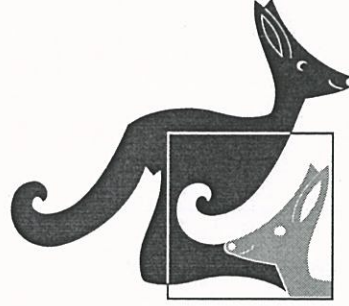


In The Name of God

Enjoy Mathematics



International Math Kangaroo 2016

Grades 9 & 10

3 Point Problems	1 to 10
4 Point Problems	11 to 20
5 Point Problems	21 to 30



دبیرخانه ریاضیات کانگورو



دانشگاه تربیت مدرس



مؤسسه
فرهنگی
فاطمی



مرکز ملی پرورش استعداد های درخشان
و دانش پژوهان جوان

3 Point Problems

1. The average of four numbers is 9. What is the fourth number if three of the numbers are 5, 9 and 12?

- (A) 6 (B) 8 (C) 9 (D) 10 (E) 36
-

2. Which of the following numbers is the closest to the result of $\frac{17 \times 0.3 \times 20.16}{999}$?

- (A) 0.01 (B) 0.1 (C) 1 (D) 10 (E) 100
-

3. On a test consisting of 30 questions, Ruth had 50% more right answers than she had wrong answers. Each answer was either right or wrong. How many correct answers did Ruth have, assuming she answered all questions?

- (A) 10 (B) 12 (C) 15 (D) 18 (E) 20
-

4. In a coordinate system four of the following points are the vertices of a square. Which point is not a vertex of this square?

- (A) (-1; 3) (B) (0; -4) (C) (-2; -1) (D) (1; 1) (E) (3; -2)
-

5. When the positive integer x is divided by 6, the remainder is 3. What is the remainder when $3x$ is divided by 6?

- (A) 4 (B) 3 (C) 2 (D) 1 (E) 0
-

6. How many weeks are the same as 2016 hours?

- (A) 6 (B) 8 (C) 10 (D) 12 (E) 16
-

7. Little Lucas invented his own way to write down negative numbers before he learned the usual way with the minus sign in front. Counting backwards, he'd write: ... 3, 2, 1, 0, 00, 000, 0000, ... What is the result of $000 + 0000$ in his notation?

- (A) 1 (B) 00000 (C) 000000 (D) 0000000 (E) 00000000
-

8. I have some strange dice: the faces show the numbers 1 to 6 as usual, except that the odd numbers are negative (-1, -3, -5 in place of 1, 3, 5). If I throw TWO such dice, which of these totals can not be achieved?

- (A) 3 (B) 4 (C) 5 (D) 7 (E) 8
-

9. How many times do two directly adjacent letters have to be exchanged in order to change the word MEGA step by step into the word GAME ?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

10. Sven wrote five different one-digit positive integers on a blackboard. He discovered that no sum of any two numbers is equal to 10. Which of the following numbers did Sven definitely write on the blackboard?

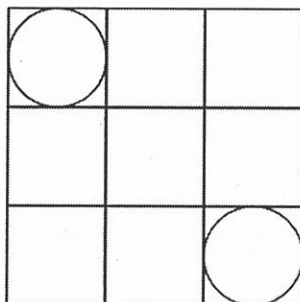
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

4 Point Problems

11. Let $a + 5 = b^2 - 1 = c^2 + 3 = d - 4$. Which one of the numbers a, b, c, d is the largest?

- (A) a (B) b (C) c
 (D) d (E) impossible to determine

12. The 3×3 table is divided into 9 unit squares, and two circles are inscribed in two of them (see picture). What is the distance between the two circles?

