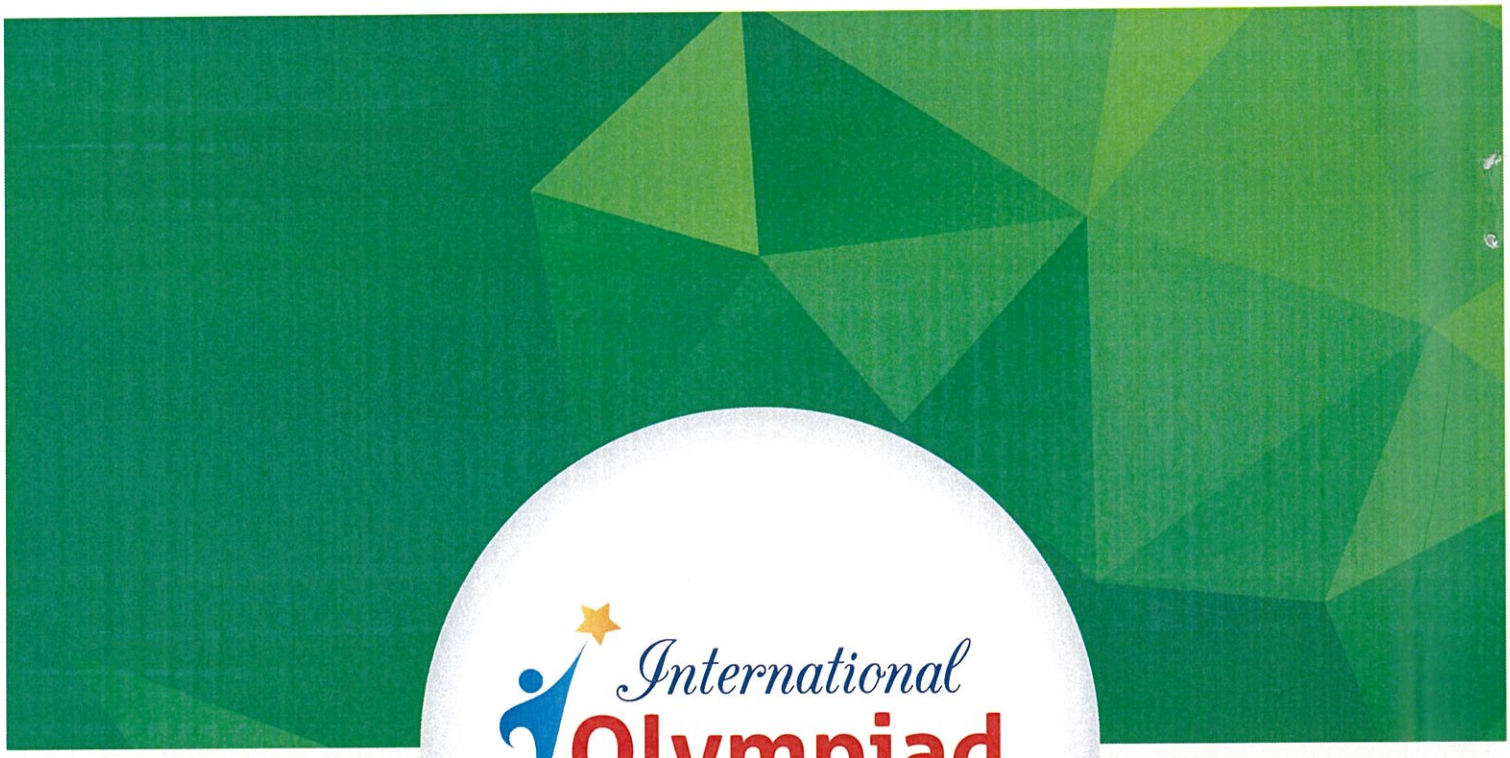


(a) $AC = AB$ (b) $AC = BC$

(c) $AQ = QC$ (d) $AB = BC$

Anwers

1.(c)	2.(b)	3.(b)	4.(b)	5.(a)	6.(b)
7.(d)	8.(b)	9.(a)	10.(a)	11.(a)	12.(c)
13.(a)	14.(b)	15.(b)	16.(a)	17.(b)	18.(c)
19.(a)	20.(c)	21.(b)	22.(a)	23.(b)	24.(c)
25.(d)	26.(d)	27.(a)	28.(c)	29.(d)	30.(b)



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