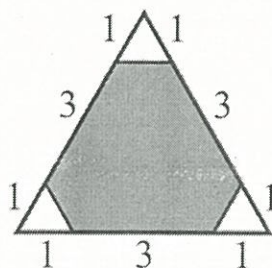


- (A) $2\sqrt{2} - 1$ (B) $\sqrt{2} + 1$ (C) $2\sqrt{2}$ (D) 2 (E) 3

13. In a tennis tournament on a knock-out basis, six of the results of the quarter-finals, the semi-finals and the final were (not necessarily in this order): Bella beat Ann, Celine beat Donna, Gina beat Holly, Gina beat Celine, Celine beat Bella and Emma beat Farah. Which result is missing?

- (A) Gina beat Bella (B) Celine beat Ann
 (C) Emma beat Celine (D) Bella beat Holly
 (E) Gina beat Emma

14. What percentage of the area of the triangle in the figure is shaded?



- (A) 80% (B) 85%
 (C) 88% (D) 90%
 (E) impossible to determine

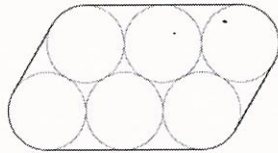
15. Jill is making a magic multiplication square using the numbers 1, 2, 4, 5, 10, 20, 25, 50 and 100. The products of the numbers in each row, in each column and in the two diagonals must all be the same. In the figure you can see how she has started. Which number should Jill place in the cell

20	1	
		?

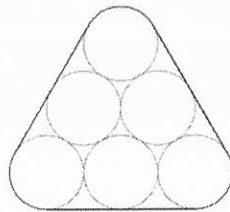
with the question mark?

- (A) 2 (B) 4 (C) 5 (D) 10 (E) 25

16. Jack wants to hold six circular pipes with diameter 2 cm together by a rubber band. Not considering other possibilities he decided between the following two options as shown in the picture. Which



of the following is true about the lengths of the rubber bands?

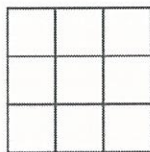


- (A) In the left picture it is π cm shorter. (B) In the left picture it is 4 cm shorter.
 (C) In the right picture it is π cm shorter. (D) In the right picture it is 4 cm shorter.
 (E) Both have the same length.

17. Eight identical envelopes contain the numbers: 1, 2, 4, 8, 16, 32, 64, 128. Eve chooses a few envelopes randomly. Alie takes the rest. Both sum up their numbers. Eve's sum is 31 more than Alie's. How many envelopes did Eve take?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

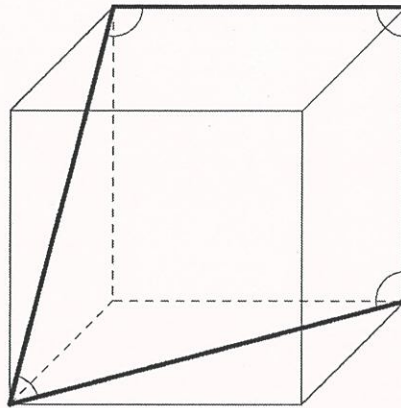
18. Peter wants to colour the cells of a 3×3 square in such a way that each of the rows, the columns and both diagonals have three cells of three different colours. What is the least number of



colours Peter could use?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

19. The picture shows a cube with four marked angles. What is the sum of these angles?



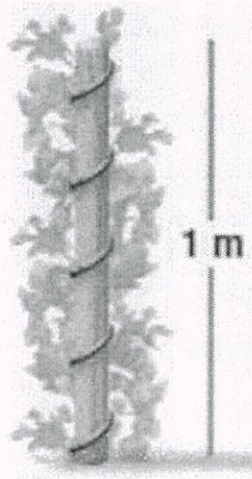
- (A) 315° (B) 330° (C) 345° (D) 360° (E) 375°

20. There are 2016 kangaroos, each of them is either grey or red and at least one of them is grey and at least one is red. For every kangaroo K we compute the fraction of the number of kangaroos of the other colour divided by the number of kangaroos of the same colour as K (including K). Find the sum of the fractions of all 2016 kangaroos.

- (A) 2016 (B) 1344 (C) 1008
 (D) 672 (E) more information is needed

5 Point Problems

21. A plant wound itself exactly 5 times around a pole with height 1 m and circumference 15 cm as shown in the picture. As it climbed, its height increased at a constant rate. What is the length of the plant?

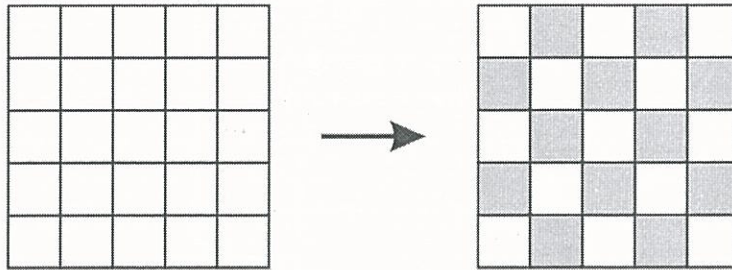


- (A) 0.75 m (B) 1.0 m (C) 1.25 m (D) 1.5 m (E) 1.75 m

22. What is the largest possible remainder that can be obtained when a two-digit number is divided by the sum of its digits?

- (A) 13 (B) 14 (C) 15 (D) 16 (E) 17

23. A 5×5 square is divided into 25 cells. Initially all its cells are white, as shown on the left. Neighbouring cells are those that share a common edge.



On each move two neighbouring cells have their colours changed to the opposite colour (e.g. white cells become black and black ones become white). What is the minimum number of moves required in order to obtain the chess-like colouring shown on the right?

- (A) 11 (B) 12 (C) 13 (D) 14 (E) 15

24. It takes 4 hours for a motorboat to travel downstream from X to Y. To return upstream from Y to X it takes the motorboat 6 hours. How many hours would it take a wooden log to be carried from X to Y by the current, assuming it is unhindered by any obstacles?

- (A) 5 (B) 10 (C) 12 (D) 20 (E) 24

25. In the Kangaroo republic each month consists of 40 days, numbered 1 to 40. Any day whose number is divisible by 6 is a holiday, and any day whose number is a prime is a holiday. How many times in a month does a single working day occur between two holidays?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

26. Two of the altitudes of a triangle are 10 cm and 11 cm. Which of the following cannot be the length of the third altitude?

- (A) 5 cm (B) 6 cm (C) 7 cm (D) 10 cm (E) 100 cm

27. Jakob wrote down four consecutive positive integers. He then calculated the four possible totals made by taking three of the integers at a time. None of these totals was a prime. What is the smallest integer Jakob could have written?

- (A) 12 (B) 10 (C) 7 (D) 6 (E) 3

28. Four sportsmen and sportswomen - a skier, a speed skater, a hockey player and a snowboarder - had dinner at a round table. The skier sat at Andrea's left hand. The speed skater sat opposite Ben. Eva and Filip sat next to each other. A woman sat at the hockey player's left hand. Which sport does Eva do?

- (A) speed skating (B) skiing
 (C) ice hockey (D) snowboarding
 (E) It's not possible to find out with the given information.

29. Dates can be written in the form DD.MM.YYYY. For example, today's date is 17.03.2016. A date is called "surprising" if all 8 digits in its written form are different. In what month will the next surprising date occur?

- (A) March (B) June (C) July (D) August (E) December
-

30. At a conference, the 2016 participants are registered from P1 to P2016. Each participant from P1 to P2015 shook hands with exactly the same number of participants as the one on their registration number. How many hands did the 2016th participant shake?

- (A) 1 (B) 504 (C) 672 (D) 1008 (E) 2015



دبیرخانه ریاضیات کانگورو

مؤسسه فرهنگی فاطمی

دبیرخانه ریاضیات کانگورو در ایران

تهران، میدان فاطمی، خیابان جویبار، خیابان میرهادی شرقی، پلاک ۱۴

تلفن: ۸۸۹۴۵۵۴۵

www.mathkangaroo.ir